Analysis of length, time and velocity scales in the Mesosphere

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Motivation and Problem Statement

- Small-scale coherent structures (ripples and KH billows) are ubiquitously found in Mesosphere
- Ripples are wave structures in OH airglow (around 85 km) with horizontal length of 5-15km and lifetime less than 45 min (*Taylor & Hapgood, 1990*)
- Kelvin-Helmholtz (KH) billow structures are present in NLC layer (about 83 km) with horizontal length of 10-20km and lifetime around 20-45min (*Baumgarten & Fritts, 2014; Fritts et. al., 2014*)
- It is not clear if these structures are of turbulent nature and how large they can grow in Mesosphere
- Are these structures affected by flow stratification?

Governing equations

- The equations of motions of a stratified, incompressible flow under the Boussinesq approximation

$$\frac{\partial \boldsymbol{u}_h}{\partial t}+\boldsymbol{u}_h\cdot\boldsymbol{\nabla}_h\boldsymbol{u}_h+\frac{F_h^2}{\alpha^2}u_z\frac{\partial \boldsymbol{u}_h}{\partial z}=-\boldsymbol{\nabla}_h p+\frac{1}{Re}\left[\frac{1}{\alpha^2}\frac{\partial^2 \boldsymbol{u}_h}{\partial z^2}+\nabla_h^2\boldsymbol{u}_h\right],$$

$$F_h^2\left[\frac{\partial u_z}{\partial t}+\boldsymbol{u}_h\cdot\boldsymbol{\nabla}_h u_z+\frac{F_h^2}{\alpha^2}u_z\frac{\partial u_z}{\partial z}\right]=-\frac{\partial p}{\partial z}-\rho+\frac{F_h^2}{Re}\left[\frac{1}{\alpha^2}\frac{\partial^2 u_z}{\partial z^2}+\nabla_h^2 u_z\right],$$

$$\boldsymbol{\nabla}_h\cdot\boldsymbol{u}_h+\frac{F_h^2}{\alpha^2}\frac{\partial u_z}{\partial z}=0,$$

$$\frac{\partial \rho}{\partial t}+\boldsymbol{u}_h\cdot\boldsymbol{\nabla}_h\rho+\frac{F_h^2}{\alpha^2}u_z\frac{\partial \rho}{\partial z}=u_z+\frac{1}{ReSc}\left[\frac{1}{\alpha^2}\frac{\partial^2 \rho}{\partial z^2}+\nabla_h^2\rho\right],$$

- here $\alpha=l_v/l_h,\ F_h=U/(l_hN),\ Re=Ul_h/\nu,\ Sc=\nu/k$

$$\boldsymbol{u}_h\to\boldsymbol{u}'/U,\ u_z\to u_z'\alpha/(UF_h^2),\ \boldsymbol{x}\to\boldsymbol{x}'/l_h,\ z\to z'/l_v,$$

$$t\to t'U/l_h,\ \rho\to\rho' g l_v/(U^2\rho_0),\ p\to p'/(U^2\rho_0)$$

- In the limit of strong stratification, i.e. $F_h=U/(l_hN)\to 0,\ Re\gg 1$
- Vertical advection term $=\mathcal{O}(F_h^2/\alpha^2)$
- Vertical diffusion terms $=\mathcal{O}(Re\alpha^2)$
- Strongly stratified turbulence regime

$$\mathcal{R}=ReF_h^2\gg 1$$

Strongly stratified turbulence

- Diffusive terms ($\mathcal{O}(Re\alpha^2)$) can be neglected compared to $\mathcal{O}(F_h^2/\alpha^2)$
- The vertical advection terms contribute to the leading dynamics $l_v \sim U/N$
- The dynamics are three-dimensional but strongly anisotropic

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- <u>Assumptions:</u>
  - $l_v \sim U/N$ - is valid at all scales in the inertial subrange
  - $l_h \sim U^3/\epsilon$ - estimation for the horizontal lengthscale (*similar to the integral lengthscale of isotropic turbulence!*)

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- Reynolds number, Froude number and buoyancy Reynolds number

$$Re = \frac{U^4}{\nu\epsilon}, \quad F_h = \frac{\epsilon}{NU^2}, \quad \mathcal{R} = \frac{\epsilon}{\nu N^2}$$

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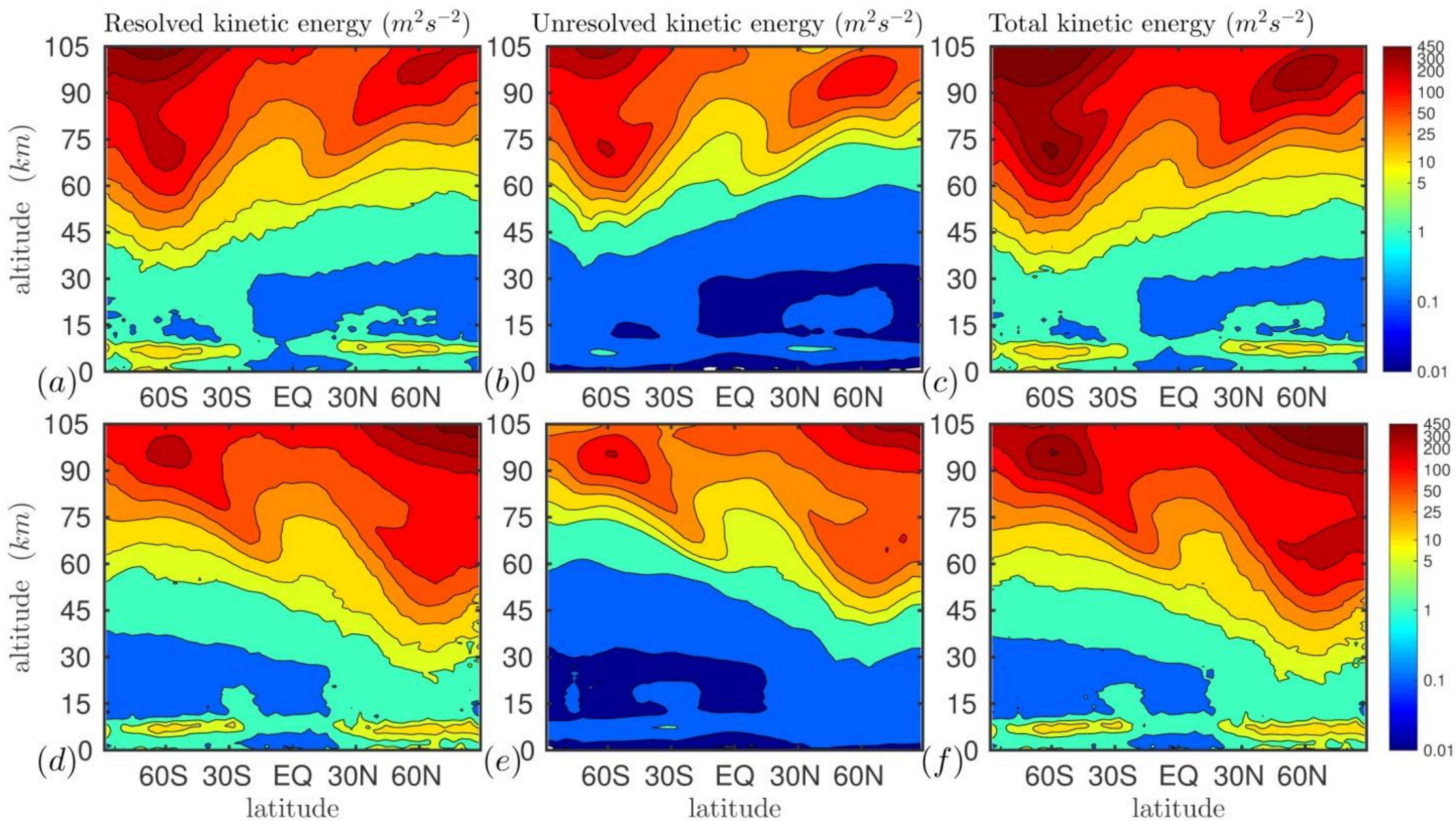
$$Re = \frac{U^4}{\nu\epsilon}, \quad F_h = \frac{\epsilon}{NU^2}, \quad \mathcal{R} = \frac{\epsilon}{\nu N^2}$$

