Downscale energy transfer from balanced to unbalanced regime

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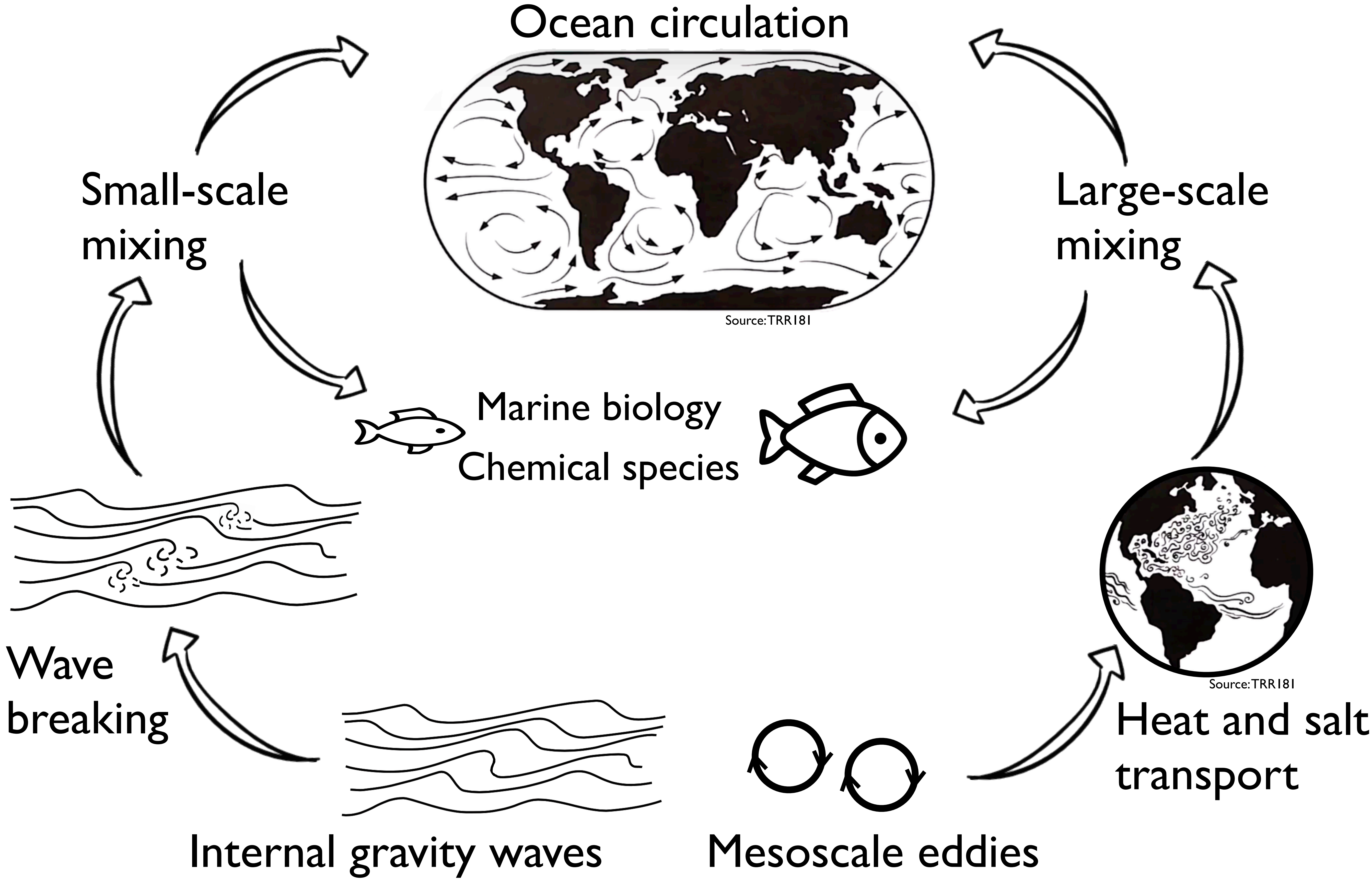
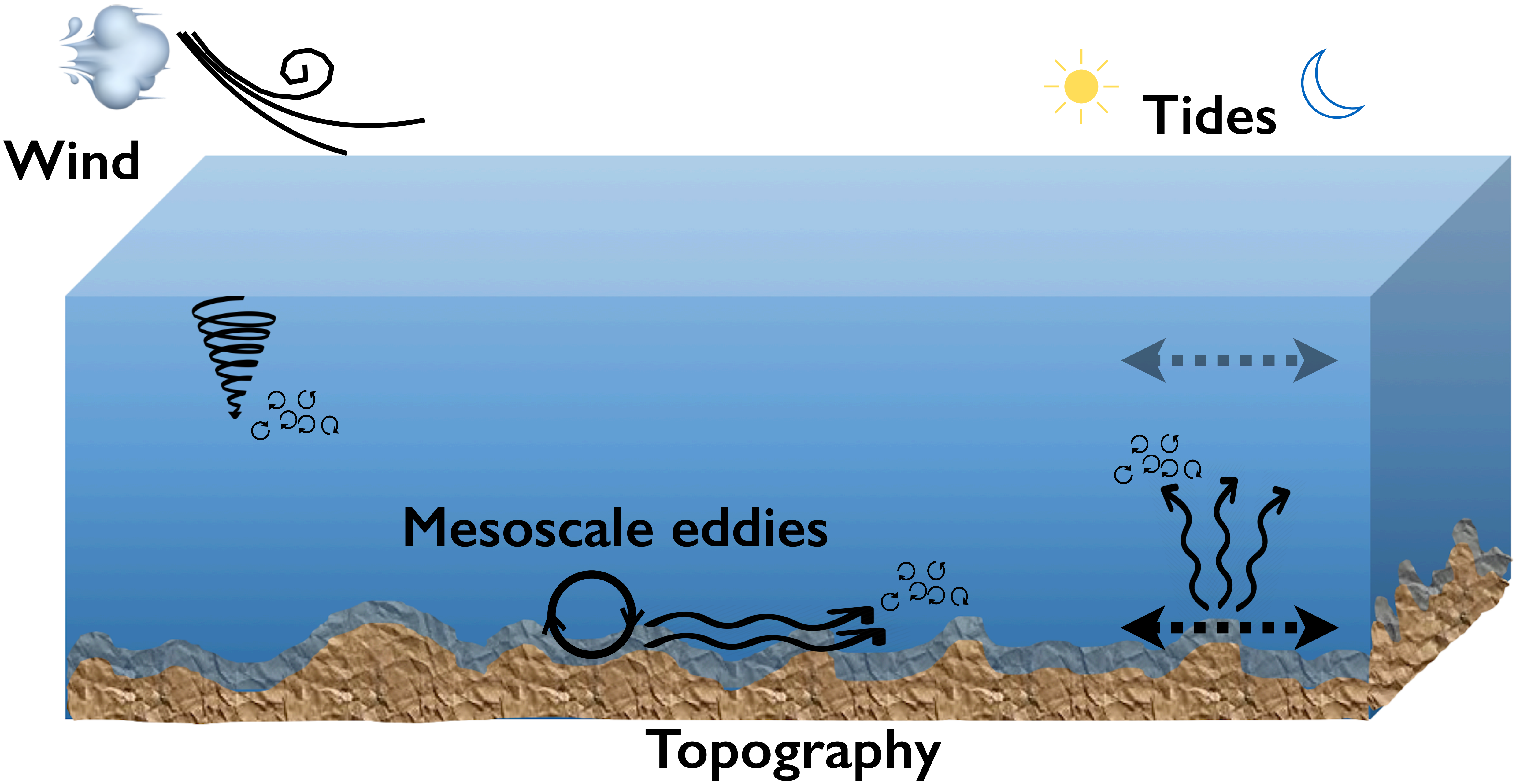
Balanced and unbalanced regime in the ocean



