

# Systematic Decomposition of the MJO and its Northern Hemispheric Extra-Tropical Response into Rossby and Inertio-Gravity Components

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In collaboration with **Nedjeljka Zagar & Damjan Jelic** (Ljubljana, Slovenia),

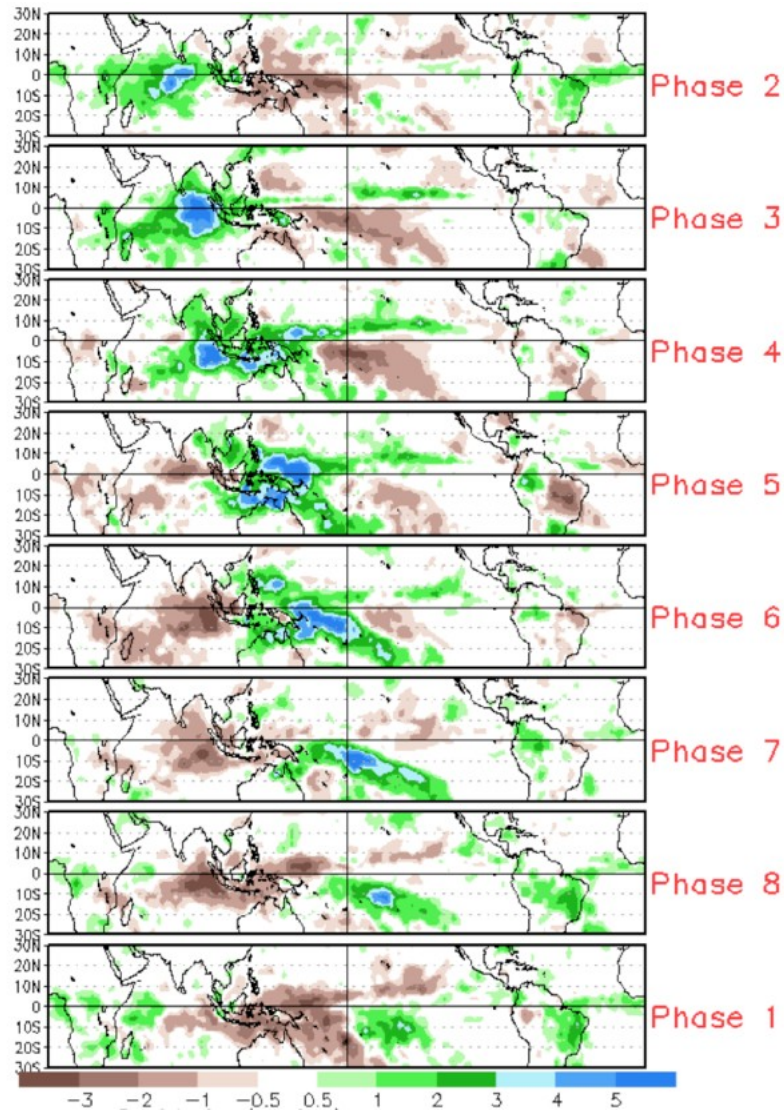
**Sukyong Lee & Steven B Feldstein** (PSU)

# Outline

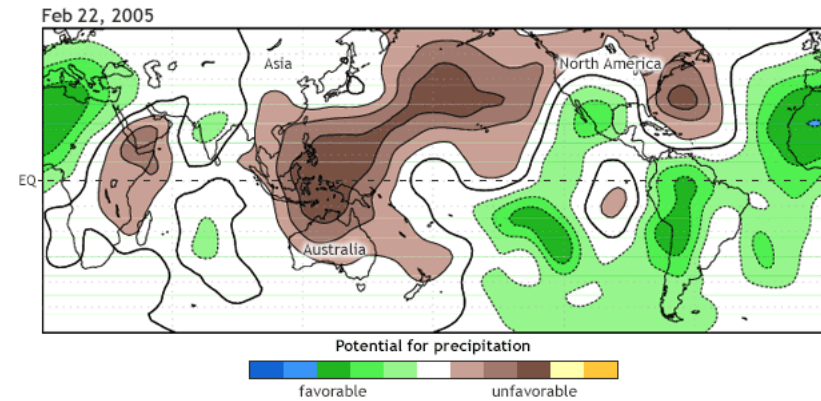
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- 1) MJO
- 2) Normal Mode Functions
- 3) Climatology
- 4) MJO decomposition
- 5) Summary

# Madden-Julian Oscillation



## Precipitation



- Eastward propagating at 4-8 m/s
- 30-60 day oscillation

Source:NOAA

# Madden-Julian Oscillation

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Impacts of the MJO:

- North Atlantic weather regimes
- North Atlantic weather forecasts
- Tropical cyclones
- Tornado outbreaks
- North American west coast winter precipitation
- North American east coast cold air outbreaks
- ...