$$\boxed{\mathcal{R} = \left(\frac{l_o}{\eta}\right)^{4/3}, \quad l_o = \epsilon^{1/2}/N^{3/2}, \quad \eta = \nu^{3/4}/\epsilon^{1/4}}$$

  - l_o - marks the transition between stratified and Kolmogorov turbulence

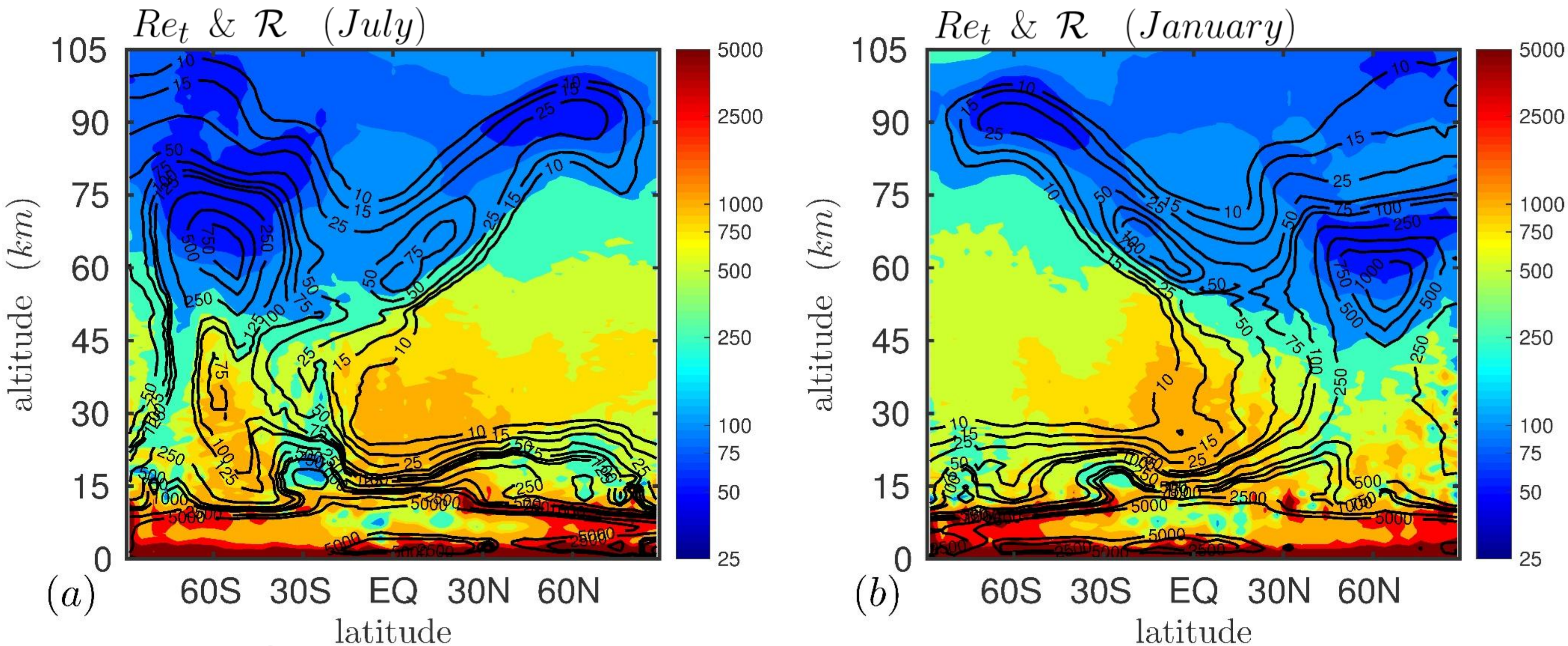
Turbulent kinetic energy simulated with KMCM



- For July
- For January

- $(a),(d)$ - Resolved mesoscale kinetic energy $k_h = \frac{1}{2}\left(\left[u^{*2}\right] + \left[v^{*2}\right]\right)$
- $(b),(e)$ - Sub-grid scale kinetic energy $k_{sgs} = \sqrt{\frac{\epsilon \nu_t}{C_\mu}}$, here $C_\mu \approx 0.09$
- $(c),(f)$ - Total kinetic energy $k_h = \frac{1}{2}\left(\left[u^{*2}\right] + \left[v^{*2}\right]\right) + \sqrt{\frac{\epsilon \nu_t}{C_\mu}}$

Turbulent and Buoyancy Reynolds numbers



- $Re_t = \dfrac{k^2}{\epsilon \nu_t}$ - Turbulent Reynolds number
- $\mathcal{R} = \dfrac{\epsilon}{\nu N^2}$ - Buoyancy Reynolds number
- Condition for small-scale turbulence: $\mathcal{R} \gg 1$
- Conditions for stratified turbulence: $Re_t \gg 1$ and $\mathcal{R} \gg 1$

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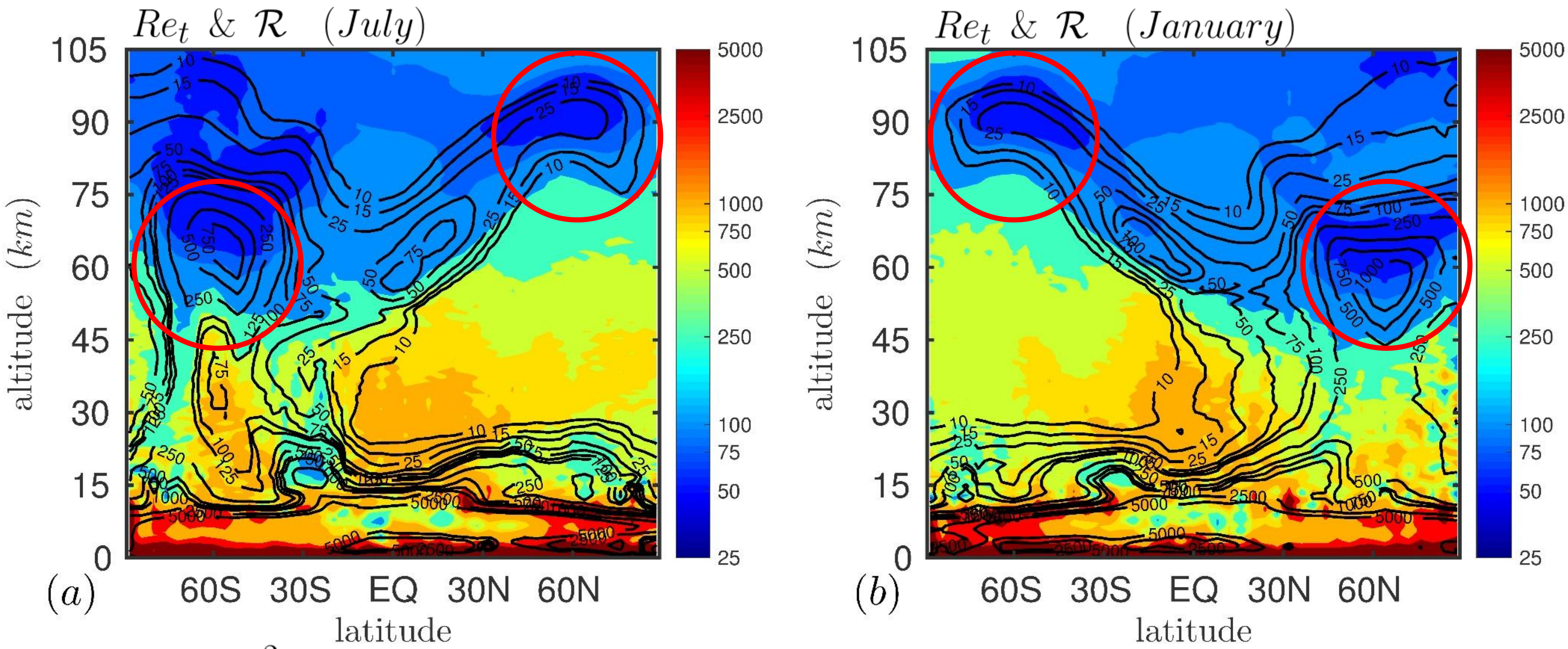

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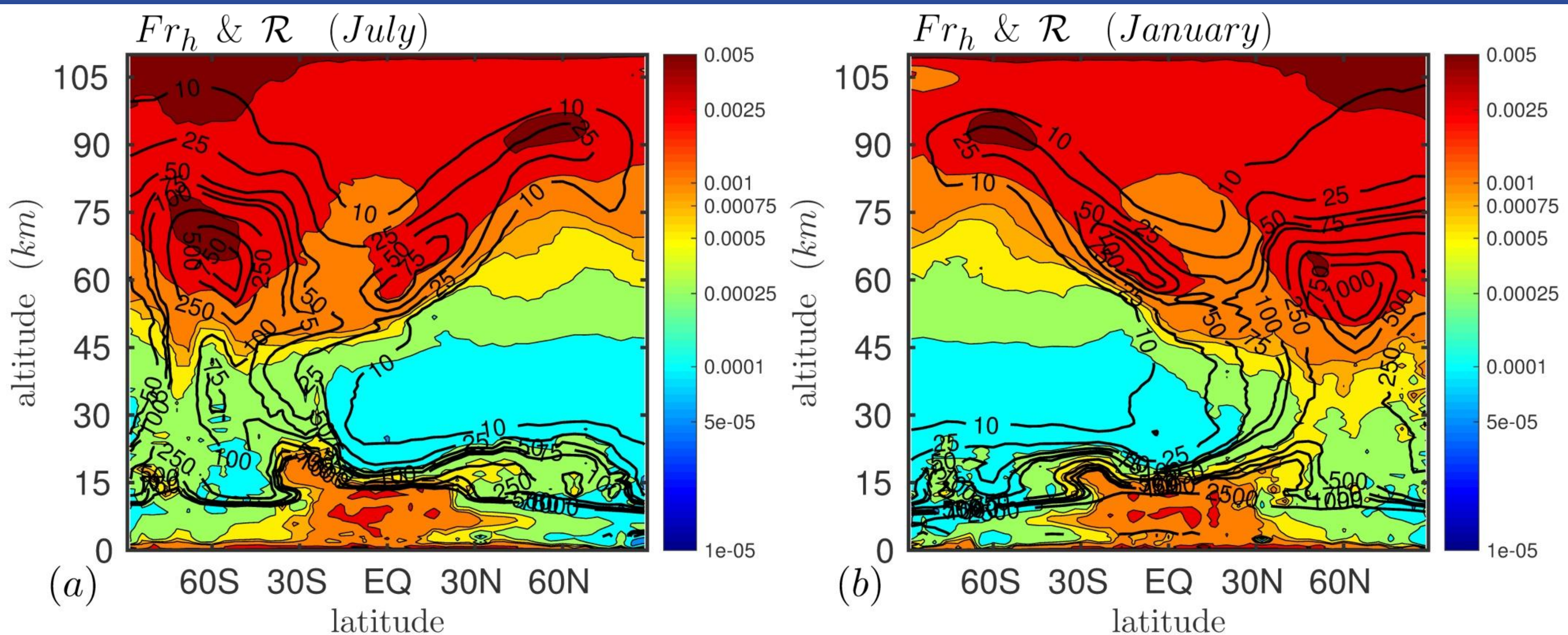
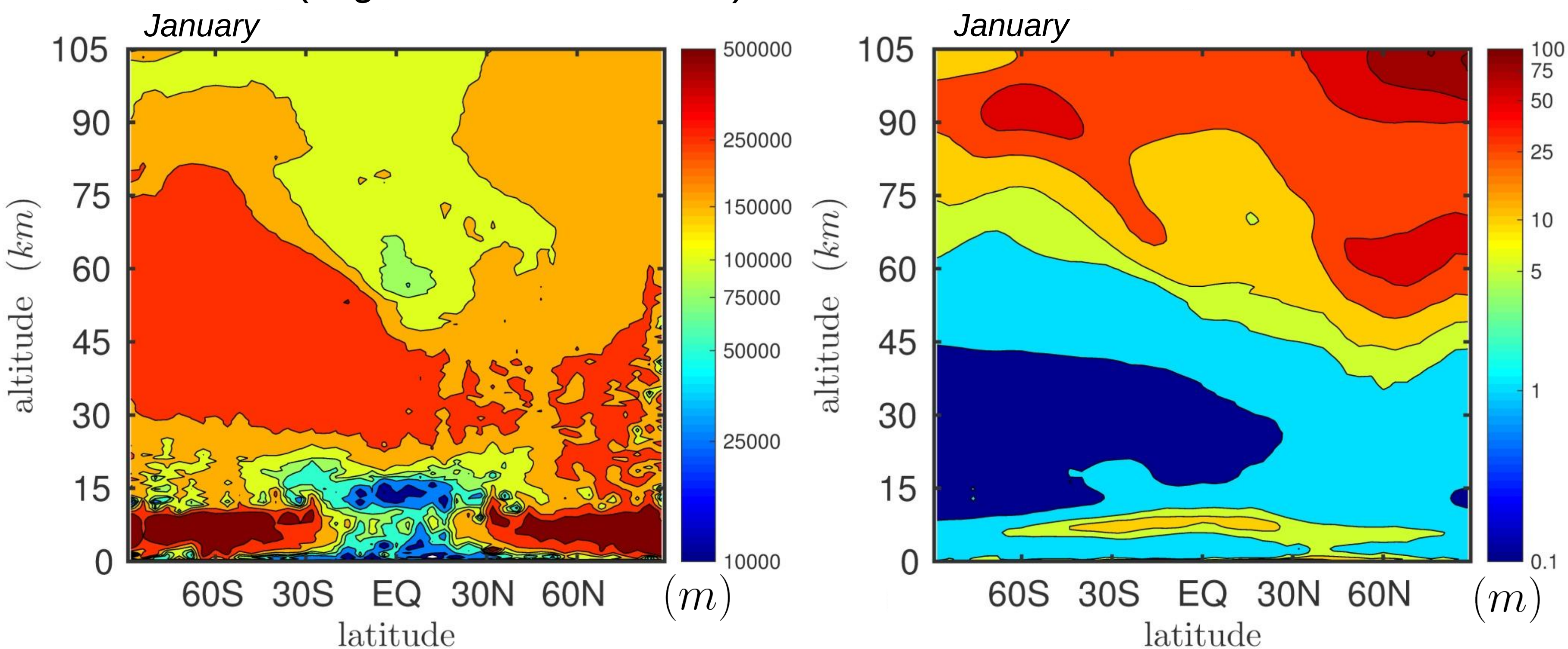
Horizontal Froude number