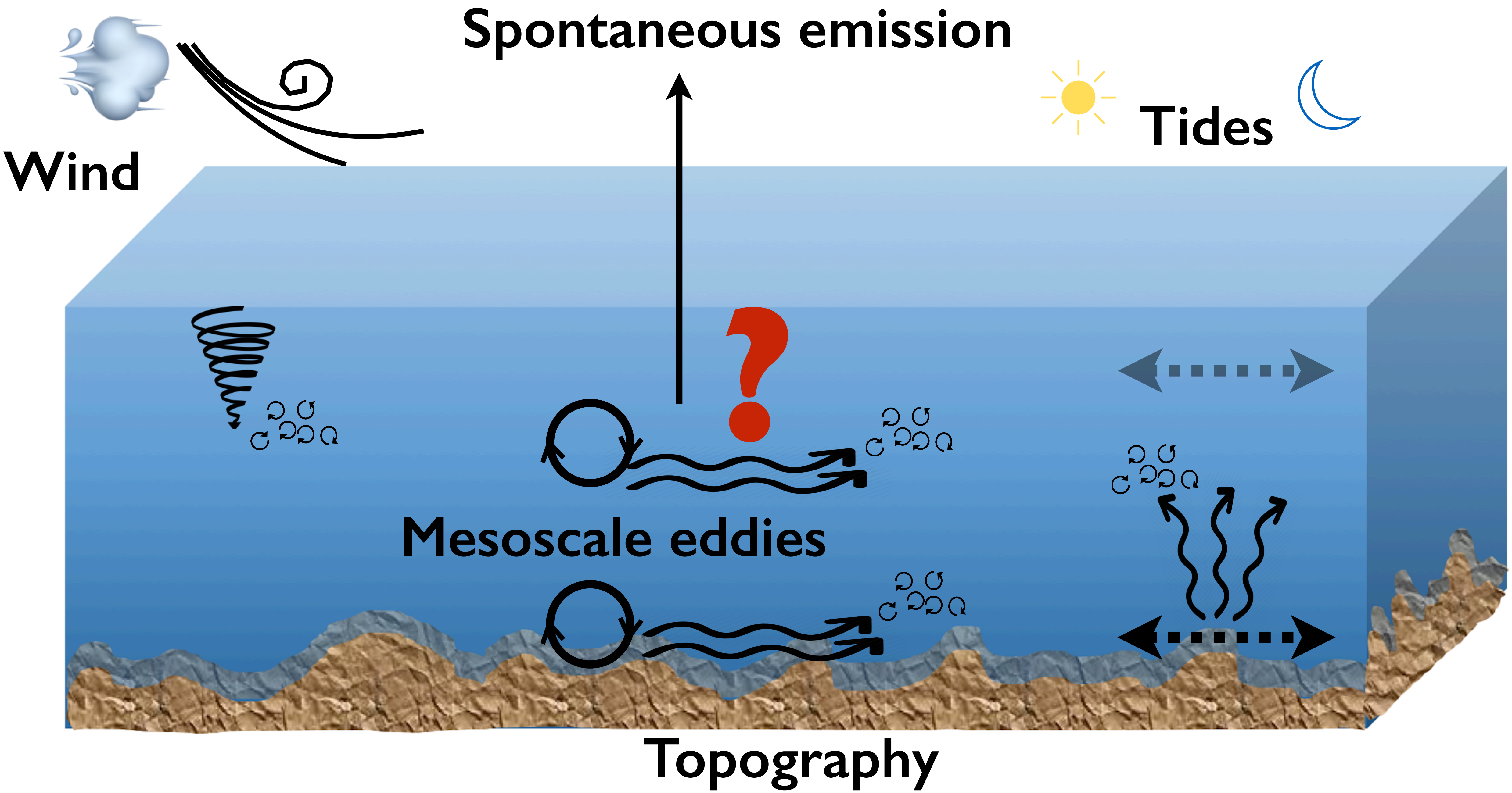
Generation of internal gravity waves



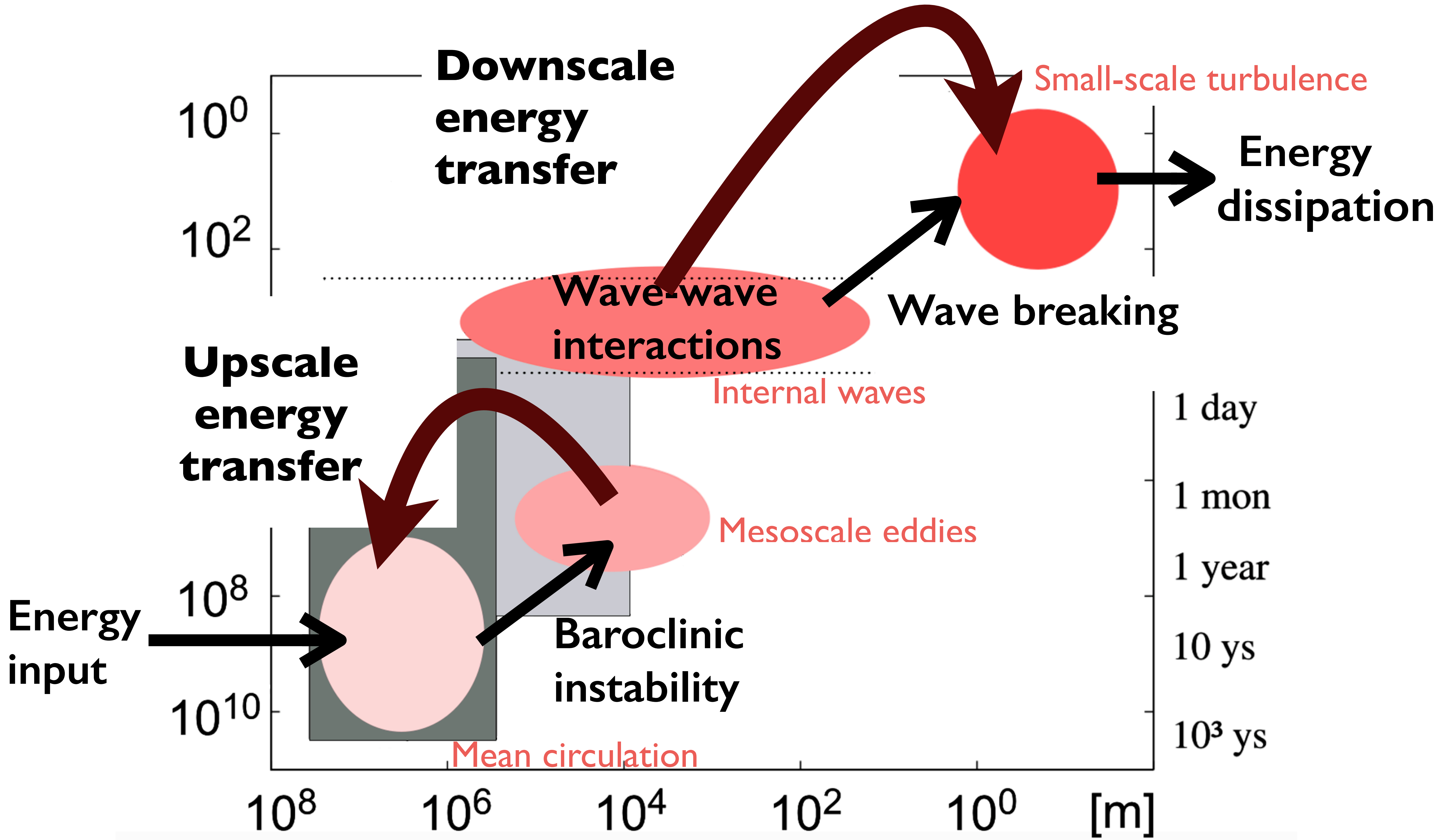
Generation of internal gravity waves

- Energy source of internal gravity waves
- Energy sink of balanced flow

} **Ocean's energy cycle**

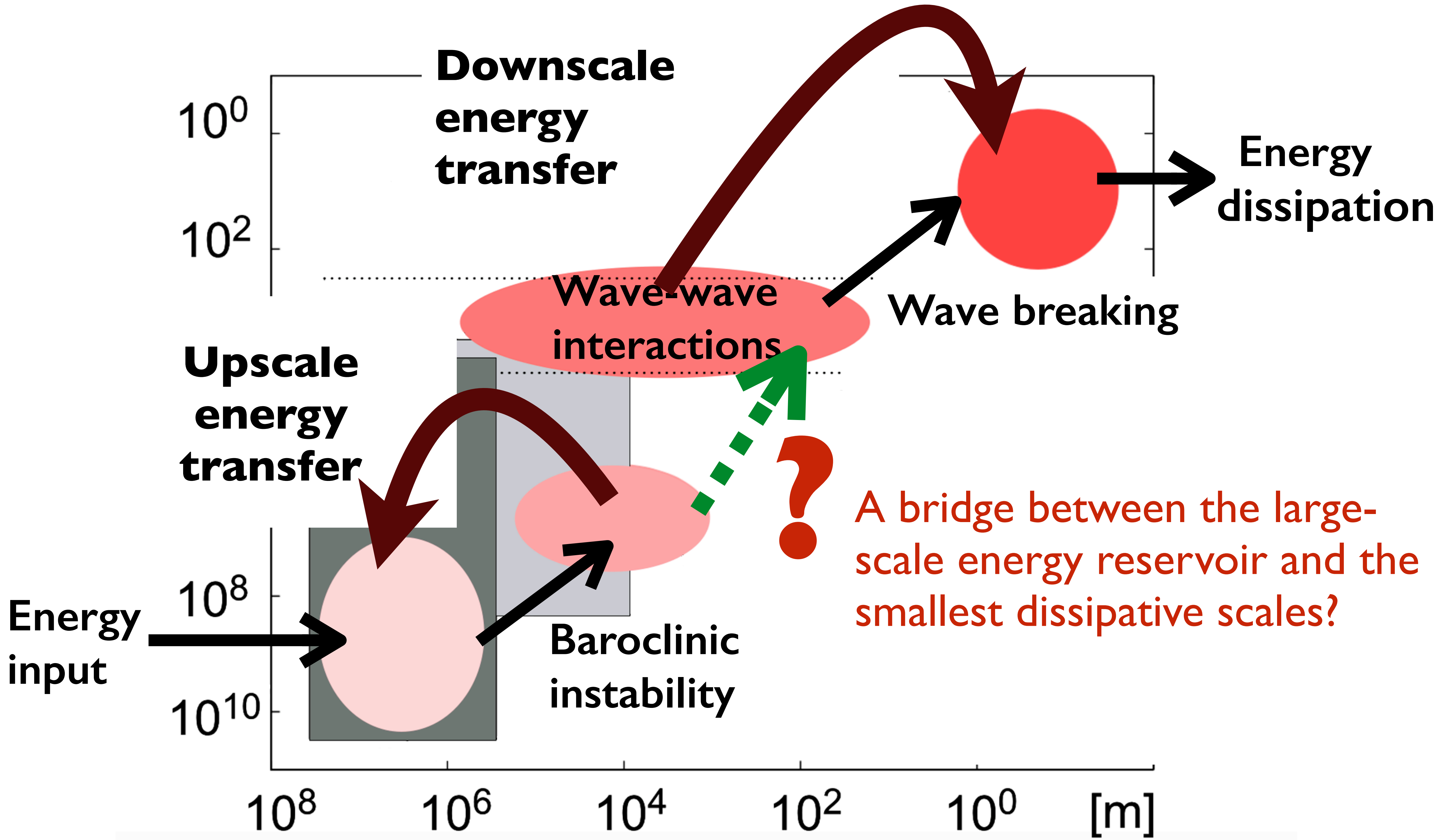


Energy pathways—what's missing?



Adapted from Eden et al. 2014

Energy pathways—what's missing?