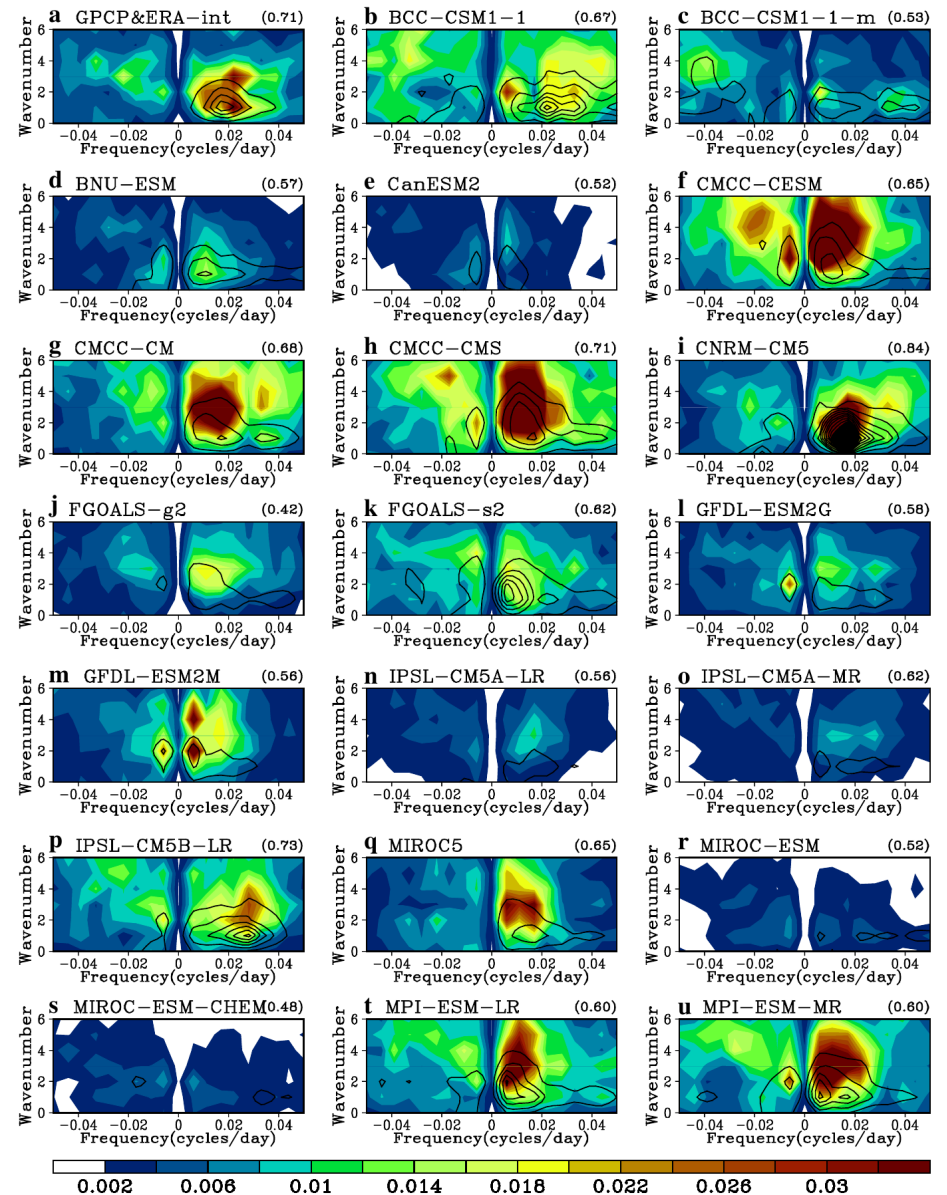


# Madden-Julian Oscillation

MJO in CMIP5 climate models

Precipitation: Colors  
850hPa U-Wind: Contours

The MJO is the “Holy Grail”  
of climate research



# Normal Mode Decomposition

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Geosci. Model Dev., 8, 1169–1195, 2015  
www.geosci-model-dev.net/8/1169/2015/  
doi:10.5194/gmd-8-1169-2015  
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## **Normal-mode function representation of global 3-D data sets: open-access software for the atmospheric research community**

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**Based on earlier work by Kasahara in the 1970s.**

# Normal Mode Decomposition

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Linearized primitive equations:

$$\frac{\partial u'}{\partial t} - 2\Omega v' \sin(\varphi) = -\frac{g}{a \cos(\varphi)} \frac{\partial h'}{\partial \lambda},$$

$$\frac{\partial v'}{\partial t} + 2\Omega u' \sin(\varphi) = -\frac{g}{a} \frac{\partial h'}{\partial \varphi},$$

$$\frac{\partial}{\partial t} \left[ \frac{\partial}{\partial \sigma} \left( \frac{g\sigma}{R\Gamma_0} \frac{\partial h'}{\partial \sigma} \right) \right] - \nabla \cdot \mathbf{V}' = 0.$$

# Normal Mode Decomposition

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Separation into vertical and horizontal structure functions:

$$[u', v', h']^T(\lambda, \varphi, \sigma, t) = [u, v, h]^T(\lambda, \varphi, t) \times G(\sigma).$$

$$\frac{d}{d\sigma} \left( \frac{\sigma}{S} \frac{dG}{d\sigma} \right) + \frac{H_*}{D} G = 0,$$

$$\frac{\partial}{\partial t} \mathbf{W} + \mathbf{LW} = 0,$$

where  $\mathbf{W}$  denotes the vector dependent variable

$$\mathbf{W} = (\tilde{u}, \tilde{v}, \tilde{h})^T$$

# Normal Mode Decomposition

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Horizontal Structure equations

$$\frac{\partial u}{\partial t} - 2\Omega \sin \phi v = -\frac{g}{a \cos \phi} \frac{\partial h}{\partial \lambda},$$

$$\frac{\partial v}{\partial t} + 2\Omega \sin \phi u = -\frac{g}{a} \frac{\partial h}{\partial \phi},$$

$$\frac{\partial h}{\partial t} + \frac{D}{a \cos \phi} \left[ \frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \phi} (v \cos \phi) \right] = 0.$$



# Normal Mode Decomposition

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$\mathbf{H}$  is eigenfunction of  $\mathbf{L}$

$$\mathbf{L} \mathbf{H}_l = i\nu_l \mathbf{H}_l ,$$

Hough harmonic functions

$$\mathbf{H}_n^k(\lambda, \phi) = \Theta_n^k(\phi) \exp(ik\lambda).$$

$$\Theta_n^k(\phi) = \begin{pmatrix} U(\phi) \\ -iV(\phi) \\ Z(\phi) \end{pmatrix},$$

# Normal Mode Decomposition

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Advantage of this approach:

- Mass and wind fields are in balance

# Normal Mode Decomposition

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Solving  $\frac{\partial}{\partial t} \mathbf{W} + \mathbf{LW} = 0$ ,

leads to two dispersion relationships

- first kind: west- and eastward Inertio-Gravity waves
  - inertial terms dominate Coriolis terms
  - unbalanced flow

$$\nu = \frac{-k}{2n(n+1)} \pm \left( -\frac{k^2}{4n^2(n+1)^2} + \frac{n(n+1)}{\frac{4a^2\Omega^2}{gD}} \right)$$

# Normal Mode Decomposition

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Solving  $\frac{\partial}{\partial t} \mathbf{W} + \mathbf{LW} = 0$ ,