- $Fr_h = \frac{U}{l_h N} = \frac{l_v}{l_h}$ - Horizontal Froude number
- Condition for small-scale turbulence: $Fr_h \sim 1$ and $\mathcal{R} \sim 10$
- Conditions for stratified turbulence: $Fr_h \ll 1$ and $\mathcal{R} \gg 1$

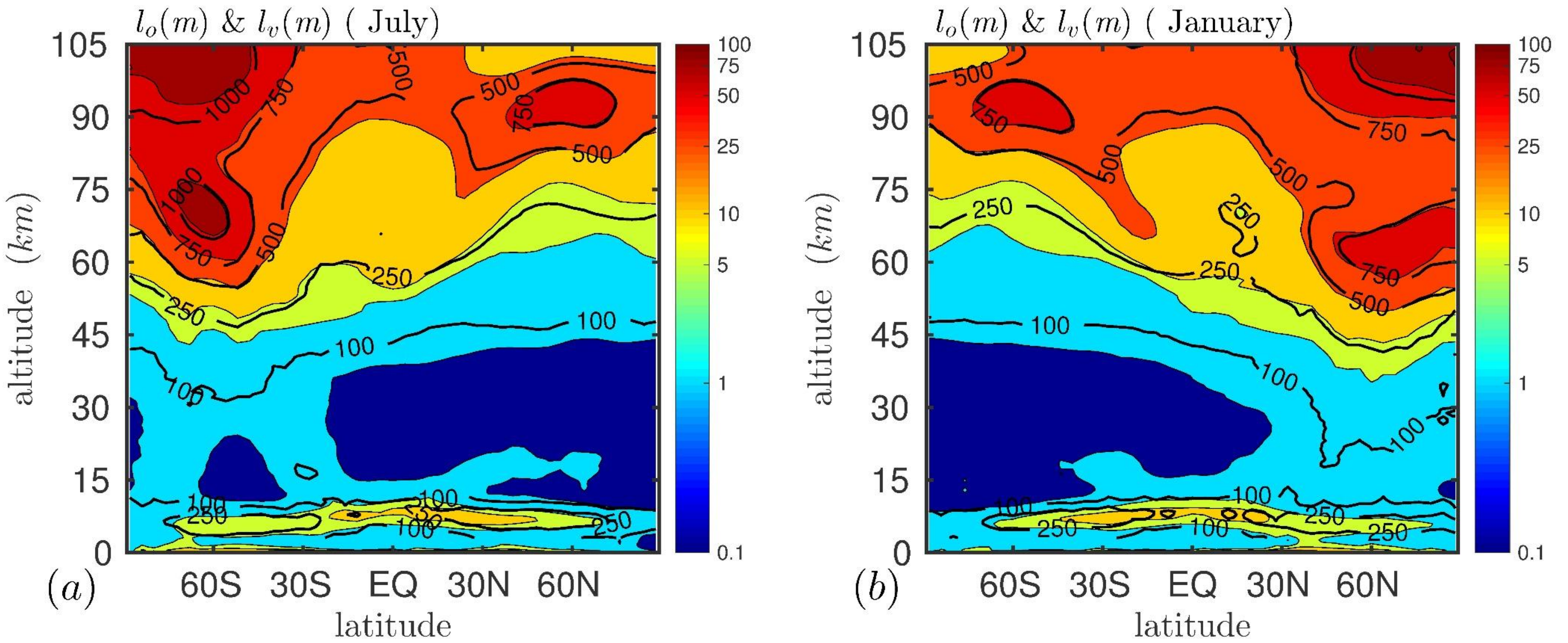
Integral and Ozmidov lengthscales

- $l_t = \frac{k^{3/2}}{\epsilon}$ - Integral lengthscale (*largest horizontal scale*)
- $l_o = \frac{\epsilon^{1/2}}{N^{3/2}}$ - Ozmidov scale



- Ozmidov scale sets a lower limit on the scales of stratified turbulence in both horizontal and vertical sizes
- Lengthscales in macroturbulent and small-scale regimes show different dynamics