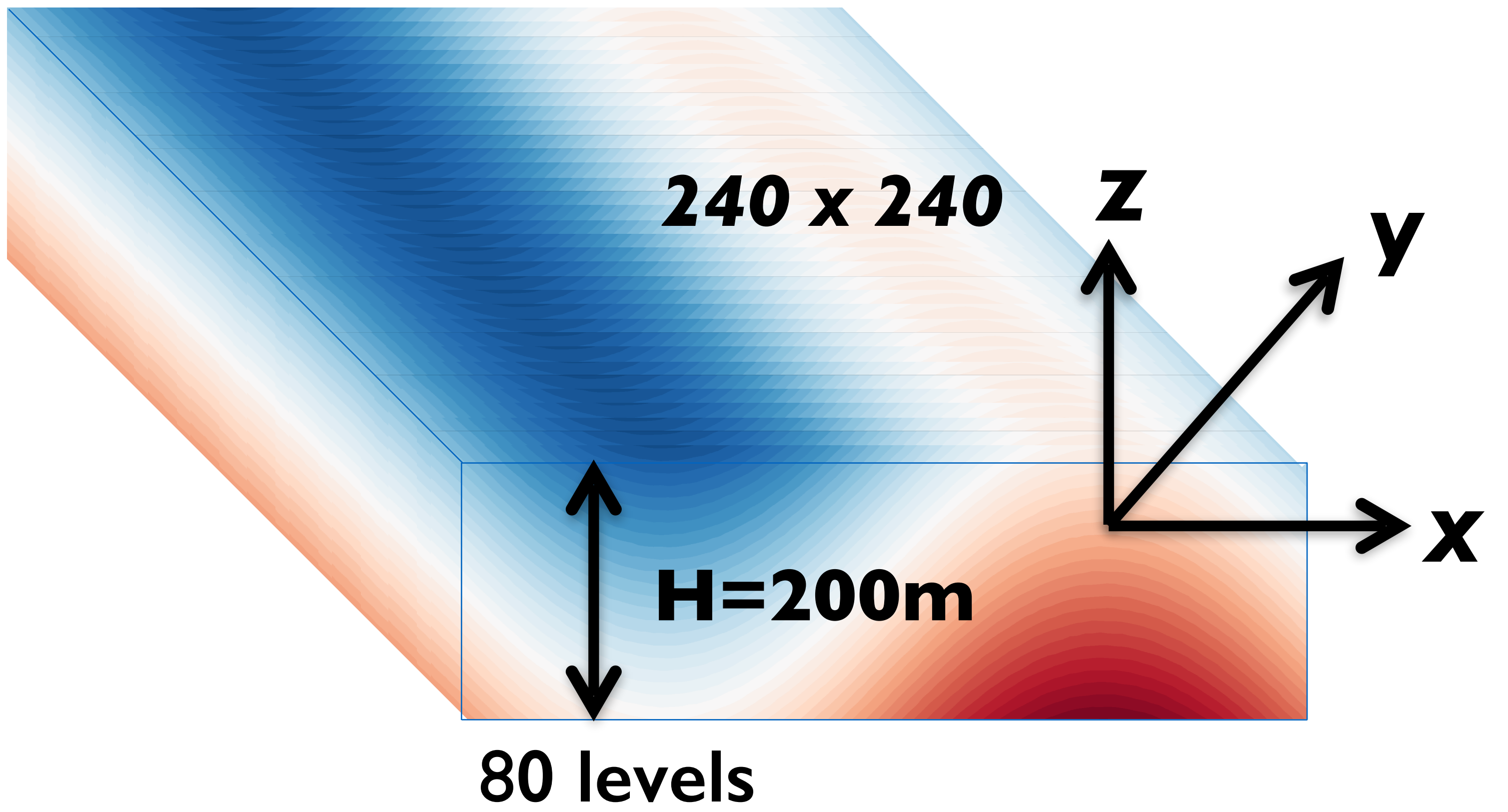
Adapted from Eden et al. 2014

Model setup: Idealized, baroclinically unstable

SETUP:

- *f*-plane
- Double periodic domain

–excludes boundary instabilities



FORCING:

- Large scales— Restoring of the zonal mean flow and buoyancy towards the initial state

DISSIPATION:

- Small scales— Biharmonic friction and vertical friction
- Large scales— Linear drag acting on the zonal mean flow

Different dynamical regimes

- Stratification—**Richardson number (Ri)**
- Rotation—**Rossby number (Ro)**

$$\text{Ri} = \frac{\text{vertical density stratification}}{\text{vertical shear of horizontal velocity}}$$

- Weakly stratified
- Strongly stratified

Ri

Ri = $O(1)$ **Ageostrophic** → **Quasi-geostrophic** **Ri >> 1**

Different dynamical regimes

- Stratification—**Richardson number (Ri)**
- Rotation—**Rossby number (Ro)**

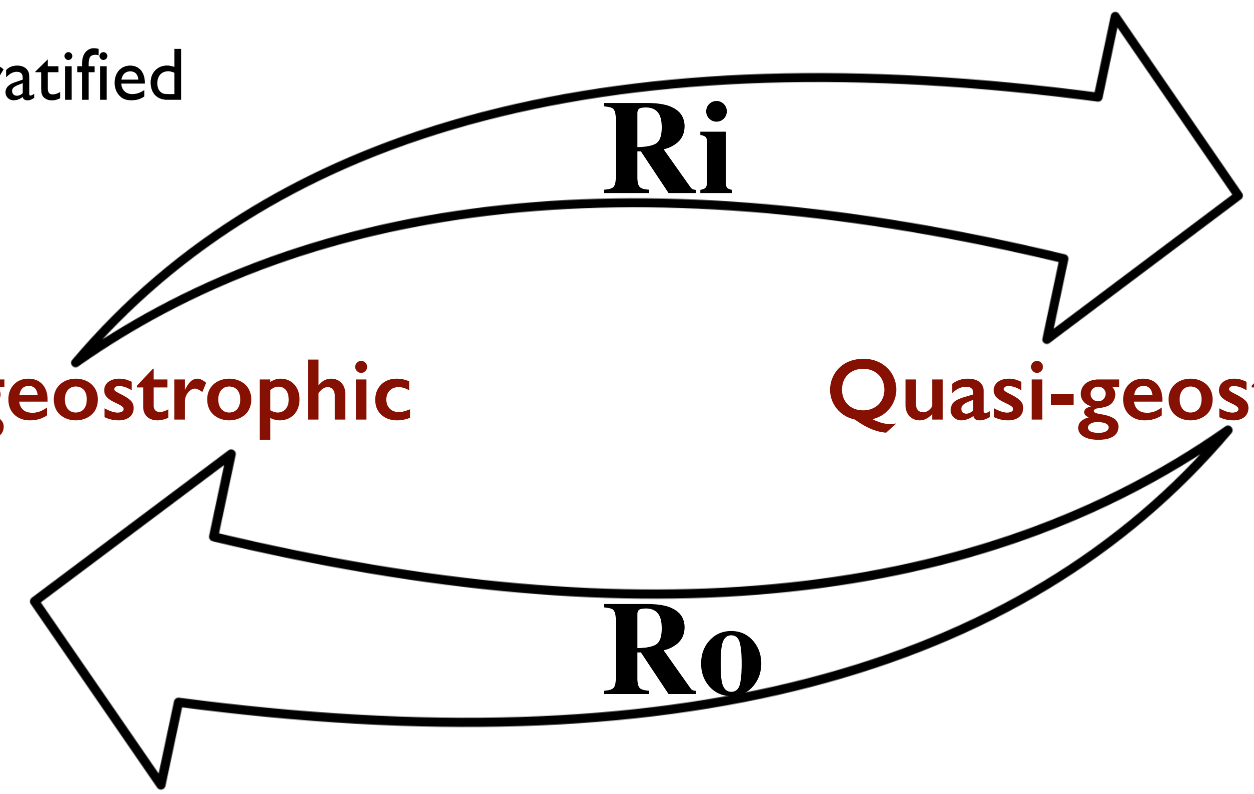
$$\text{Ri} = \frac{\text{vertical density stratification}}{\text{vertical shear of horizontal velocity}} \qquad \text{Ro} = \frac{\text{flow frequency}}{\text{frequency of rotation}}$$

- Weakly stratified
- Strongly stratified

Ri = O(1)

