IMPACT OF FILAMENTS ON THE RELATIVE DISPERSION OF SURFACE DRIFTERS IN THE BENGUELA UPWELLING REGION

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Overview

• Surface drifter experiment in the Benguela upwelling region
• (Relative) dispersion of pairs of drifters:
  fixed times analysis: mean squared pair separations
  fixed scale analysis: Finite Size Lyapunov Exponents (FSLEs)
• Mean squared pair separations: Different dispersion regimes
• Anomalous diffusion
• Do Finite Size Lyapunov Exponents yield consistent results?
• Analysis of drifter subsets: dependence on deployment location
• Do our data show local (Richardson) dynamics?
• Conclusion
Upwelling water front off Namibia’s coast with filament

SST along ship track. Red dots: triplet-deployments

36 surface drifters were released in 4 groups (12/12/6/6 drifters) of triplets, with initial distance of 100 m – 500 m. Deployment locations: southern boundary (27°S, 12°E) S, northern boundary (26°S, 13°E) N.
Drifter trajectories from cruise M132 in the area off Namibia's coast in November/December 2016.
Drifter trajectories from cruise M132 in the area off Namibia’s coast in November/December 2016
Lagrangian analysis

Pair separation statistics yield information about the dominant physical mechanisms acting at the different scales of motion (e.g. chaotic advection, turbulence, diffusion) in specific regions

- fixed time measure:
  Mean squared pair separation \( < s^2 (t) > \)

- Fixed scale measure:
  Finite size Lyapunov exponents \( \lambda(\delta) = \frac{1}{\langle \tau(\delta) \rangle} \ln(\alpha) \)

\( \langle \tau(\delta) \rangle = \) averaged time for drifter-pairs, initially separated by distance \( \delta \), to reach distance \( \alpha \delta \)

What dispersion regimes can be detected (at what scales)?
Do the two methods yield consistent results?
Lagrangian root mean squared pair separation $< s^2(t) >^{1/2}$

- Nonlocal behavior for $s < \text{Rossby Radius } R_1 (\approx 30 \text{ km})$

<table>
<thead>
<tr>
<th>Pair Separations</th>
<th>Nonlocal</th>
<th>Local</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; s^2 &gt;$</td>
<td>$s \sim e^{t/\tau}$, $\tau \approx 0.6 \text{ days}$</td>
<td>$&lt; s^2 &gt; = C\epsilon t^3$</td>
</tr>
<tr>
<td>$C\epsilon$</td>
<td>$\approx 4.4 \times 10^{-9} \text{ W/kg}$</td>
<td></td>
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<tr>
<td>Diffusive</td>
<td>$&lt; s^2 &gt; = 2\kappa t$, $\kappa \approx 3.7 \times 10^4 \text{ m}^2/\text{s}$</td>
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</table>
Normal and anomalous diffusion

- **Brownian motion** (Einstein, Ann. d. Phys., 1905)
- Deviation (diffusional anomalies)
  \[
  \alpha > 1 \text{ (superdiffusion), } \kappa(t) \to t^\gamma
  \]
  \[
  \alpha < 1 \text{ (subdiffusion), } \kappa(t) \to 0
  \]

experimentally observed in:
- \(\alpha > 1\) floating tracers in ocean (Osborne et al., Tellus `89)
  rotating fluid flows (Solomon.. Phys. Rev. Lett. `93),
  sea ice (Gabrielsi, Badin, Kaleschke, J. o. Geoph. Res. `14)
- \(\alpha < 1\) polymer network (Amblard et al., Phys. Rev. Lett. `96)
  charge transport (Scher & Montroll Phys. Rev. B)

``Rules of thumb`` for physical mechanism:
- \(\alpha > 1\): persistance of free paths (``Levy Flights``)
- \(\alpha < 1\): geometric constraints (disorder or fractal)

\[< r^2(t) > \sim t^\alpha\] (Dräger & Klafter
Phys. Rev. Lett. `00)
Finite Size Lyapunov Exponents

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<tr>
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<th>pair separations</th>
<th>FSLE (expected)</th>
</tr>
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<tr>
<td>I. Nonlocal</td>
<td>$s \sim e^{t/\tau}$</td>
<td>$\lambda(\delta) \sim \text{const}$</td>
</tr>
<tr>
<td>II. Local</td>
<td>$&lt; s^2 &gt; \sim t^3$</td>
<td>$\lambda(\delta) \sim \delta^{-2/3}$</td>
</tr>
<tr>
<td>III. Diffusive</td>
<td>$&lt; s^2 &gt; \sim t$</td>
<td>$\lambda(\delta) \sim \delta^{-2}$</td>
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Controversial results for small scales:
FSLEs $\lambda(\delta)$ point to local behavior, $<s^2(t)>$ to nonlocal behavior
Different deployment scenarios:

- Northern release
- Southern release
Dependence on deployment location

**Northern release**

\[
\langle s^2(t) \rangle^{1/2} \propto t^{3/2}
\]

**Southern release**

\[
\langle s^2(t) \rangle^{1/2} \propto t^{3/2}
\]

Consistent results for \( \langle s^2(t) \rangle \) and \( \lambda(\delta) \), when the methods are applied to the two releases *separately*
Do our data show local (Richardson) dynamics?