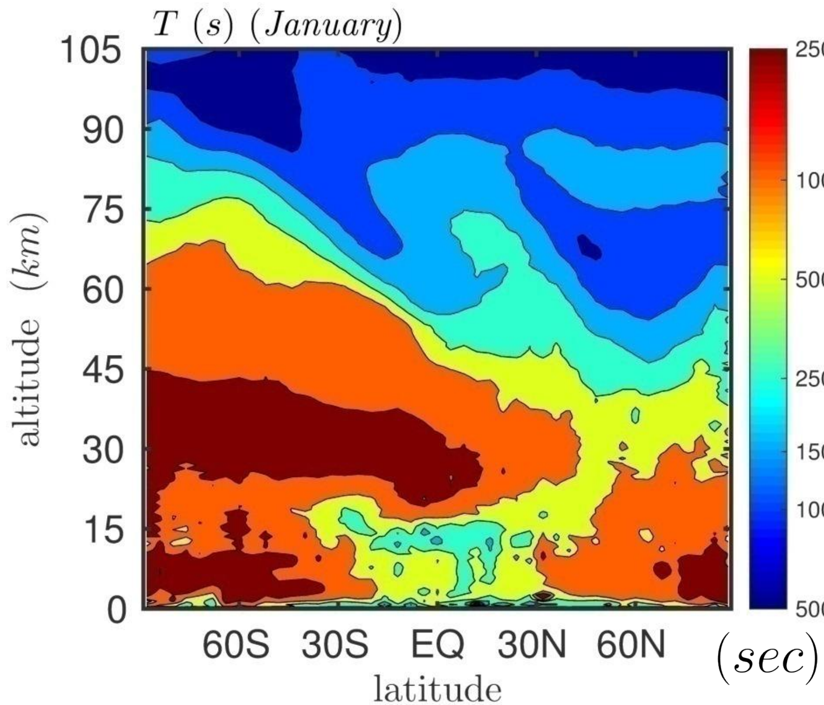
Ozmidov and vertical lengthscales

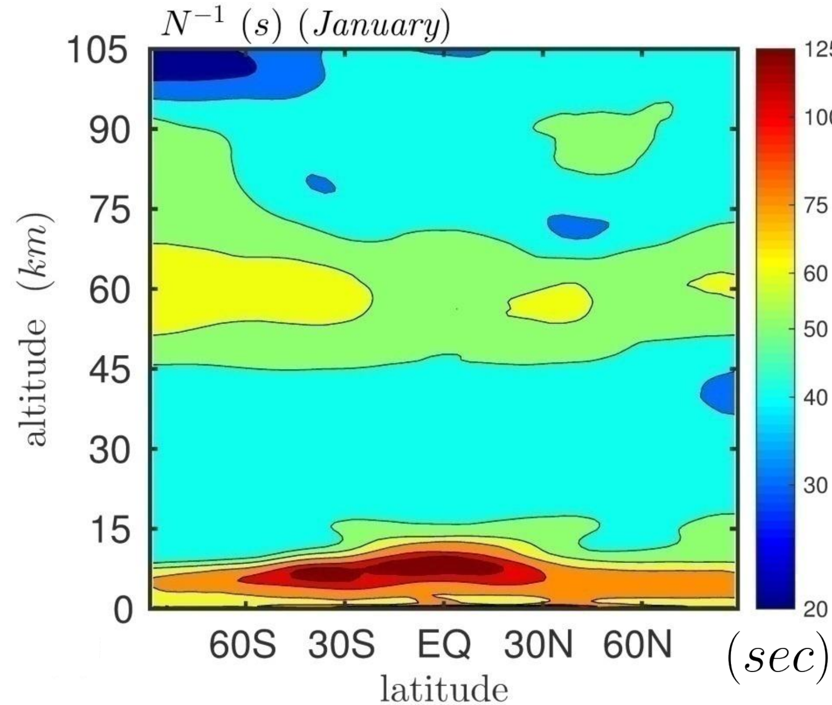


- l_o - sets a lower limit on the scales in ***vertical*** sizes
- l_v - sets a lower limit on the scales in ***horizontal*** sizes

Turbulent and buoyancy timescales

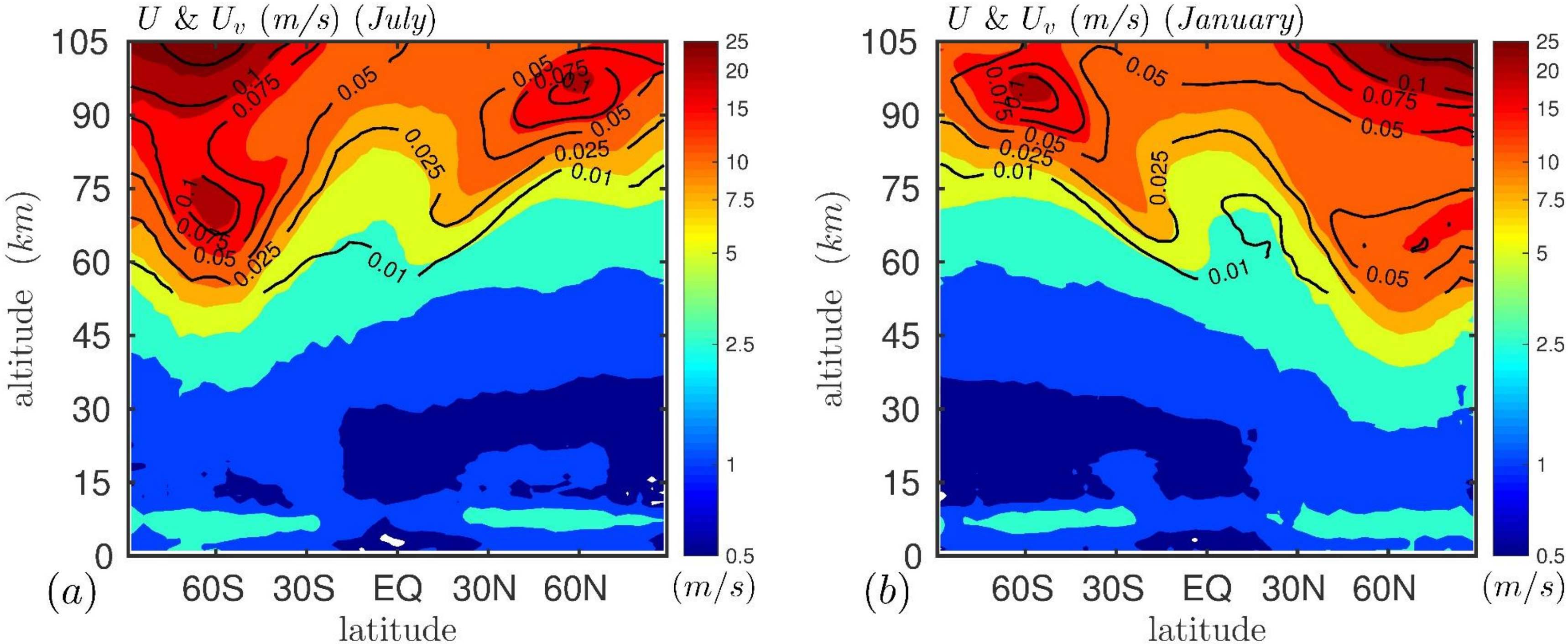
- $T = \frac{k}{\epsilon}$ - Eddy turnover time
- $\tau_O = \frac{1}{N}$ - Buoyancy timescale





- Above 75km large-scale structures live up to 4 hours
- Small-scale structures live not more than a minute in Mesosphere

Velocity scales



- Horizontal velocity: $U = [\boldsymbol{u'_h} \cdot \boldsymbol{u'_h}]/\sqrt{2} = k^{1/2}$
- From continuity equation: $U_v \sim U l_v / l_h$

Small-scale turbulence regime has different velocity scales

Small-scale turbulent rms velocity

- When $l_v \to l_o$: $Fr_v = \left.\frac{U}{Nl_v}\right|_{l_v \to l_o} = \frac{U}{Nl_o} = \frac{U}{U_{l_o}}$