Ro < 1

Ageostrophic



Quasi-geostrophic

Ri >> 1

Ro<< 1

Different dynamical regimes

Ri = $O(1)$

Ageostrophic

Ri >> 1

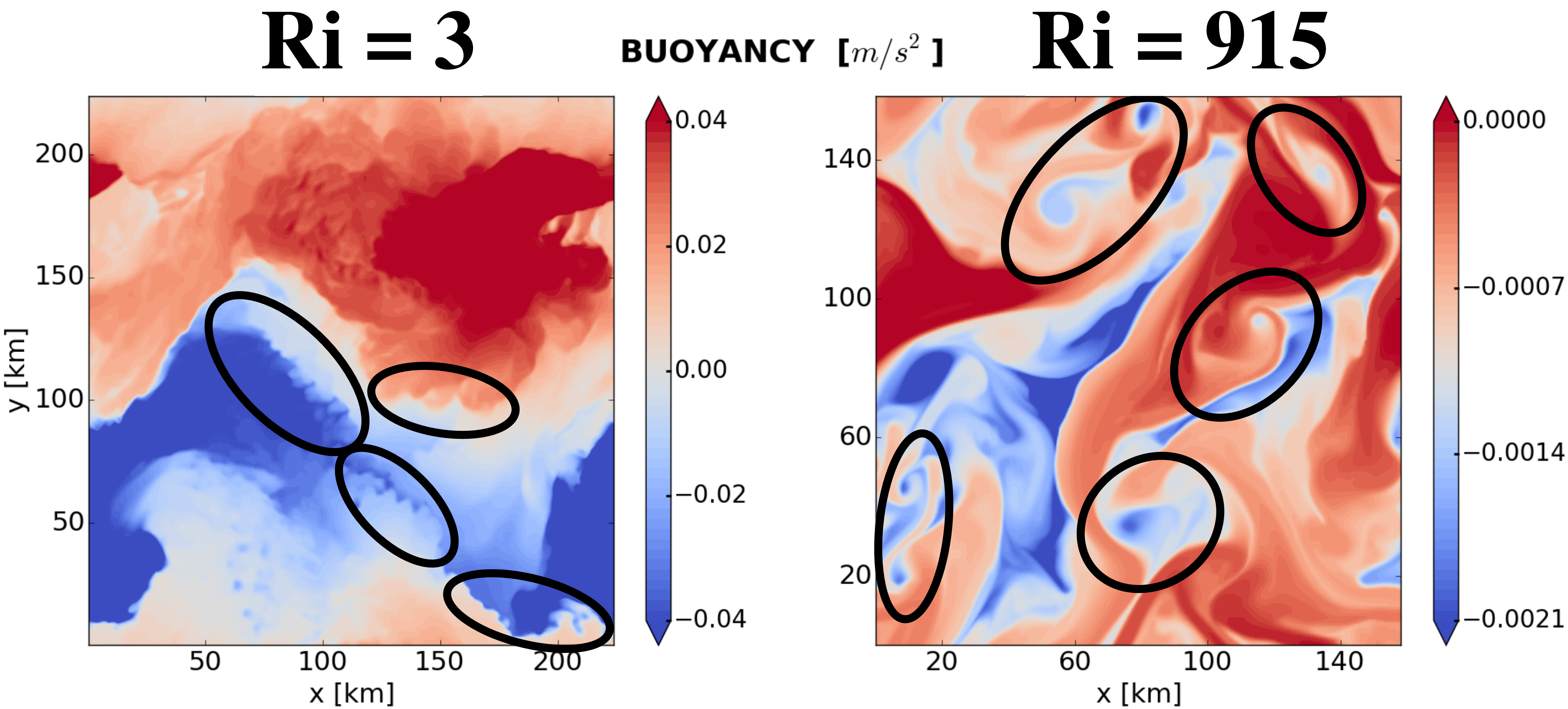
Quasi-geostrophic

- Ri sets the background state in the model.
- Hence, the flow dynamics: from Ageostrophic (Ri = $O(1)$) to Quasi-geostrophic (Ri >> 1).
- Different flow simulations with Ri = **3, 13, 327, and 917**

Different dynamical regimes

Ri = $O(1)$ **Ri** Ri >> 1

Ageostrophic → Quasi-geostrophic



Different dynamical regimes

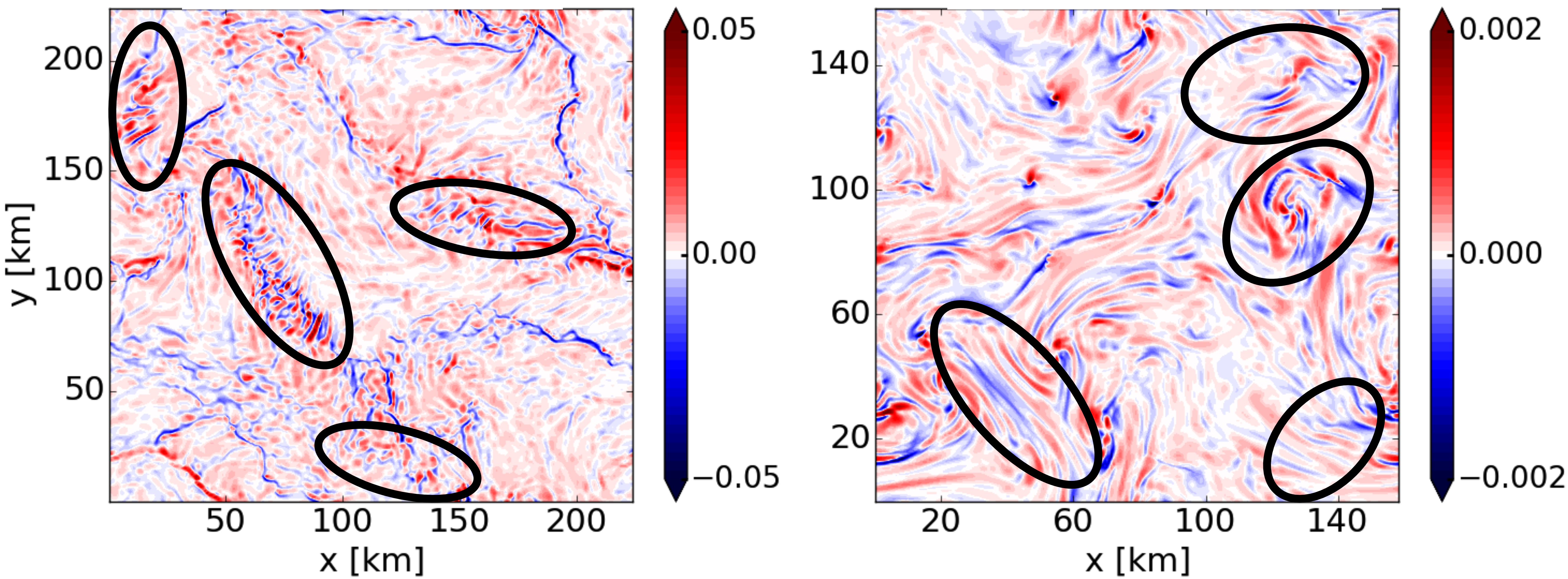
$\mathrm{Ri} = O(1)$

Ageostrophic

$\mathrm{Ri} \gg 1$

Quasi-geostrophic

Ri = 3 VERTICAL VELOCITY $[m/s]$ **Ri = 915**



Disentangling unbalanced and balanced regimes

O' mighty ocean!

Thou singeth tale of tangled waves
Waves thee maketh of all scales

Speaketh but, O' mighty ocean!

In thy wavy world of fast and slow
How doth thee split up fast from slow?

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Initialization

- treat the **initial data** to eliminate gravity waves

→ **Non-linear initialization by Machenhauer (1977)**

Decomposition of modes

Hydrostatic
and
Boussinesq equations

$$\partial_t \boldsymbol{x} = i\underbrace{\mathscr{L} \cdot \boldsymbol{x}}_{\text{Linear terms}} + \underbrace{\mathscr{N}(\boldsymbol{x})}_{\text{Non-linear terms}}$$

Fourier transformation

$$\begin{aligned} \partial_t u_n &= -ikp_n + fv_n \\ \partial_t v_n &= -ilp_n - fu_n \\ \partial_t p_n &= -(iku_n + ilv_n)c_n^2 \end{aligned}$$

Linearized equations

$$\tilde{\boldsymbol{x}} = \begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{p} \end{pmatrix} \quad \mathscr{L} = \begin{pmatrix} 0 & -if & -k \\ if & 0 & -l \\ -kc_n^2 & -lc_n^2 & 0 \end{pmatrix}$$

Decomposition of modes

Hydrostatic
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Linear decomposition

$$\mathscr{N} = 0$$

Non-linear decomposition

$$\mathscr{N} \neq 0$$

Decomposition of modes

$$\mathscr{L} = \begin{pmatrix} 0 & -if & -k \\ if & 0 & -l \\ -kc_n^2 & -lc_n^2 & 0 \end{pmatrix}$$

	Balanced component	**Unbalanced component**
Eigenvalues	$\omega^0 = 0$	$\omega^\pm = \pm\sqrt{f^2 + c_n^2 k_h^2}$
Eigenvectors	$\boldsymbol{q}^0 \;\; \boldsymbol{p}^0$	$\boldsymbol{q}^\pm \;\; \boldsymbol{p}^\pm$
Projection	$\mathscr{B} = \boldsymbol{q}^0 \cdot \boldsymbol{p}^0$	$\mathscr{G}^\pm = \boldsymbol{q}^\pm \cdot \boldsymbol{p}^\pm$

Decomposition of modes

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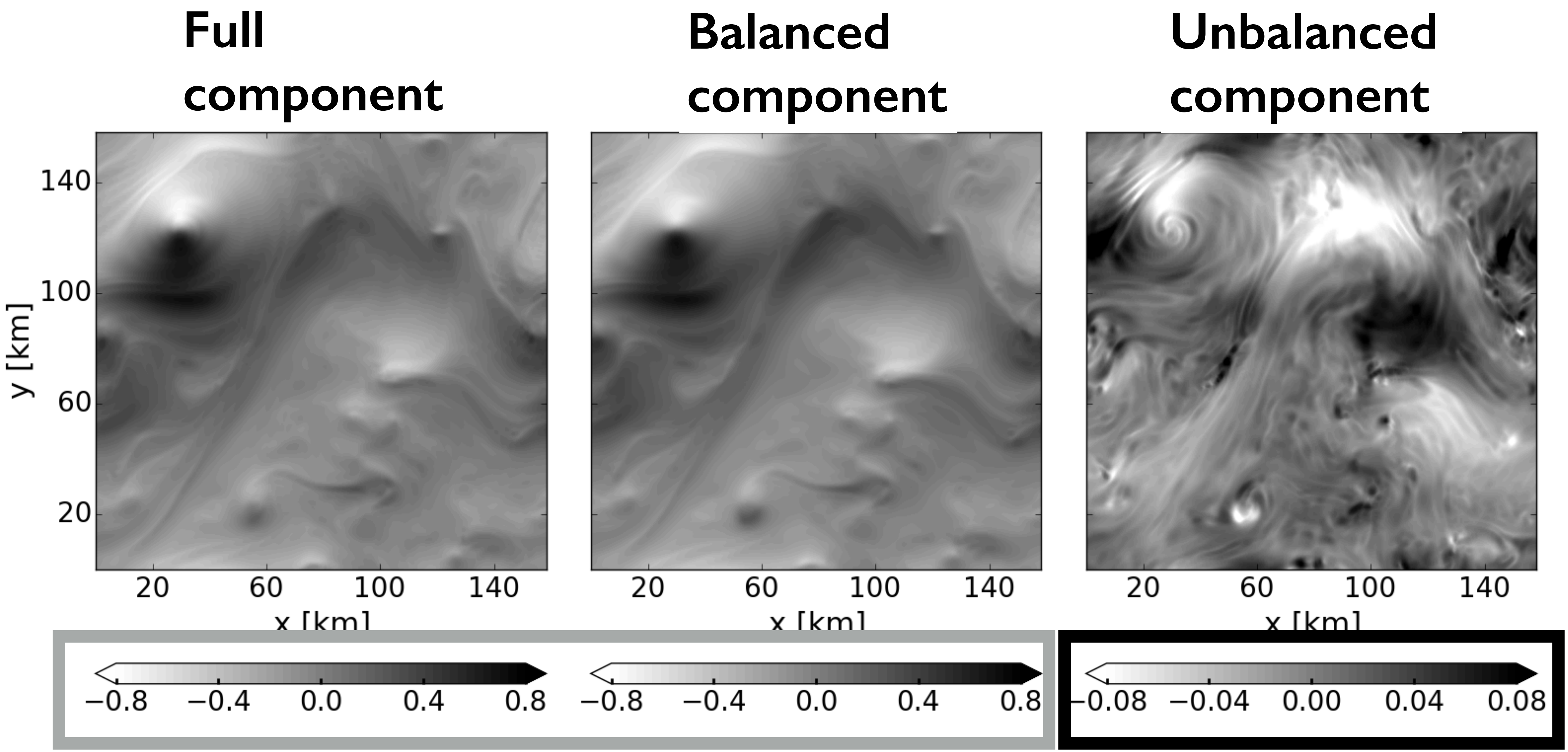
Projection	$\mathscr{B} = \boldsymbol{q}^0 \cdot \boldsymbol{p}^0$	$\mathscr{G}^\pm = \boldsymbol{q}^\pm \cdot \boldsymbol{p}^\pm$
Decomposed modes	$\tilde{\boldsymbol{x}}_B = \mathscr{B} \cdot \tilde{\boldsymbol{x}}$	$\tilde{\boldsymbol{x}}_G = \mathscr{G}^\pm \cdot \tilde{\boldsymbol{x}}$