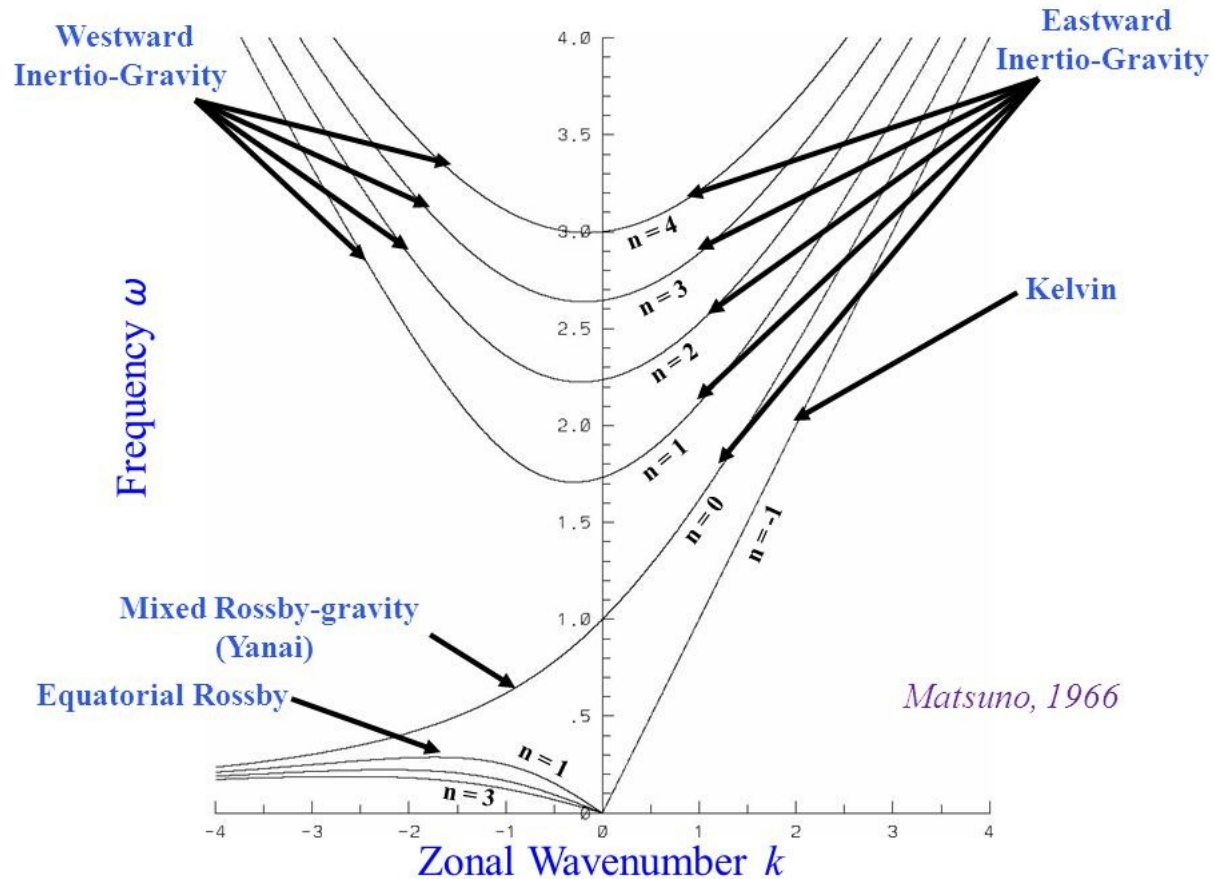
leads to two dispersion relationships

- second kind: westward Rossby-Haurwitz waves
  - Coriolis terms dominate
  - balanced flow

$$\nu = \frac{-2\Omega k}{n(n+1)}$$

# Equatorial Waves

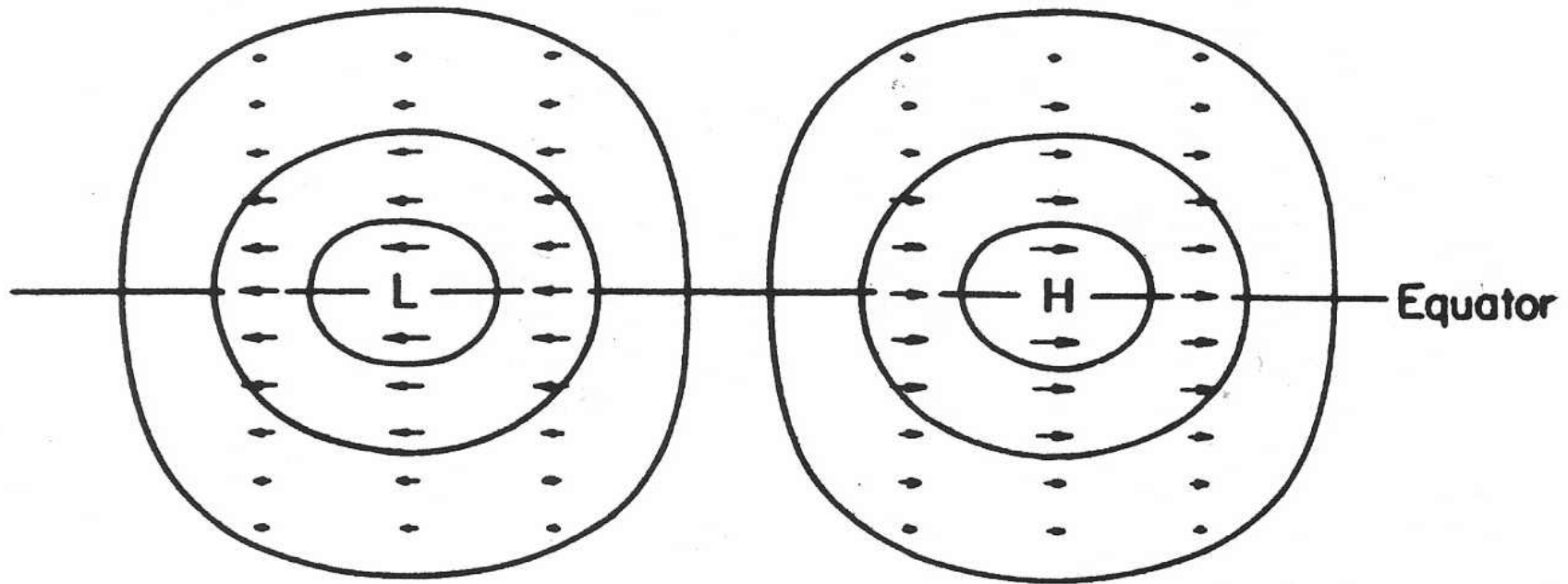
Theoretical Dispersion Relationships for Shallow Water Modes on Eq.  $\beta$  Plane