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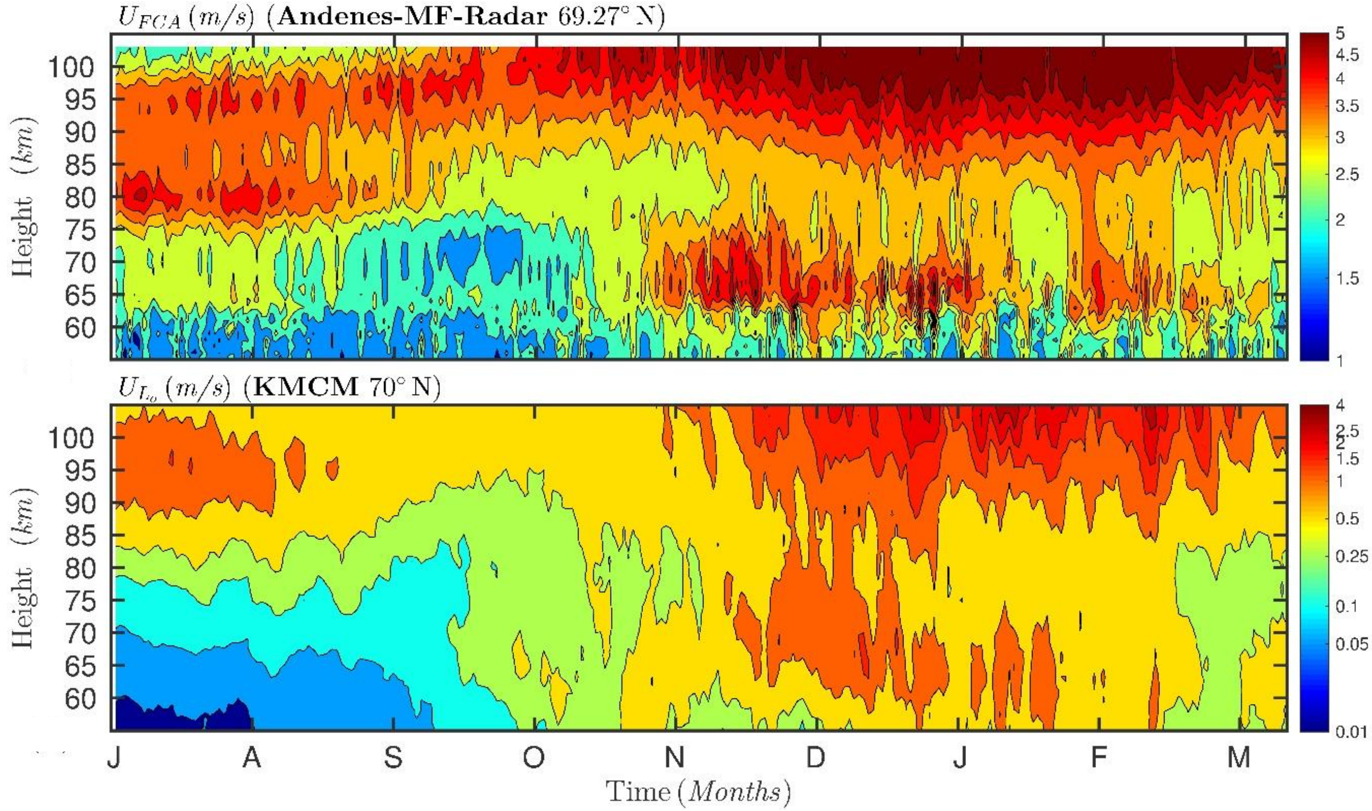
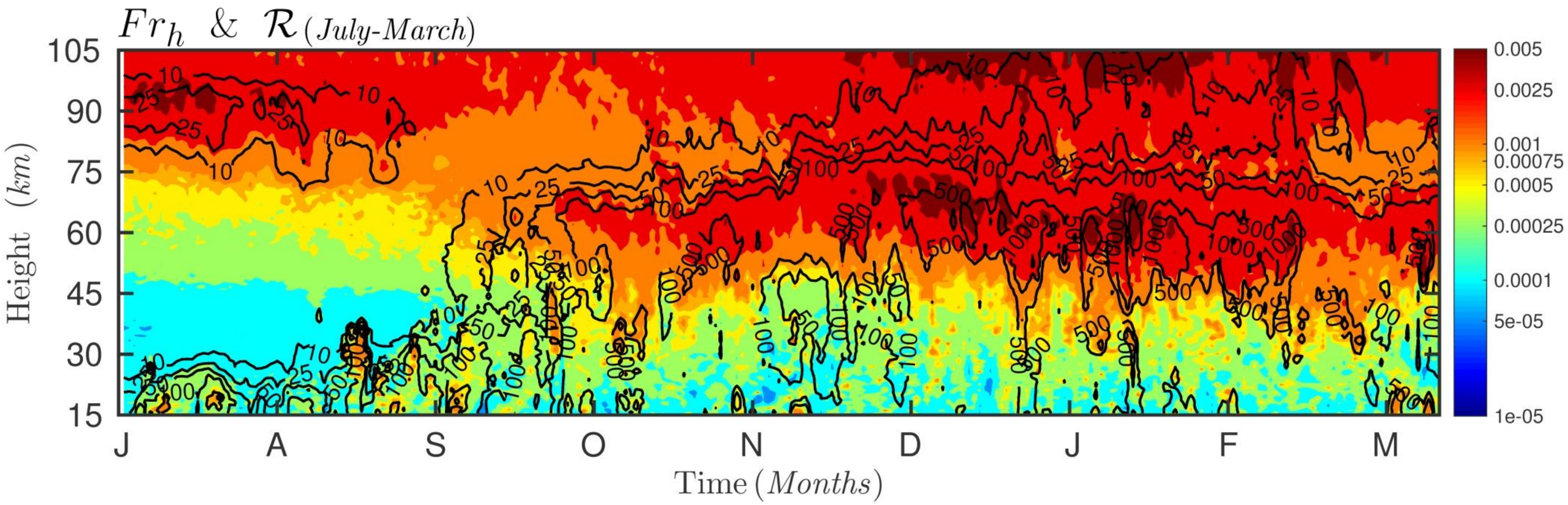


Strongly stratified turbulence



- Conditions for strongly stratified turbulence: $Fr_h \ll 1$ and $\mathcal{R} \gg 1$

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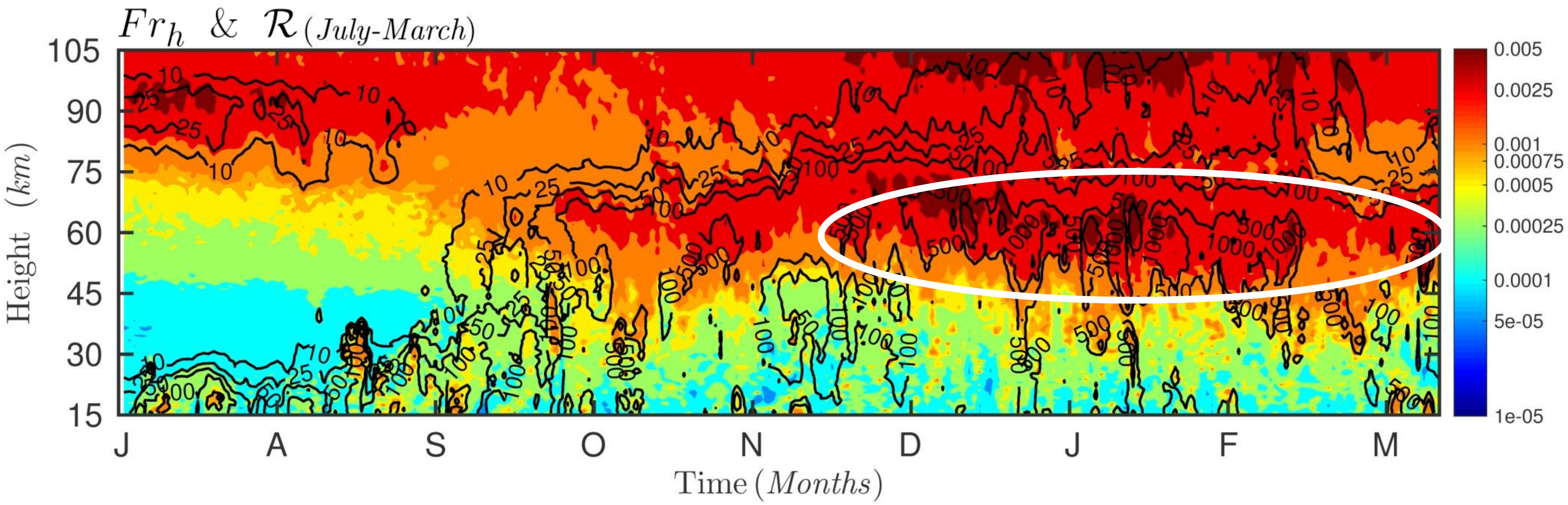

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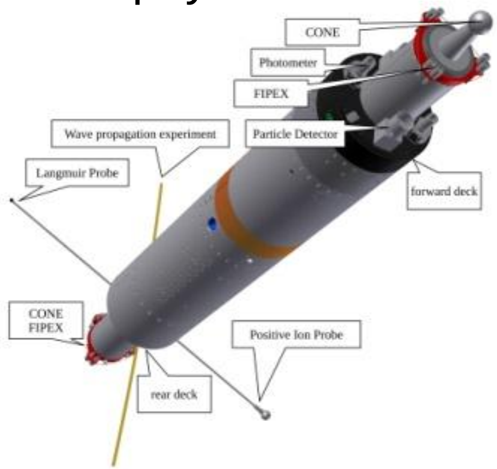
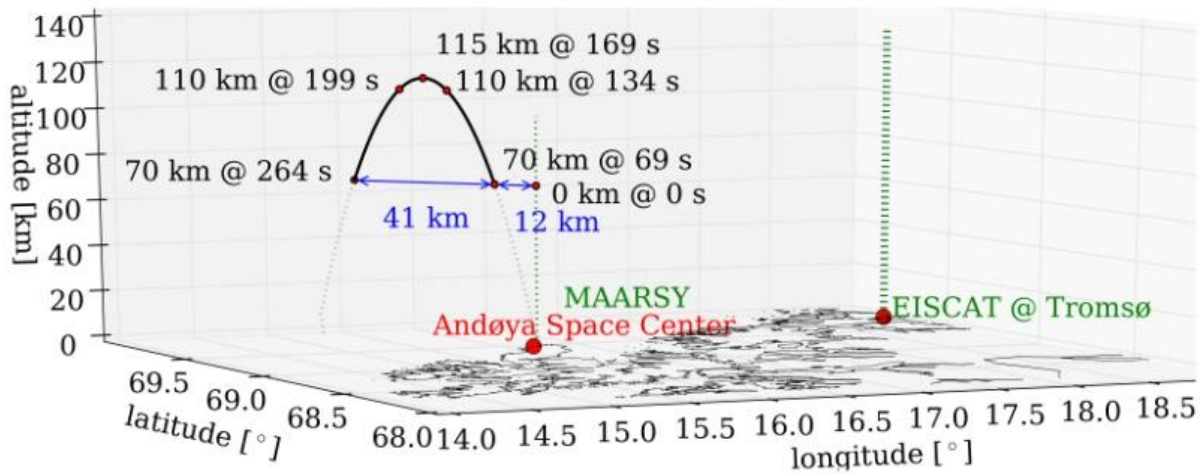
WAve propagation and **DIS**sipation project

- WADIS payload:



Instrument	Parameter measured	Status
CONE (NP)	neutral density	success
CONE (EP)	electron density	failed
PIP	positive ion density	success
Wave prop.	abs. electron density	success
LP	electron density	success
	neutral aerosols	success
FIPEX	oxygen density	success
Photmeters	oxygen density	success
PD	charged aerosols	qualitative

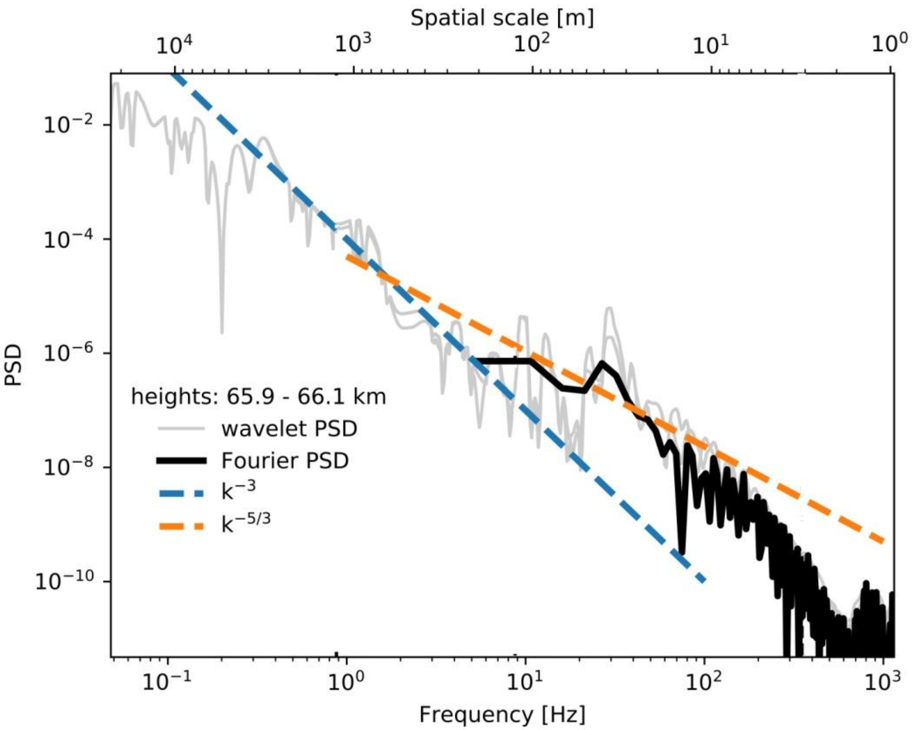
- Geometry of WADIS campaign

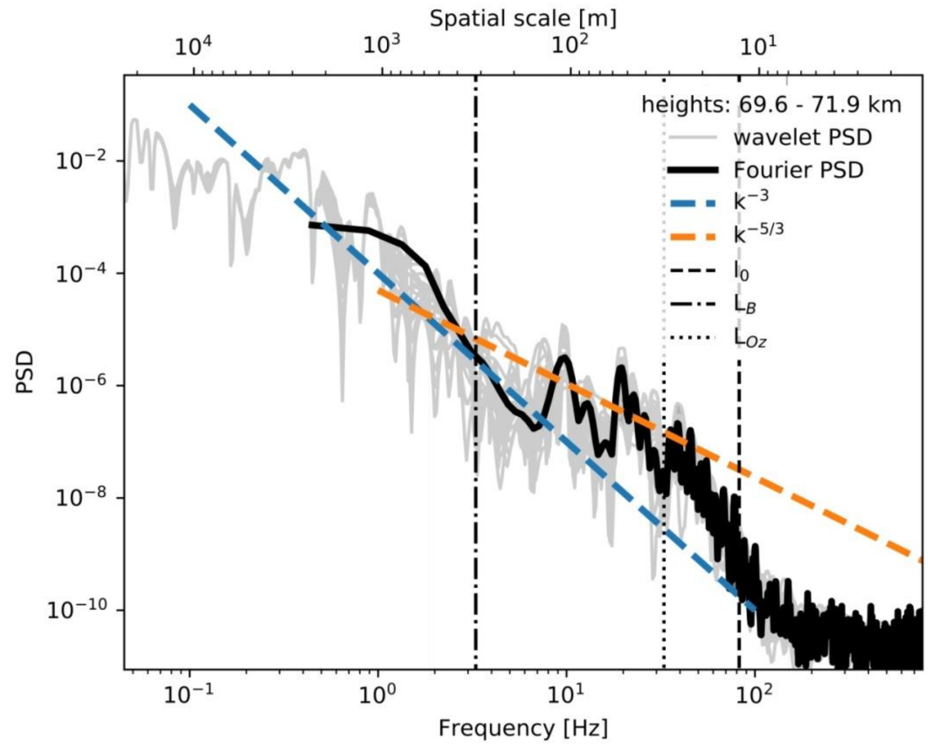


- WADIS project addresses the question of the variability of turbulence in Mesosphere and Lower Thermosphere

