Systematic Decomposition of the MJO and its Northern Hemispheric Extra-Tropical Response into Rossby and Inertio-Gravity Components

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Outline

1) MJO

2) Normal Mode Functions

3) Climatology

4) MJO decomposition

5) Summary
Madden-Julian Oscillation

Precipitation

- Eastward propagating at 4-8 m/s
- 30-60 day oscillation

Source: NOAA
Madden-Julian Oscillation

Impacts of the MJO:
• North Atlantic weather regimes
• North Atlantic weather forecasts
• Tropical cyclones
• Tornado outbreaks
• North American west coast winter precipitation
• North American east coast cold air outbreaks
• ...

...
Madden-Julian Oscillation

MJO in CMIP5 climate models

Precipitation: Colors
850hPa U-Wind: Contours

The MJO is the "Holy Grail" of climate research

Ahn et al. 2017
Normal Mode Decomposition

Based on earlier work by Kasahara in the 1970s.

Normal-mode function representation of global 3-D data sets: open-access software for the atmospheric research community

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Normal Mode Decomposition

Linearized primitive equations:

$$\frac{\partial u'}{\partial t} - 2\Omega v' \sin(\varphi) = -\frac{g}{a \cos(\varphi)} \frac{\partial h'}{\partial \lambda},$$

$$\frac{\partial v'}{\partial t} + 2\Omega u' \sin(\varphi) = -\frac{g}{a} \frac{\partial h'}{\partial \varphi},$$

$$\frac{\partial}{\partial t} \left[ \frac{\partial}{\partial \sigma} \left( \frac{g \sigma}{R \Gamma_0} \frac{\partial h'}{\partial \sigma} \right) \right] - \nabla \cdot \mathbf{V}' = 0.$$
Normal Mode Decomposition

Separation into vertical and horizontal structure functions:

$$\begin{bmatrix} u', v', h' \end{bmatrix}^T(\lambda, \varphi, \sigma, t) = [u, v, h]^T(\lambda, \varphi, t) \times G(\sigma).$$

$$\frac{d}{d\sigma} \left( \frac{\sigma}{S} \frac{dG}{d\sigma} \right) + \frac{H^*}{D} G = 0,$$

$$\frac{\partial}{\partial t} W + L \cdot W = 0,$$

where $W$ denotes the vector dependent variable

$$W = (\tilde{u}, \tilde{v}, \tilde{h})^T$$
Normal Mode Decomposition

Horizontal Structure equations

\[ \frac{\partial u}{\partial t} - 2\Omega \sin\phi \ v = - \frac{g}{a \cos\phi} \ \frac{\partial h}{\partial \lambda}, \]

\[ \frac{\partial v}{\partial t} + 2\Omega \sin\phi \ u = - \frac{g}{a} \ \frac{\partial h}{\partial \phi}, \]

\[ \frac{\partial h}{\partial t} + \frac{D}{a \cos\phi} \left[ \frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \phi} (v \cos\phi) \right] = 0. \]
Normal Mode Decomposition

\( \mathbf{H} \) is eigenfunction of \( \mathbf{L} \)

\[ \mathbf{L} \, \mathbf{H}_l = i \nu_l \, \mathbf{H}_l , \]

Hough harmonic functions

\[ \mathbf{H}_n^k (\lambda, \phi) = \Theta_n^k (\phi) \exp(i k \lambda). \]

\[ \Theta_n^k (\phi) = \begin{pmatrix} U(\phi) \\ -i V(\phi) \\ Z(\phi) \end{pmatrix}, \]
Normal Mode Decomposition

Advantage of this approach:
• Mass and wind fields are in balance
Normal Mode Decomposition

Solving \( \frac{\partial}{\partial t} W + LW = 0 \),

leads to two dispersion relationships

- first kind: west- and eastward Inertio-Gravity waves
- inertial terms dominate Coriolis terms
- unbalanced flow

\[
\nu = \frac{-k}{2n(n+1)} \pm \left( -\frac{k^2}{4n^2(n+1)^2} + \frac{n(n+1)}{4a^2\Omega^2 gD} \right)
\]
Normal Mode Decomposition

Solving \( \frac{\partial}{\partial t} W + LW = 0 \),

leads to two dispersion relationships

- second kind: westward Rossby-Haurwitz waves
- Coriolis terms dominate
- balanced flow

\[ \nu = \frac{-2\Omega k}{n(n + 1)} \]
Equatorial Waves

Theoretical Dispersion Relationships for Shallow Water Modes on Eq. β Plane

Matsuno, 1966
Kelvin Wave

- Non-dispersive
- Balances Coriolis forces against a wave-guide

Fig. 11.15 Plan view of horizontal velocity and height perturbations associated with an equatorial Kelvin wave. (Adapted from Matsuno, 1966.)
Equatorial Rossby Wave
Inertio-Gravity Wave

Gravity waves which are affected by Coriolis force
Figure 4. Vertical structure functions for (a) the first seven vertical modes and (b) modes 10, 15, 20, 30, 40 50 and 60, derived using the 60 model levels of ERA Interim; (c) same as (a) but for the 21 model levels closest to the standard 21 pressure levels.
Climatology

![Graph showing energy distribution vs. zonal wave number]
MJO: Rossby Flow
MJO: Inertio-Gravity Flow
Contours correspond to the zonal winds every 0.5 m/s (blue for negative and red for positive speeds) and shades to the geopotential height (in meters).
MJO: Extra-Tropics

Rossby Flow
MJO: Extra-Tropics

Inertio-Gravity
Summary

• New tool to decompose flow fields into Rossby and Inertio-Gravity components
• Major IG MJO component is the Kelvin mode
• Rossby flow is more dominant for MJO
  • Rossby components 93% of kinetic energy
  • IG components: 7% of kinetic energy
• IG flow propagates also into the extra-tropics

Reference: