TRR181 – Newsletter Report

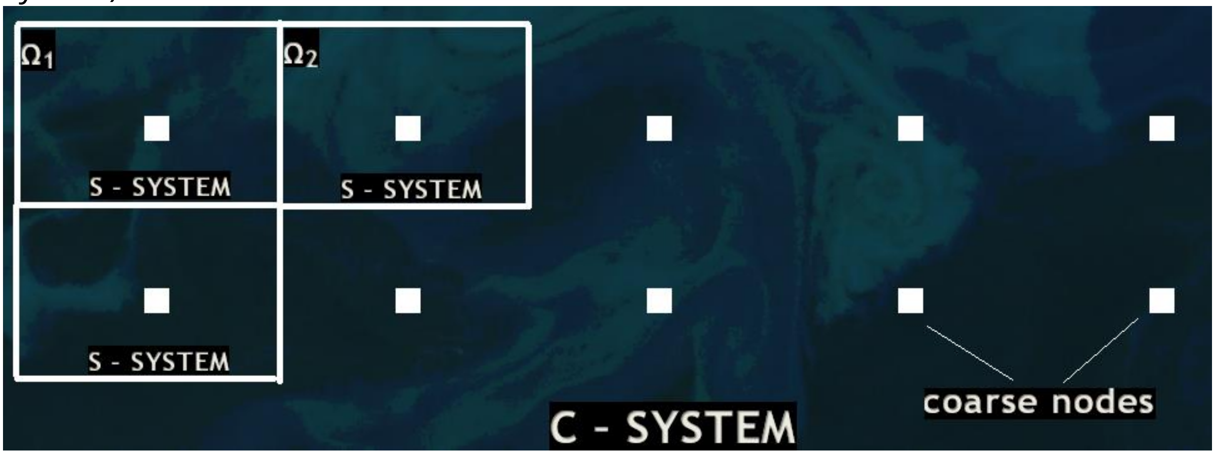
Stochastic superparametrization (SSP) for the ocean models. Due to the coarse resolution, ocean circulation models cannot resolve all important effects of mesoscale eddies. There are different ways to parameterize these effects without making expensive simulations on a fine grid. We will focus on SSP proposed in, e.g., [1-3] for quasi-geostrophic ocean models (QG). We found a straightforward way [4] to explain the main idea. Assume a simple single layer model

$$\partial_t \zeta + [\psi, \zeta] + \beta \partial_x \psi = F, \quad \zeta = \Delta\psi, \qquad (1)$$

where ζ, ψ are the related vorticity and the stream function, $[\psi, \zeta]$ is the Jacobian operator, and, for simplicity we omit a dissipation operator $D\zeta$. Let us decompose ζ, ψ on the coarse mesh and subscale variables $\zeta = \zeta^c + \zeta^s$, $\psi = \psi^c + \psi^s$, and $F = F^c + F^s$ on the physical and stochastic forcing. The stochastic source F^s emulates various uncertainties of subscales. Substituting them into (1) we obtain

$$\partial_t \zeta^c + \partial_t \zeta^s + [\psi^c, \zeta^c] + [\psi^c, \zeta^s] + [\psi^s, \zeta^c] + [\psi^s, \zeta^s] + \beta \partial_x \psi^c + \beta \partial_x \psi^s = F^c + F^s \quad (2)$$

and $\zeta^c + \zeta^s = \Delta\psi^c + \Delta\psi^s$. Now let us split the equation into two systems describing the evolution of coarse and subscale mesh variables (c-system and s-system).



This spitting is non-rigorous, so we have some freedom of choice. Nevertheless, we need to take into account the arguments: 1) c-system should contain all coarse mesh variables and some terms (CT) connecting c-system with s-system, otherwise c-system will be uncoupled from s-system; 2) s-system should be linear, otherwise the combined computational cost would be higher than the cost of simulating of the entire system on the fine grid; 3) the linear combinations of fine mesh variables have little effect on c-system because they are fast and their linear combinations are also fast. Hence, we obtain exact two systems from (2):

c-system $\partial_t \zeta^c + [\psi^c, \zeta^c] + [\psi^s, \zeta^s] + \beta \partial_x \psi^c = F^c, \quad \zeta^c = \Delta\psi^c;$

s-system $\partial_t \zeta^f + [\psi^c, \zeta^s] + [\psi^s, \zeta^c] + \beta \partial_x \psi^s = F^s, \quad \zeta^s = \Delta \psi^s.$

S-system is linear with constant coefficients since all coarse mesh variables are constants in the local boxes Ω_n, see Fig. Hence, s-system admits an explicit solution. Taking this solution, we compute $CT = [\psi^s, \zeta^s]$ and substitute it into c-system at each coarse time step. For solving s-system we also need to know the initial data. These data can be selected to be satisfying some a-priori statistical information taken from independent fine-grid simulations (or physical predictions, or real-world observations, or simply random). In any case, the initial data and F^s are parameters for tuning the systems. Now we are working on adapting SSP for applying it to ocean primitive equations (PE). We have already obtained c-system and s-system for PE, and the explicit solution of s-system which is more complicated than for QG. We are working also on simulations. Because PE are written in terms of the velocities, there are some possibilities for improving s-system which can make the coefficients not only constant or introduce other non-homogeneous defects. We expect that in this case some recent methods of homogenization of non-uniform media [5] or fast algorithms of solving non-uniform systems, based on integral continued fractions [6], can be helpful.

[1] Efficient stochastic superparameterization for geophysical turbulence; I. Grooms, A.J. Majda; *Proceedings of the National Academy of Sciences*, 2013, **110** (12), 4464-4469

[2] Stochastic superparameterization in quasigeostrophic turbulence; I. Grooms, A.J. Majda; *Journal of Computational Physics*, 2014, **271**, 78-98

[3] Stochastic superparameterization in a quasigeostrophic model of the Antarctic Circumpolar Current; I. Grooms, A.J. Majda, K.S. Smith; *Ocean Modeling*, 2015, **85**, 1-15

[4] Toward consistent subgrid momentum closures in ocean models; S. Danilov, A. Kutsenko, M. Oliver; book chapter; *in preparation*

[5] Wave Propagation and Homogenization in 2d and 3d Lattices: A Semi-Analytical Approach; A.A. Kutsenko, A.J. Nagy, X. Su, A.L. Shuvalov, A.N. Norris; *Q J Mechanics Appl Math,* 2017, **70**, 131-151

[6] Application of matrix-valued integral continued fractions to spectral problems on periodic graphs with defects; A.A. Kutsenko; *submitted*