Vertical spectra I

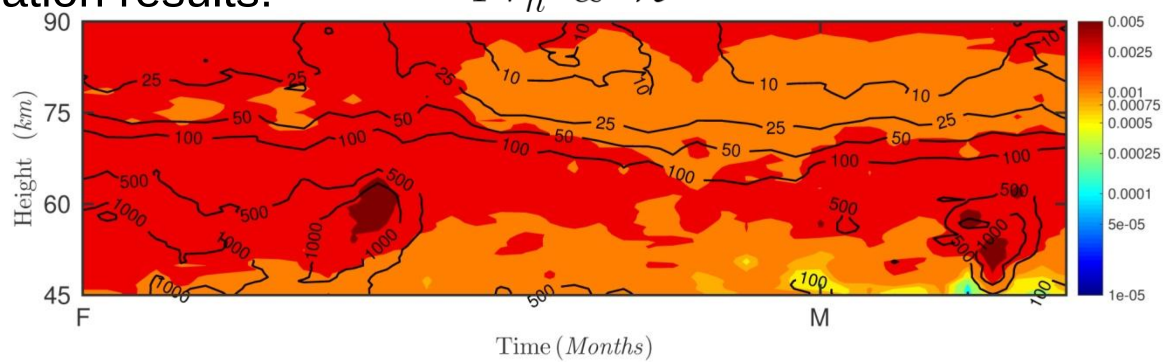
- Vertical spectra of kinetic energy from WADIS campaign:





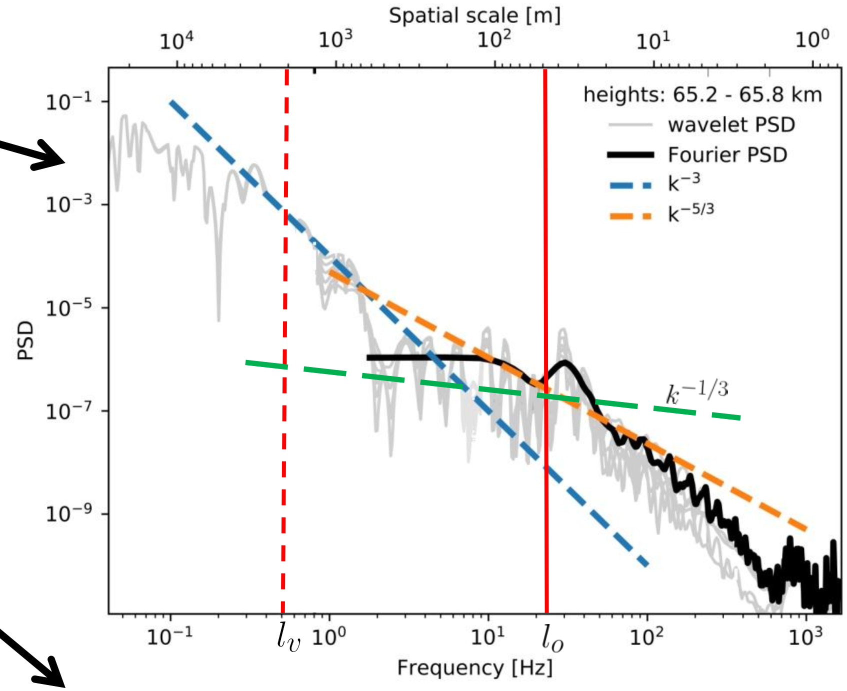
- KMCM simulation results:

Fr_h & $\mathcal{R}$



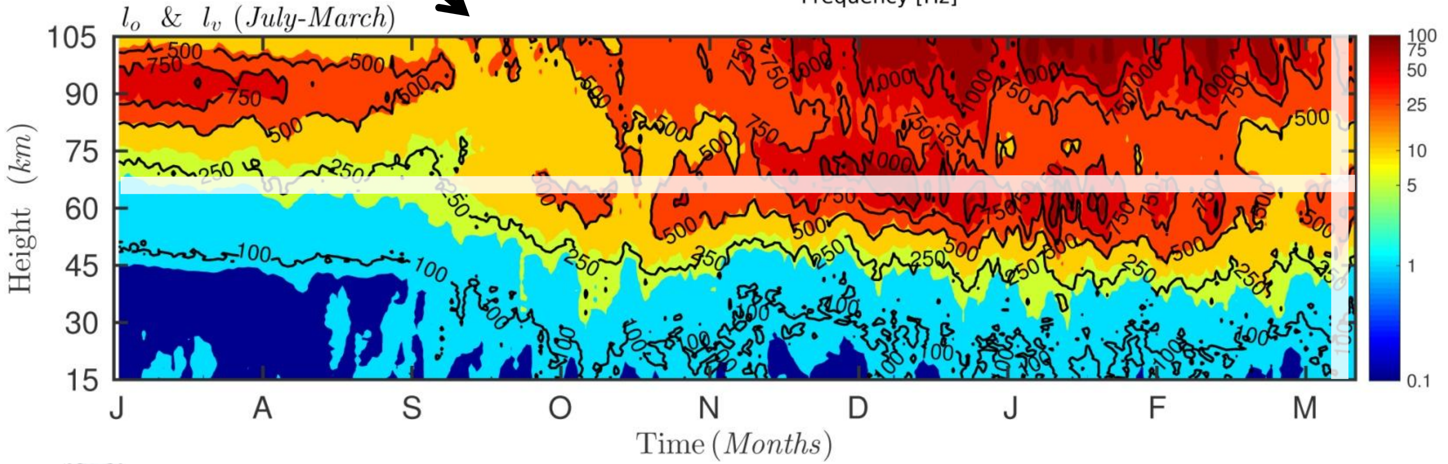
Vertical spectra II

- WADIS sounding rocket campaign



- $Re_b \approx 900, \;\; Fr_h \approx 0.01$

- KMCM simulations



Conclusions

- Buoyancy Reynolds number indicates where gravity wave breaking events are taking place
- Strongly stratified turbulence regime is found in winter Mesosphere
- Results for turbulent rms velocity are in a good agreement with radar observations
- Vertical spectra obtained from in-situ rocket measurements was predicted by KMCM simulations
- For the first time KMCM simulations, radar observations and in-situ rocket measurements facilitate a joint analysis of small-scale turbulence in the Mesosphere