$$\boxed{\tilde{\boldsymbol{x}} = \tilde{\boldsymbol{x}}_B + \tilde{\boldsymbol{x}}_G}$$

Fourier space

Linear decomposition

$$\mathcal{N} = 0$$



Non-linear decomposition

$$\mathscr{N} \neq 0$$

$$\partial_t \tilde{\boldsymbol{x}}_B = \tilde{\mathscr{N}}(\boldsymbol{x}_B) \neq 0$$

Non-linear balanced mode

Machenhauer (1977):

$$\mathscr{G} \cdot \partial_t \tilde{\boldsymbol{x}}^*_B = 0$$

$$\rightarrow \mathscr{G} \cdot \boxed{\tilde{\boldsymbol{x}}^*_B} = i(\mathscr{L} \cdot \mathscr{G})^{-1} \cdot \mathscr{G} \cdot \tilde{\mathscr{N}}\boxed{(\tilde{\boldsymbol{x}}^*_B)}$$

Leith (1980):

$$\boxed{\tilde{\boldsymbol{x}}^*_B = \tilde{\boldsymbol{x}}_B + i(\mathscr{L} \cdot \mathscr{G})^{-1} \cdot \mathscr{G} \cdot \tilde{\mathscr{N}}(\boldsymbol{x}_B)}$$

Quasi-geostrophic balanced state

Non-linear decomposition

$$\mathscr{N} \neq 0$$

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Non-linear balanced mode

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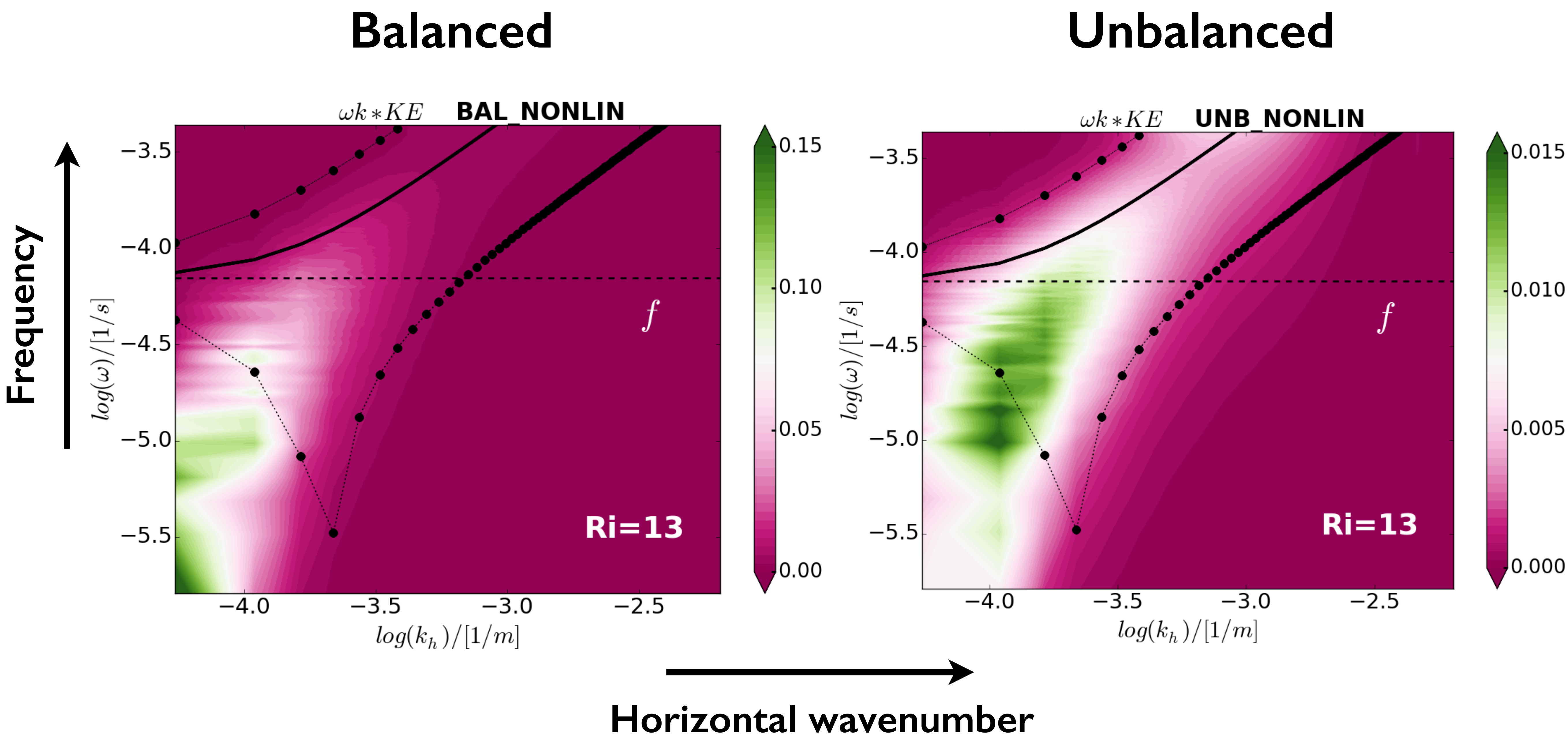
Quasi-geostrophic balanced state

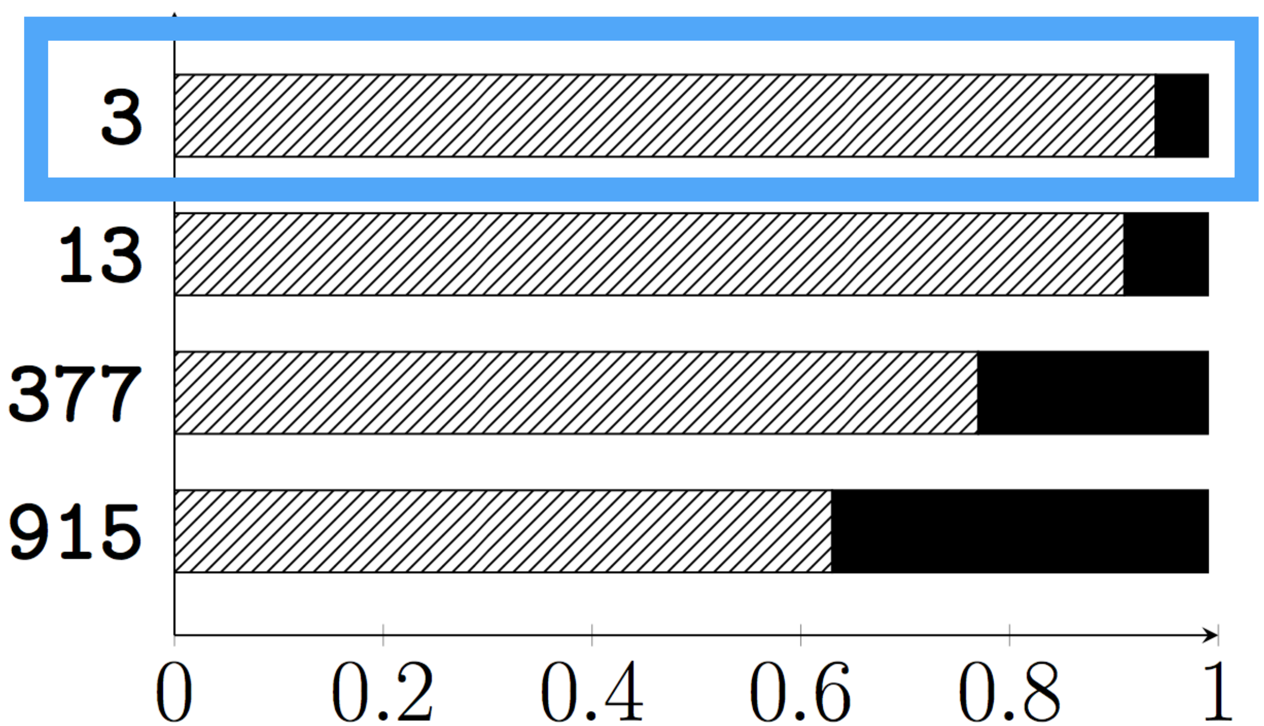
Non-linear unbalanced mode = Full state vector — Non-linear balanced mode

Non-linear decomposition



Dissipation

Ri Unbalanced component (non-linear)



- The unbalanced mode shows a preferred dissipation through small-scale dissipation **for all Ri**