# Kelvin Wave

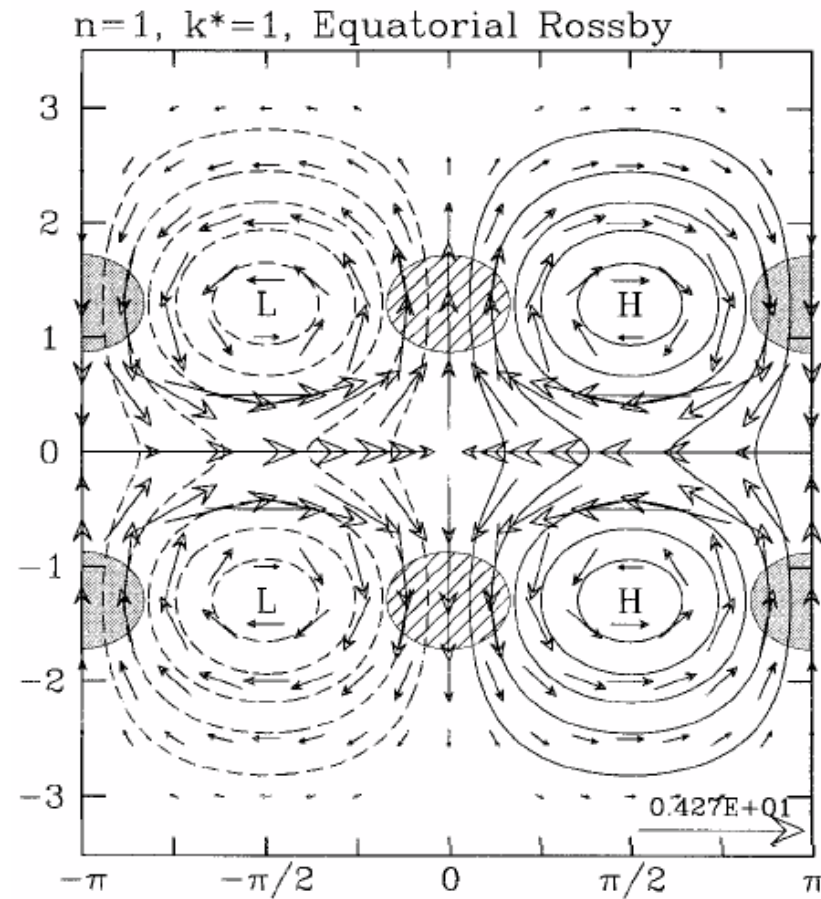
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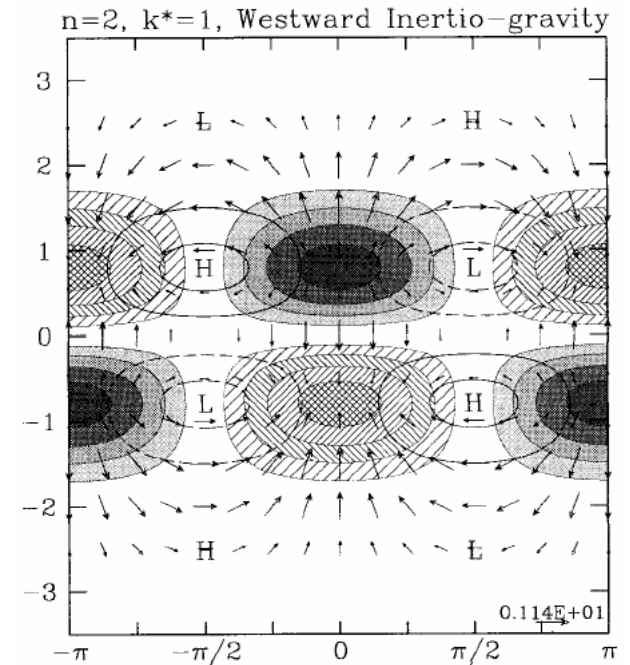
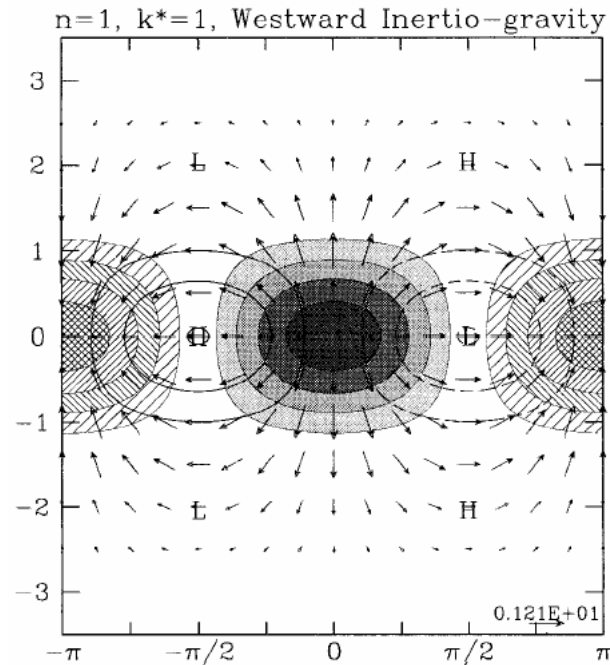
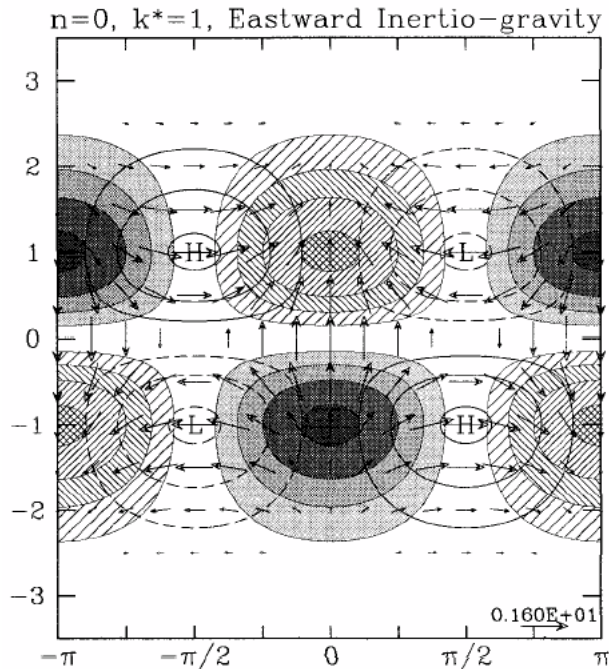
**fig. 11.15** Plan view of horizontal velocity and height perturbations associated with an equatorial Kelvin wave. (Adapted from Matsuno, 1966.)