Below or above $R_1$? For both releases?

\[ <s^2(t)> \sim \epsilon t^3 \] for $s > R_1$ (\textit{northern release} & whole ensemble)

\[ \lambda(\delta) \sim \delta^{-2/3} \] for $\delta \ll R_1$ (\textit{southern release} & whole ensemble)

Hallmark of local transport: anomalous broad distribution: 

Would be confirmed by data collapse of moments $< s^n >$

\[ p(s|t) = \frac{s}{3} \left( \frac{b}{t} \right)^3 \exp \left[ \frac{-bs^\beta}{t} \right] \] , $\beta = 2/3$

\begin{itemize}
  \item Northern release
    \begin{itemize}
      \item $<s^4(t)>^{1/4}$
      \item $<s^2(t)>^{1/2}$
      \item $<s(t)>$
      \item $<s^0.5(t)>^2$
    \end{itemize}
  \item Southern release
    \begin{itemize}
      \item $<s^4(t)>^{1/4}$
      \item $<s^2(t)>^{1/2}$
      \item $<s(t)>$
      \item $<s^0.5(t)>^2$
    \end{itemize}
\end{itemize}

$\rightarrow$ no intermittent Richardson dynamic

(Probably)

No
Conclusions

• The Lagrangian squared pair separations \( < s^2 > \) averaged over all trajectories show a nonlocal behavior for the first 4 days followed by a (local) Richardson regime \( (t^3\text{- law}) \) and a diffusive regime \( < s^2 > \sim t \).

• In contrast the Finite Size Lyapunov-exponents (FSLE) \( \lambda(\delta) \) are not constant for small \( \delta \), as expected for nonlocal transport, but decay as \( \delta^{-2/3} \).

• The inconsistency originates in the strong influence of the location of deployment leading to a mixture of dynamics:
  \( < s^2(t) > \) is dominated by the northern (inertial) release due to scale separation
  \( \lambda(\delta) \) reveals the small scale behavior \textit{within} the filament (southern release)

• While the anomalous PDF (northern release) supports local transport for \( s > R_1 \) the narrow PDF (southern release) for small \( s \) questions submesoscale local dispersion.

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Thank you!

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The moments $\langle s^n(t) \rangle^{1/n}$ can be derived analytically straightforwardly from the theoretical forecast for $P(s|t)$ for the local (5)) and the diffusive regime (7), leading to

$$\langle s^n(t) \rangle^{1/n} = \begin{cases} 
i_n(t/b)^{3/2} & \text{for } t < t_x(n) \\ k_n(2\kappa t)^{1/2} & \text{for } t > t_x(n). \end{cases} \quad (8)$$

Here $i_n = (0.5 \Gamma((n+2)/3))^{1/n}$ and $k_n = \Gamma(n+2)^{1/n}$ where $\Gamma(x)$ is the Gamma-function and with $b$ defined in Eq. (5).

As the transition time $t_x(n)$ has to fullfill $i_n(t_x(n)/b)^{3/2} = k_n(2\kappa t_x(n))^{1/2}$, it is given by

$$t_x(n) = (2\kappa b^3)^{1/2} \gamma_n \quad (9)$$

with $\gamma_n = k_n/i_n$. Equations (8) and (9) indicate hat the transition time $t_x(n)$ from the local to the diffusive dispersion regime differs for the different moments $\langle s^n(t) \rangle^{1/n}$. 

\[ \text{Titel, Autor, Datum} \]
Local submesoscale features within the filament?

→ no intermittent Richardson dynamic
## Mean squared pair separation: theoretical prediction

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<td>II. local: (Richardson) s ( \approx ) eddies</td>
<td>( &lt; s^2 &gt; = \epsilon t^3 ) ( \epsilon = \text{energy transfer rate} )</td>
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<td>III. diffusive s &gt; eddies</td>
<td>( &lt; s^2 &gt; = 2\kappa t ) ( \kappa = \text{relative diffusivity} )</td>
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\( < s^2 (t) > \): averaged squared separation of drifter-pairs, \( \lambda(\delta) \): FSLE
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