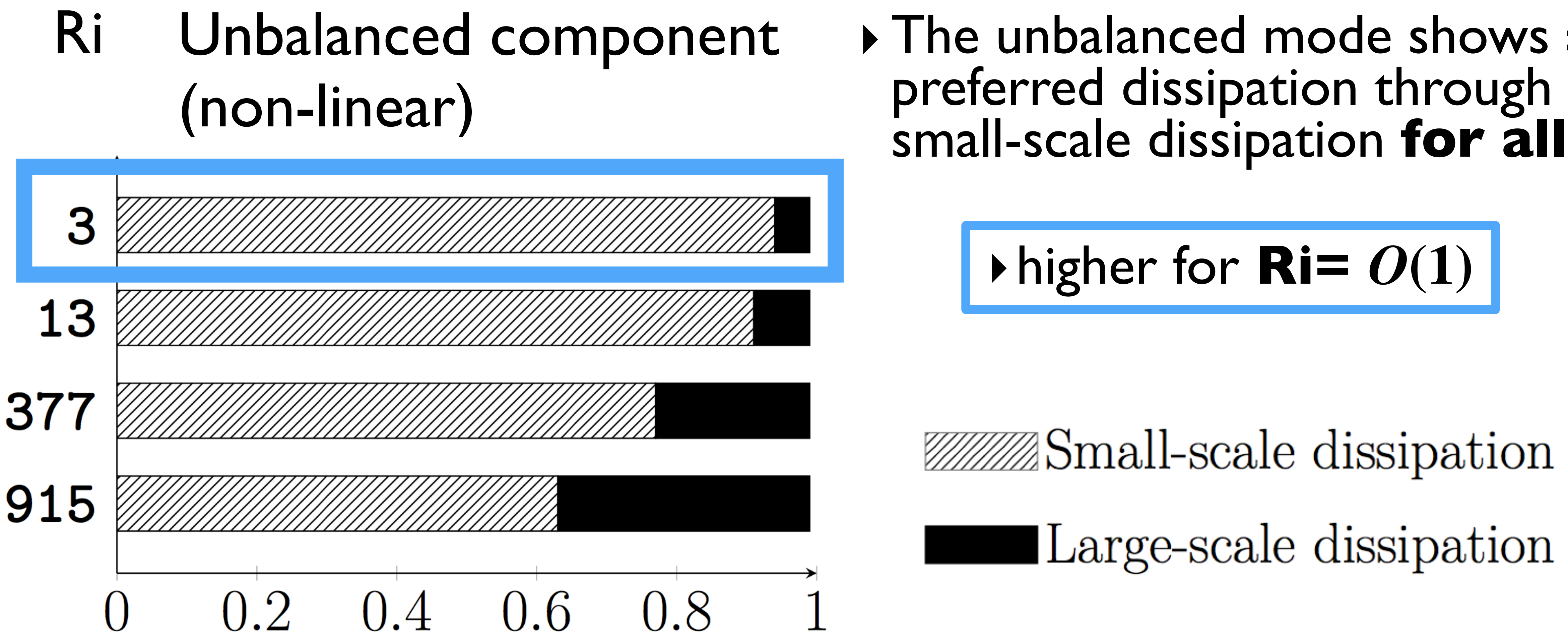
- higher for **Ri=** $\mathcal{O}(1)$

Small-scale dissipation

Large-scale dissipation

Dissipation

Gravity waves dissipate predominantly through small-scale dissipation for all Ri.

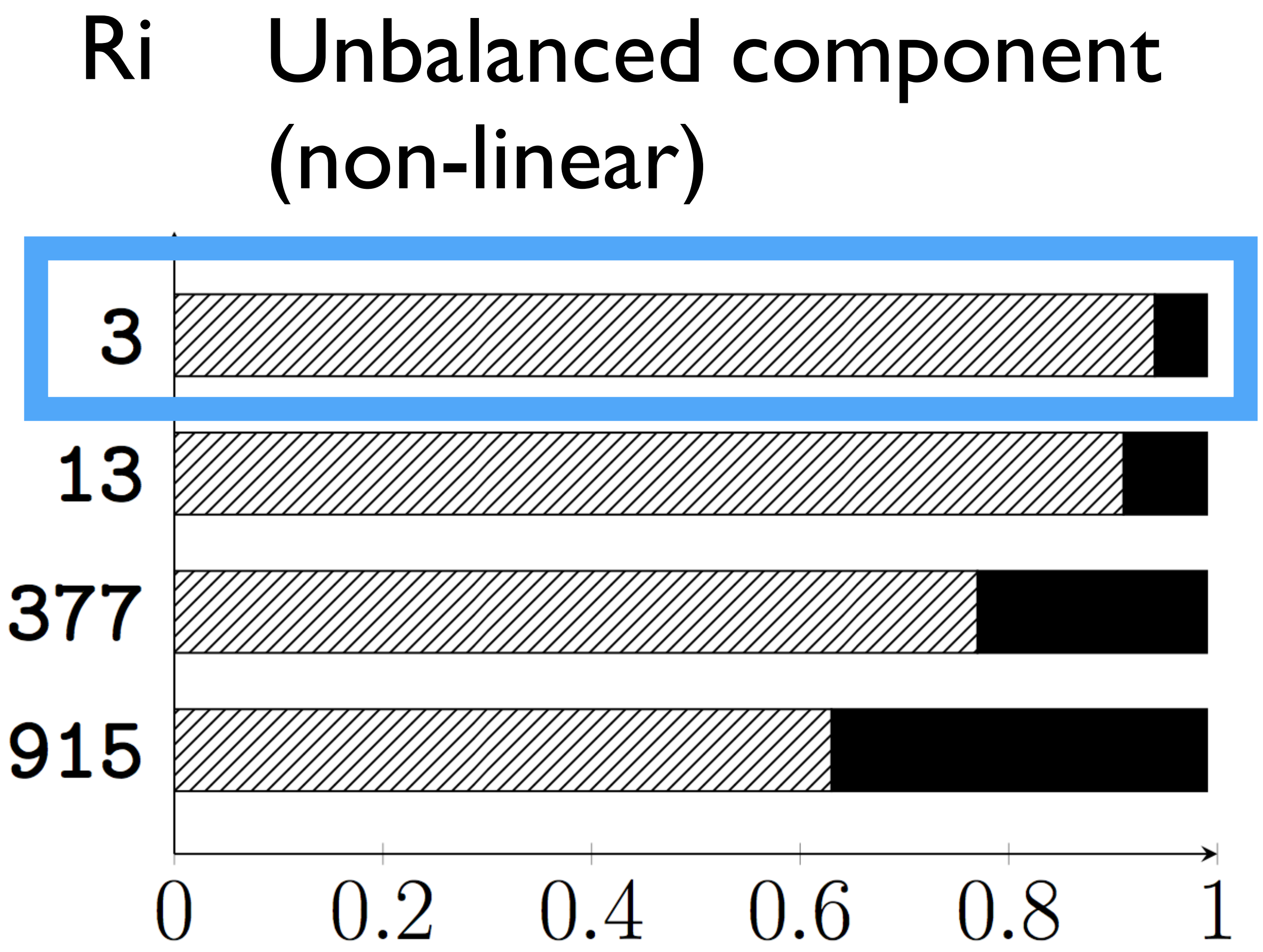


- The unbalanced mode shows a preferred dissipation through small-scale dissipation **for all Ri**
  - higher for **Ri=** $O(1)$

Dissipation

Ageostrophic baroclinic instability can generate internal gravity waves during the downscale energy transfer.

Gravity waves dissipate predominantly through small-scale dissipation for all Ri.



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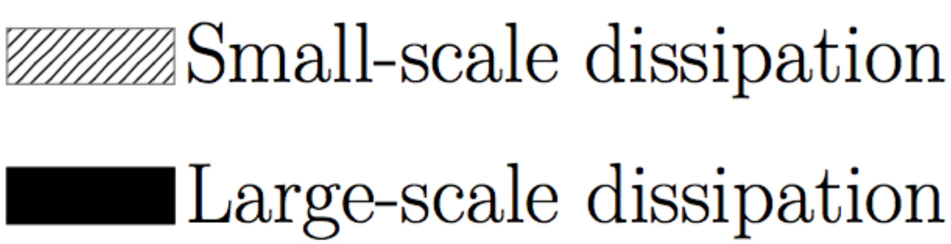
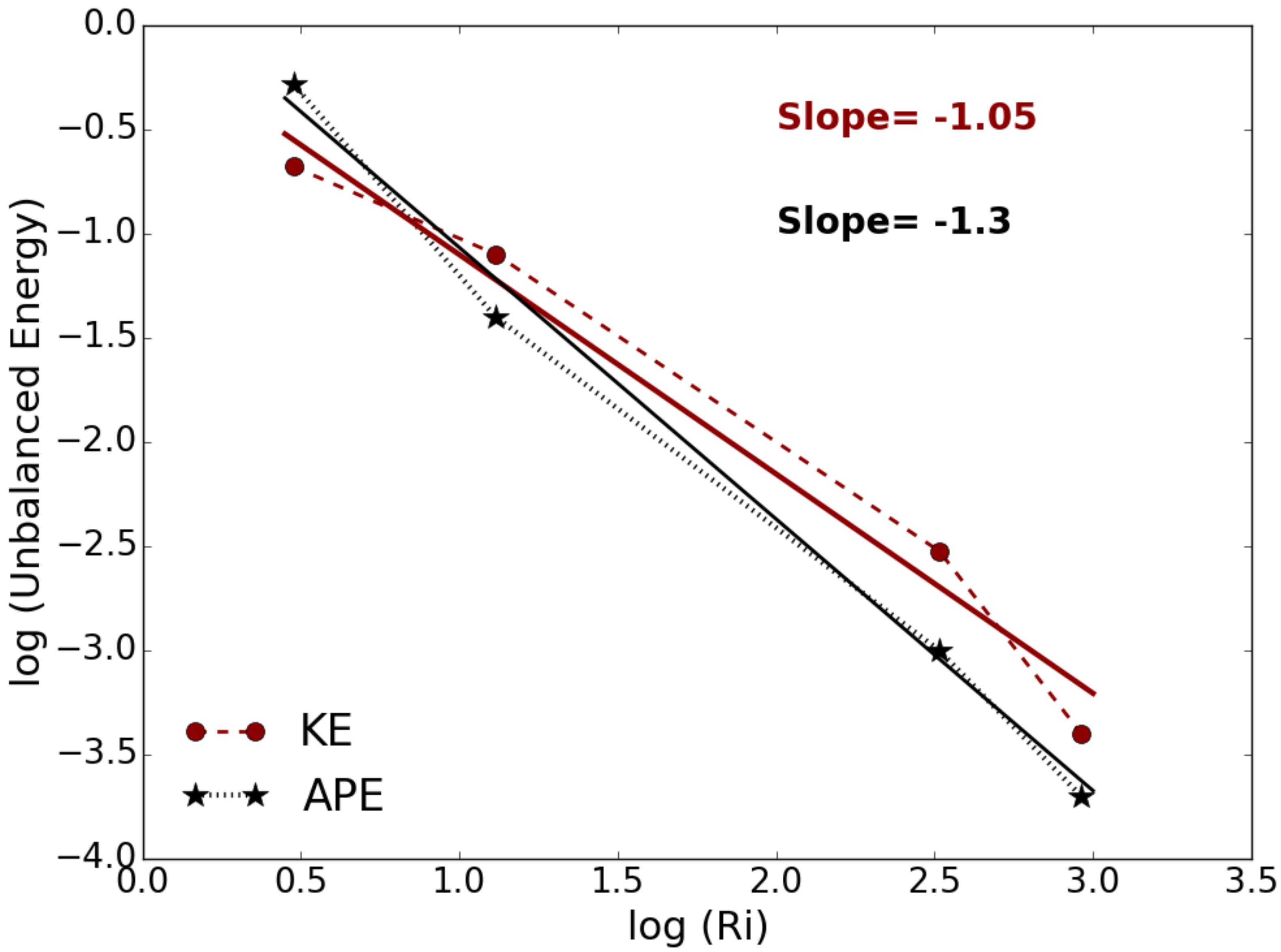


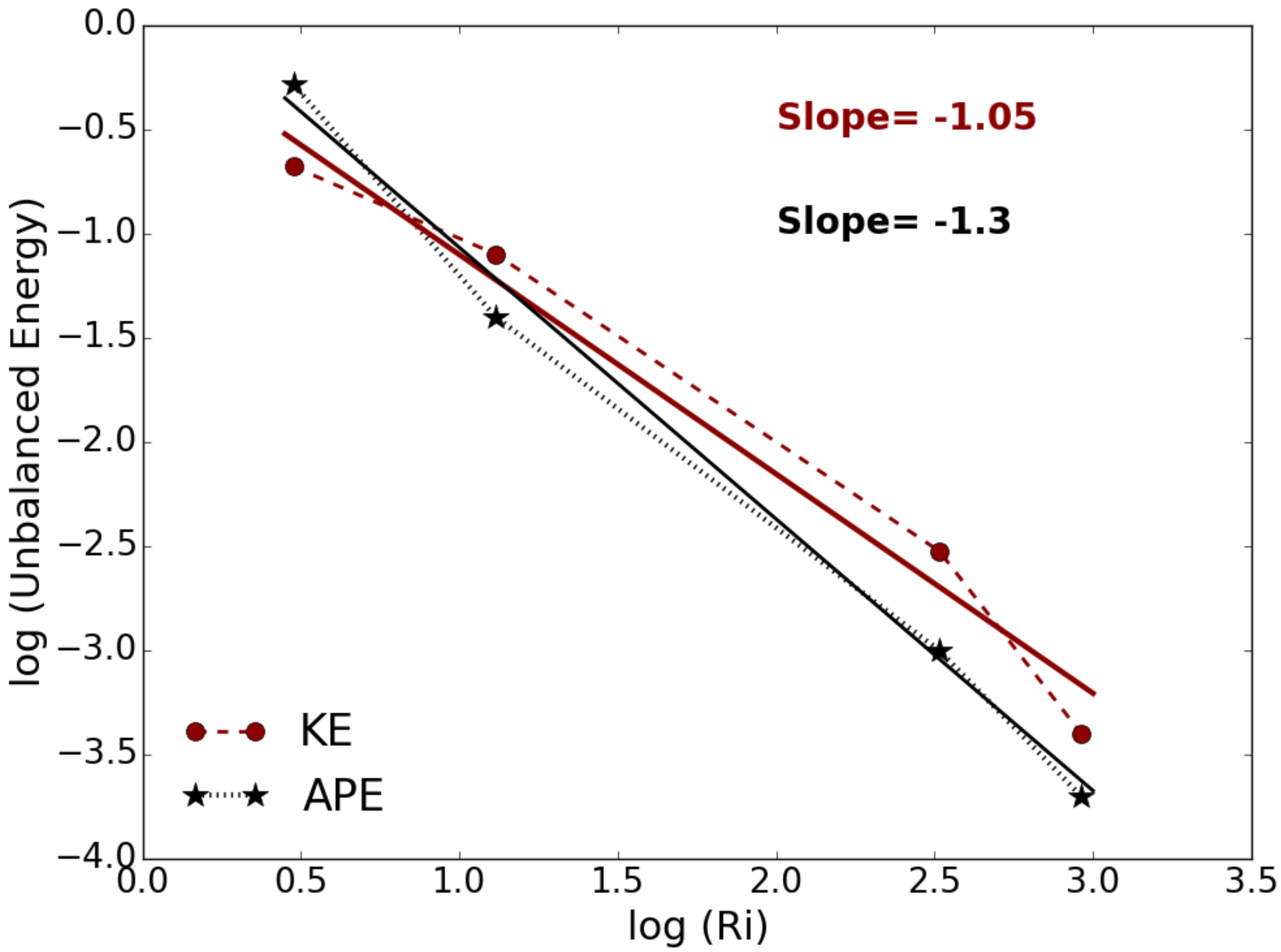
Gravity wave emission: power law



Chouksey, Eden, and Brüggemann, 2018: Internal gravity wave emission in different dynamical regimes (under revision in JPO)

Gravity wave emission: power law

The kinetic energy tied to the unbalanced mode scales with Rossby number as Ro^2.



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Wave emission: theoretical results

Kinetic equation for wave energy:

$$\partial_t E_0^{s_0} = \frac{4}{n_0^{s_0}} \int d\boldsymbol{k}_1 \sum_{s_1,s_2} C^{s_0,s_1,s_2}_{\boldsymbol{k}_0,\boldsymbol{k}_1,\boldsymbol{k}_2} \Big(n_1^{s_1} n_2^{s_2} \boxed{E_1^{s_1} E_2^{s_2}} (C^{s_0,s_2,s_1}_{\boldsymbol{k}_0,\boldsymbol{k}_2,\boldsymbol{k}_1})^*$$
$$- n_0^{s_0} n_1^{s_1} \boxed{E_0^{s_0} E_1^{s_1}} C^{s_2,-s_1,s_0}_{\boldsymbol{k}_2,-\boldsymbol{k}_1,\boldsymbol{k}_0} - n_0^{s_0} n_2^{s_2} \boxed{E_0^{s_0} E_2^{s_2}} C^{s_1,-s_2,s_0}_{\boldsymbol{k}_1,-\boldsymbol{k}_2,\boldsymbol{k}_0} \Big) \Delta(\omega_1^{s_1} + \omega_2^{s_2} - \omega_0^{s_0})$$
$$- 4E_0^{s_0} \int d\boldsymbol{k}_1 \sum_{s_1=\pm} n_1^{s_1} E_1^{s_1} \sum_{s_2=\pm} C^{s_0,s_0,s_2}_{\boldsymbol{k}_0,\boldsymbol{k}_0,0} C^{s_2,s_1,-s_1}_{0,\boldsymbol{k}_1,-\boldsymbol{k}_1} \Delta(s_2 f) + c.c.$$

s = ±,0

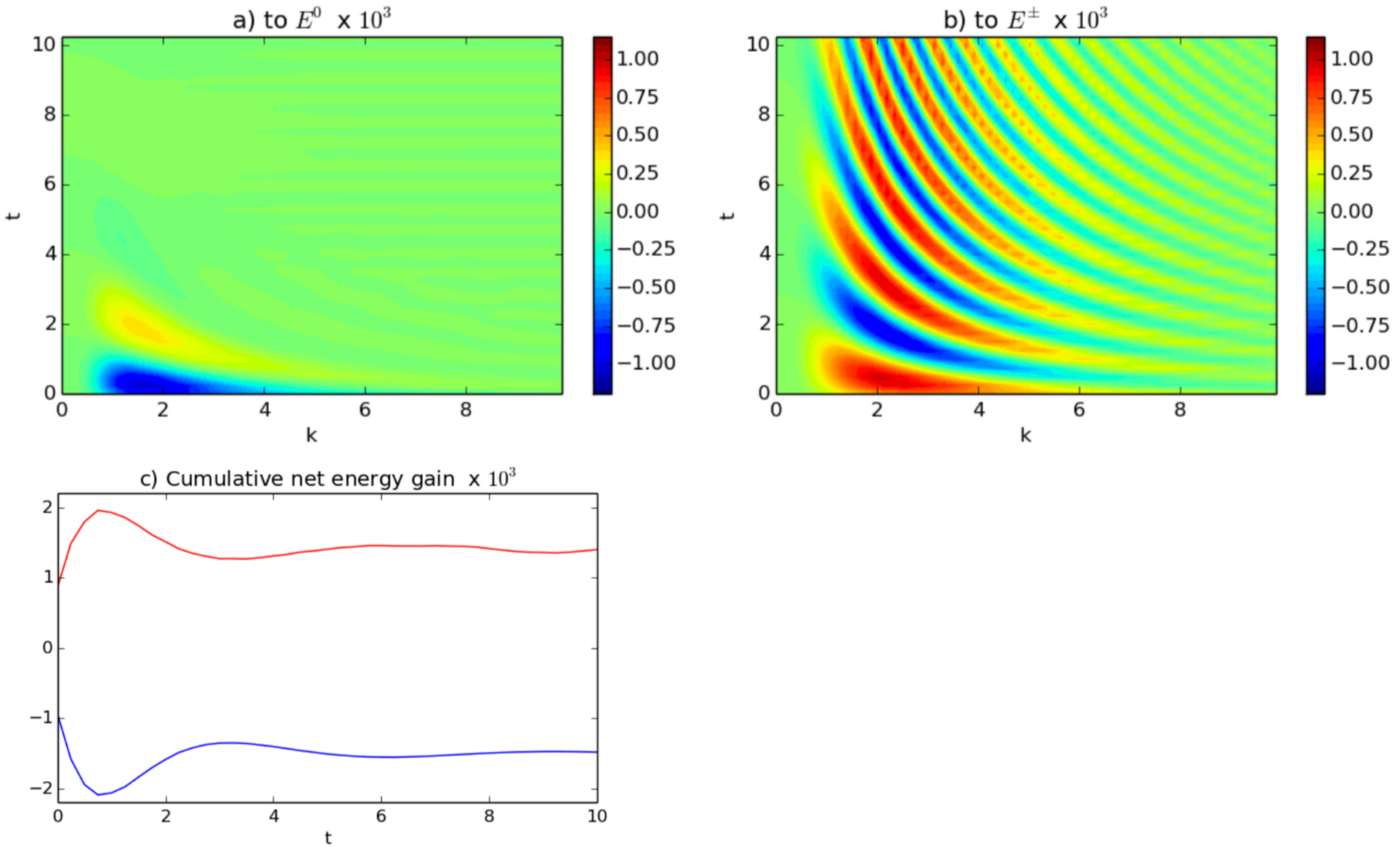
- Initially vanishing gravity waves (**E**$^{\pm}$)
- Initially vanishing Rossby waves (**E**^{0})