- Non-dispersive
- Balances Coriolis forces against a wave-guide

# Equatorial Rossby Wave

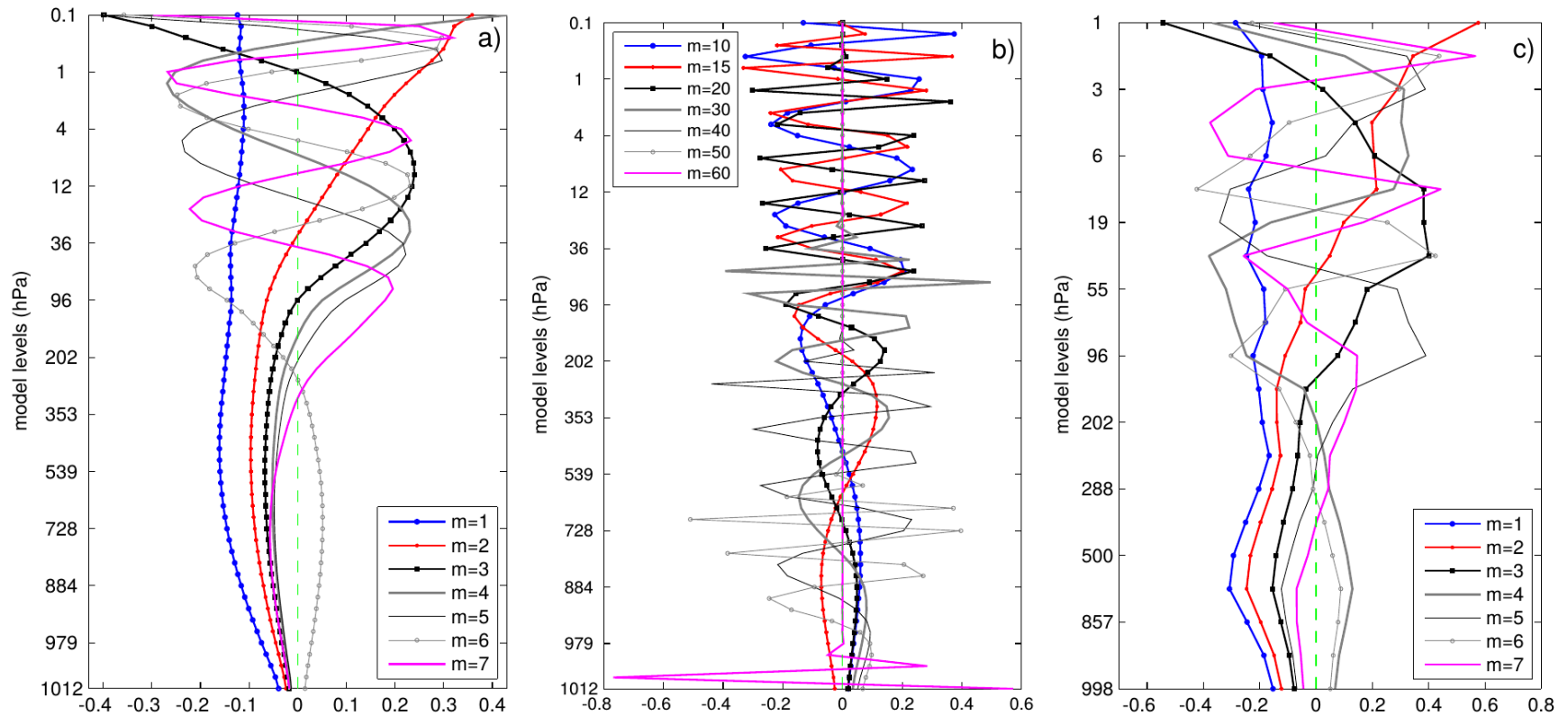


# Inertio-Gravity Wave



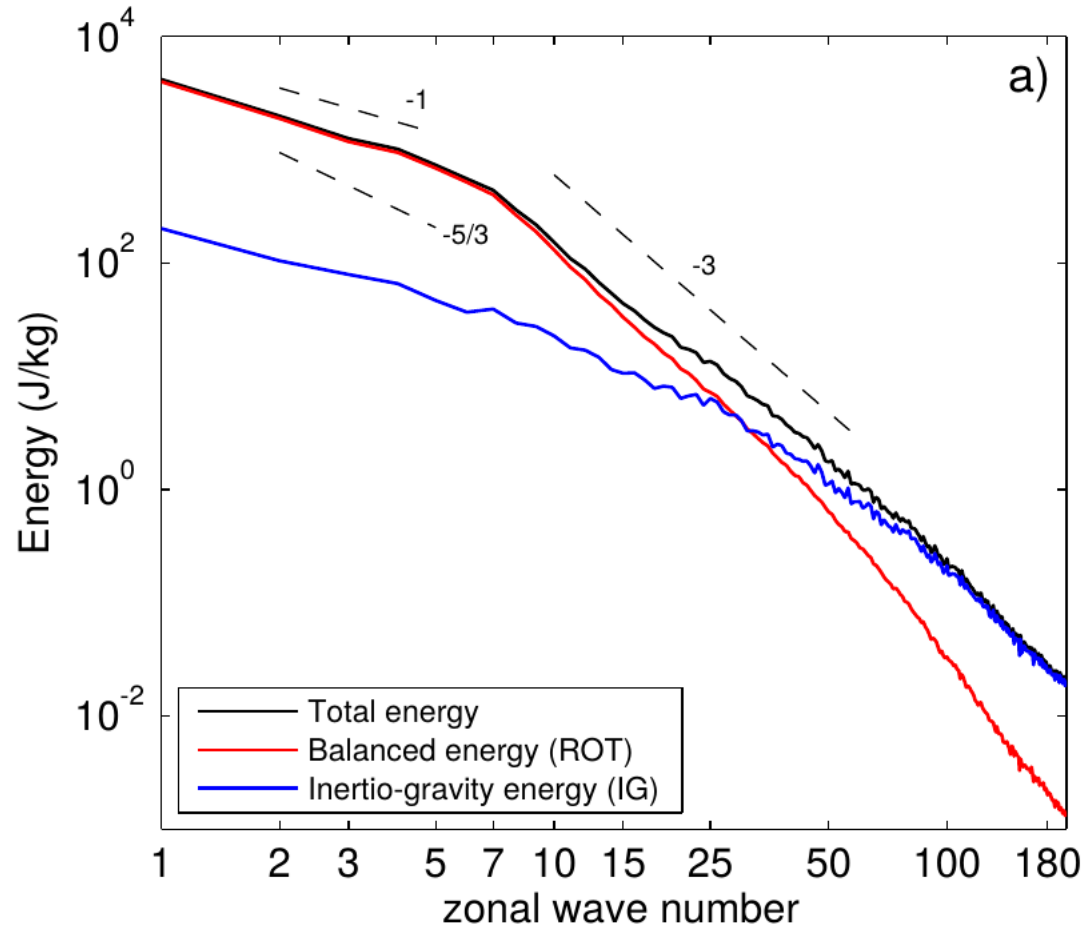
Gravity waves which are affected by Coriolis force

# Climatology



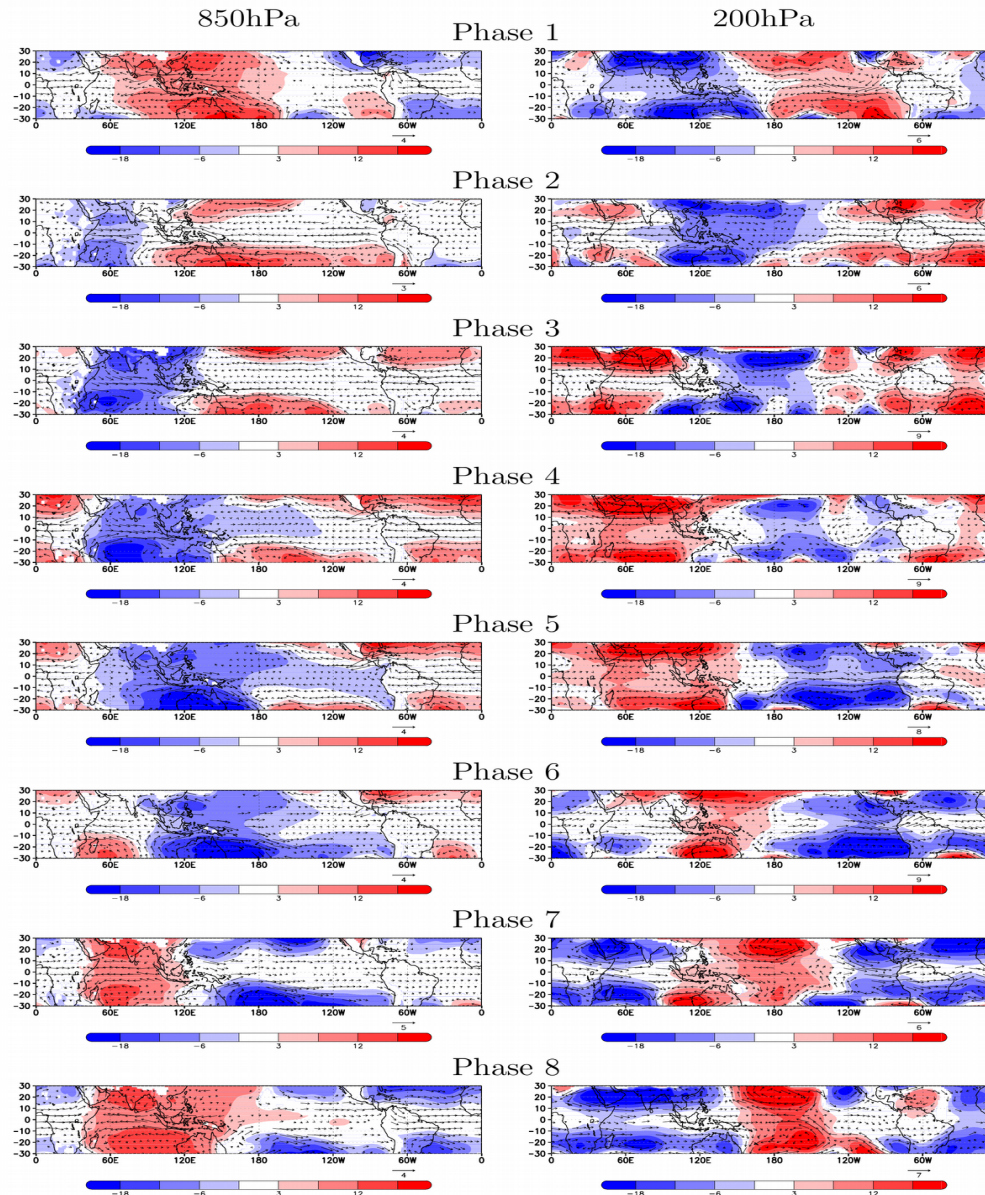
**Figure 4.** Vertical structure functions for (a) the first seven vertical modes and (b) modes 10, 15, 20, 30, 40 50 and 60, derived using the 60 model levels of ERA Interim; (c) same as (a) but for the 21 model levels closest to the standard 21 pressure levels.