Eden, Chouksey, and Olbers, 2018: Mixed Rossby-gravity wave-wave interactions. (submitted)

Wave emission: theoretical evaluation

Rossby waves (Balanced)

Gravity waves (Unbalanced)



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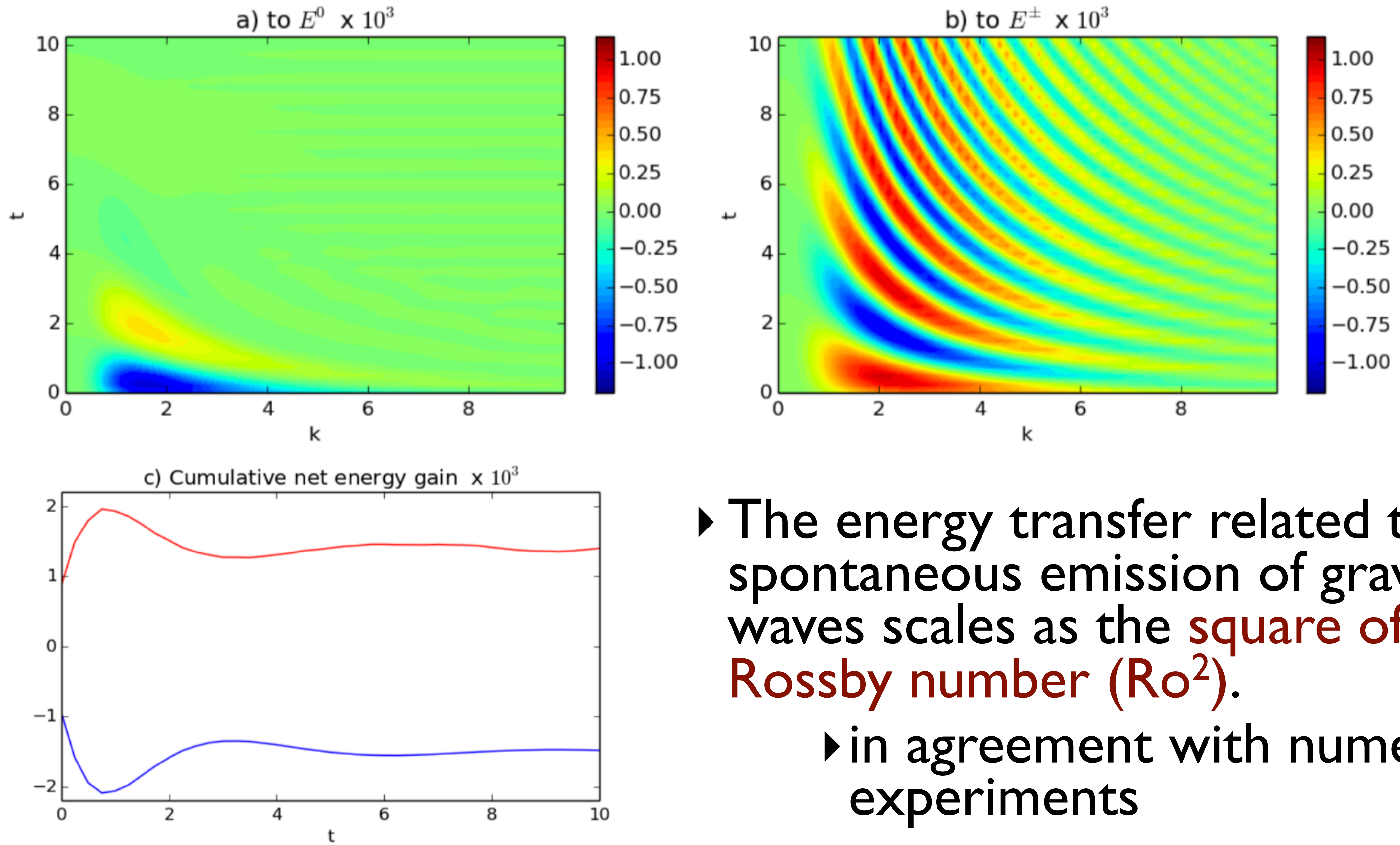
Wave emission: theoretical evaluation

Rossby waves (Balanced) | **Gravity waves (Unbalanced)**

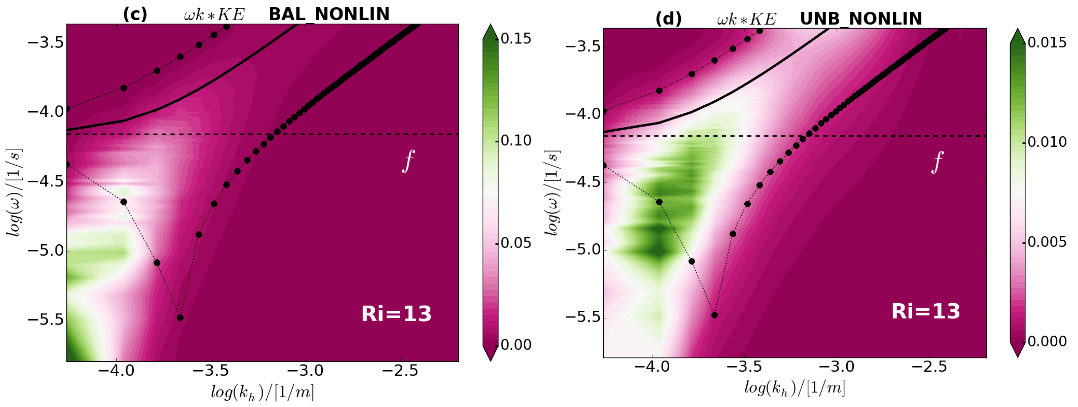


- The energy transfer related to the spontaneous emission of gravity waves scales as the square of the Rossby number (Ro^2).
  - in agreement with numerical experiments

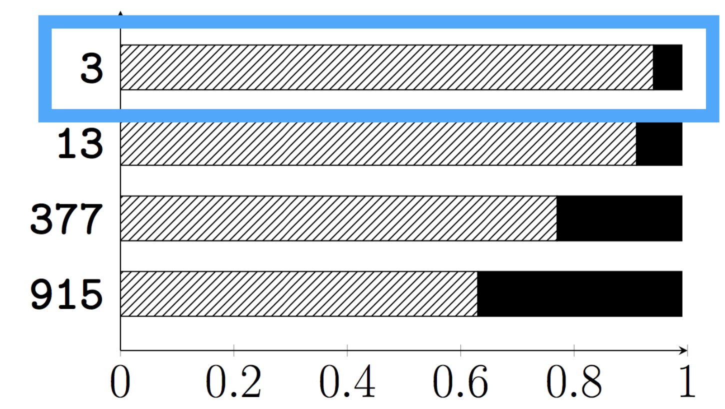
Eden, Chouksey, and Olbers, 2018: Mixed Rossby-gravity wave-wave interactions. (submitted)

Conclusions

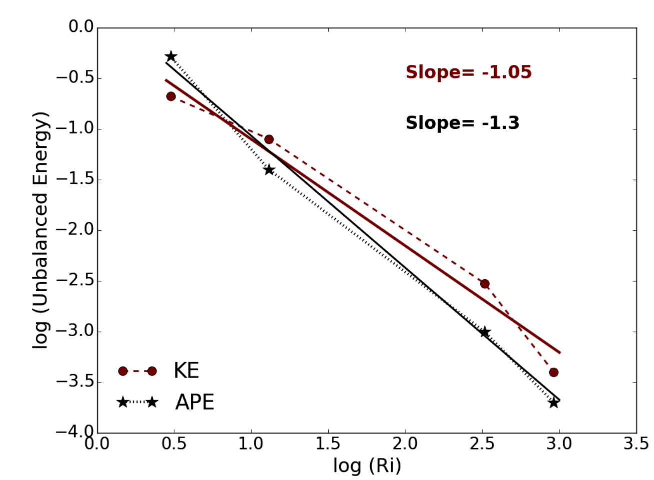
- Gravity wave activity is much more pronounced for a Ri = $O(1)$ regime than for a Ri >>1 regime, identified using non-linear initialization technique as a diagnostic.



- Gravity waves dissipate predominantly through small-scale dissipation for all Ri and hence during the downscale energy transfer.



- The kinetic energy tied to gravity wave emission scales with Rossby number as Ro^2, in agreement with theoretical results.



Thank you!