# Climatology

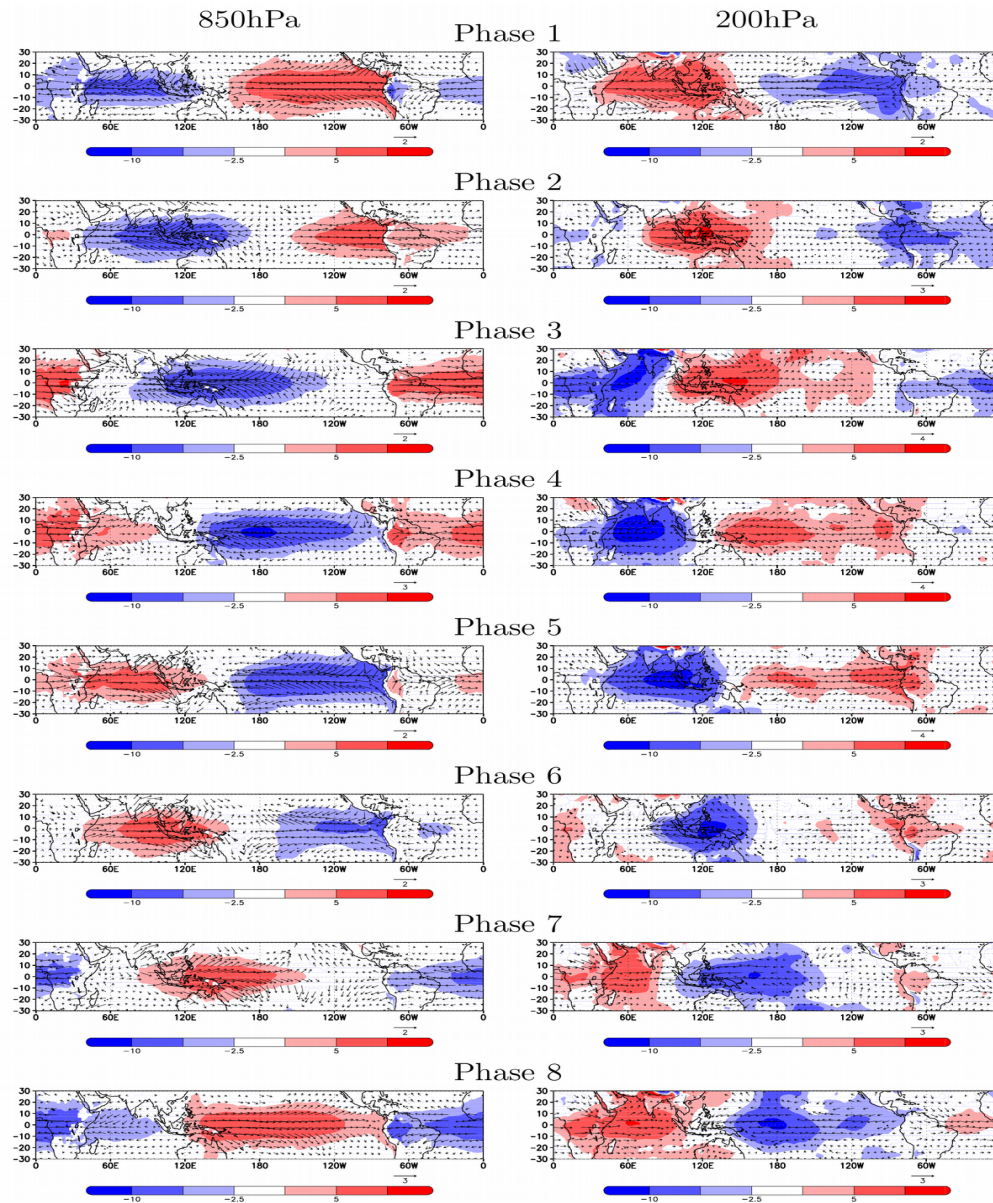




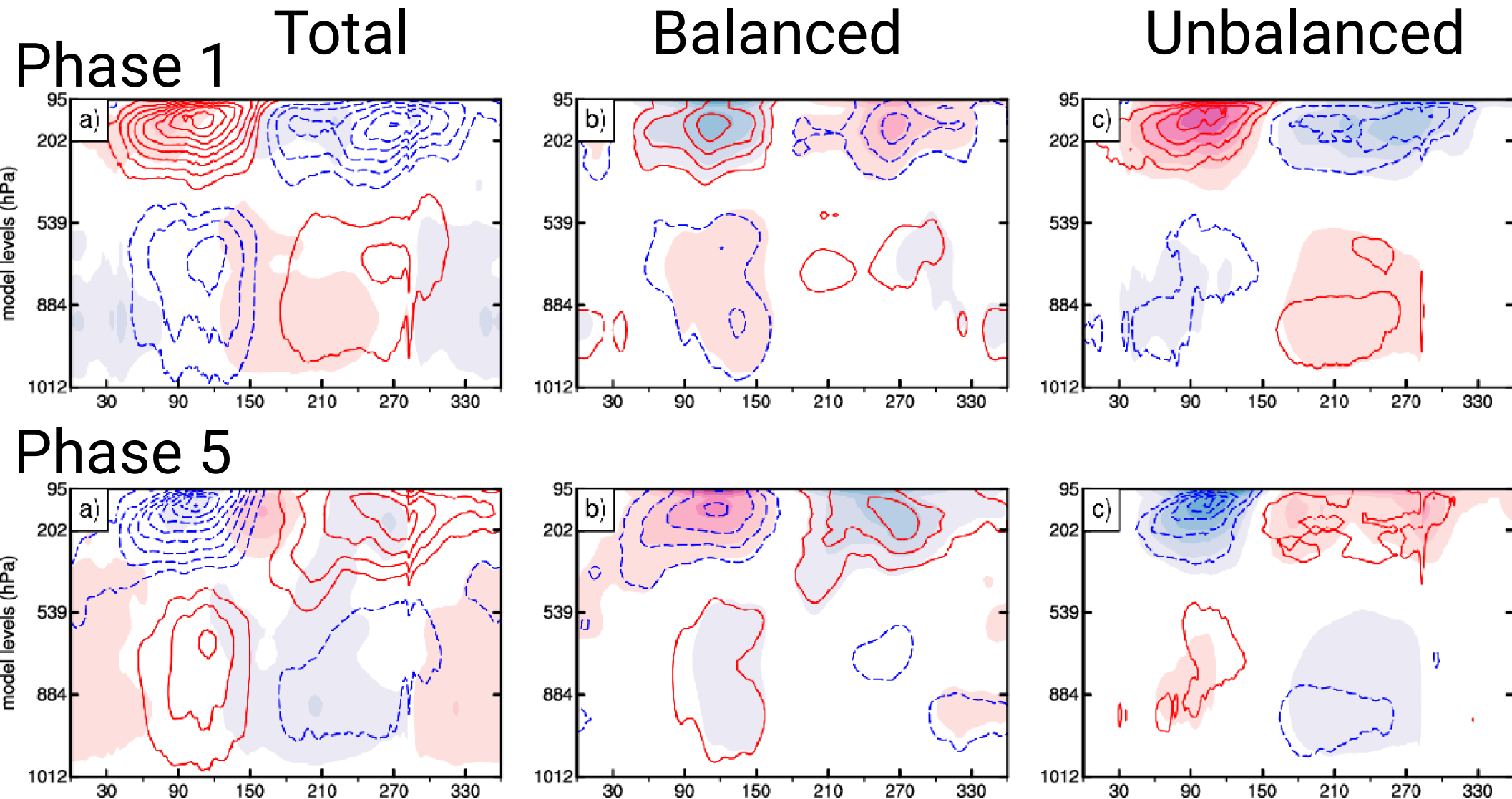
# MJO: Rossby Flow



# MJO: Inertio-Gravity Flow



# MJO

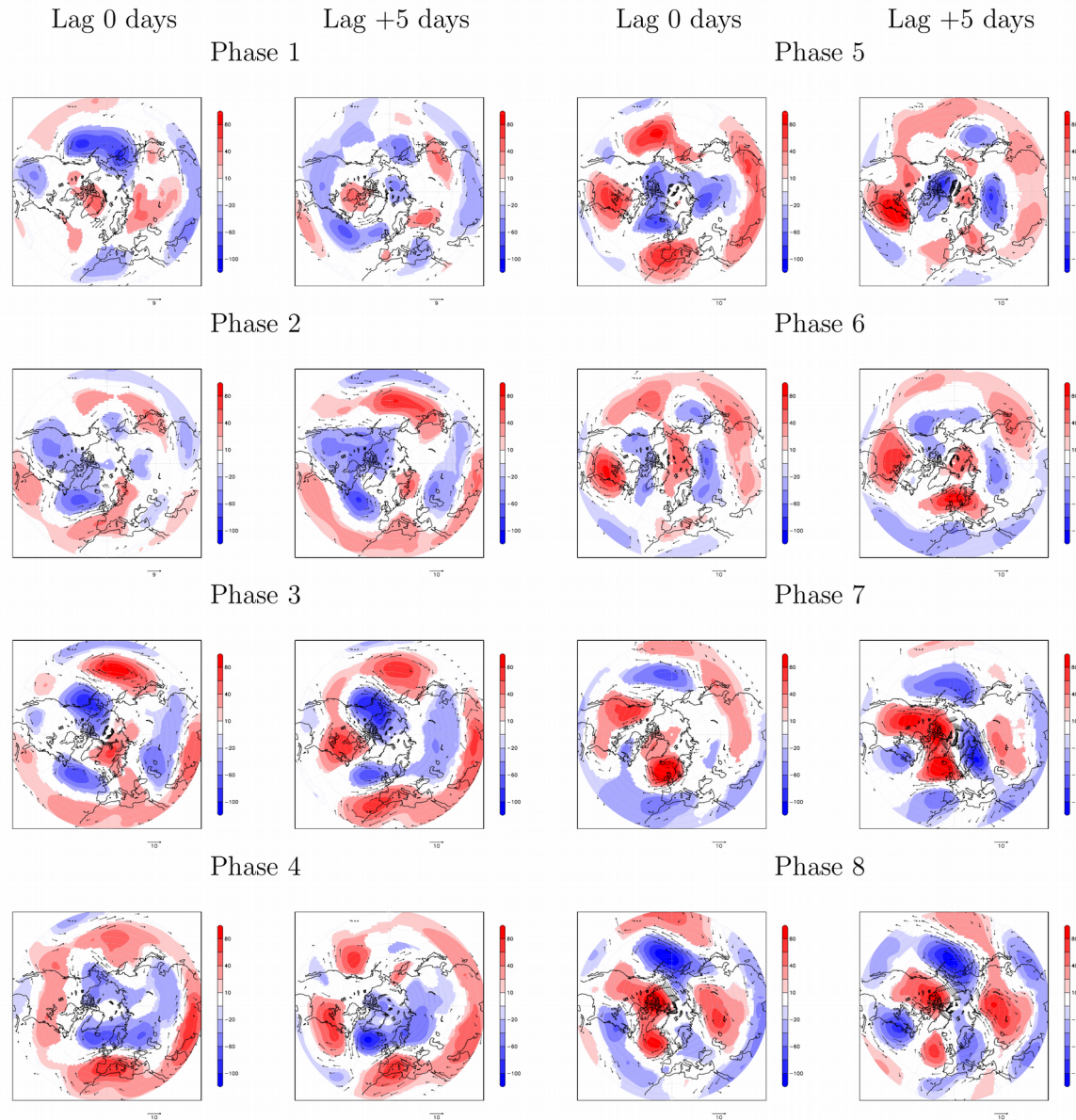


Contours correspond to the zonal winds every 0.5 m/s (blue for negative and red for positive speeds) and shades to the geopotential height (in meters).



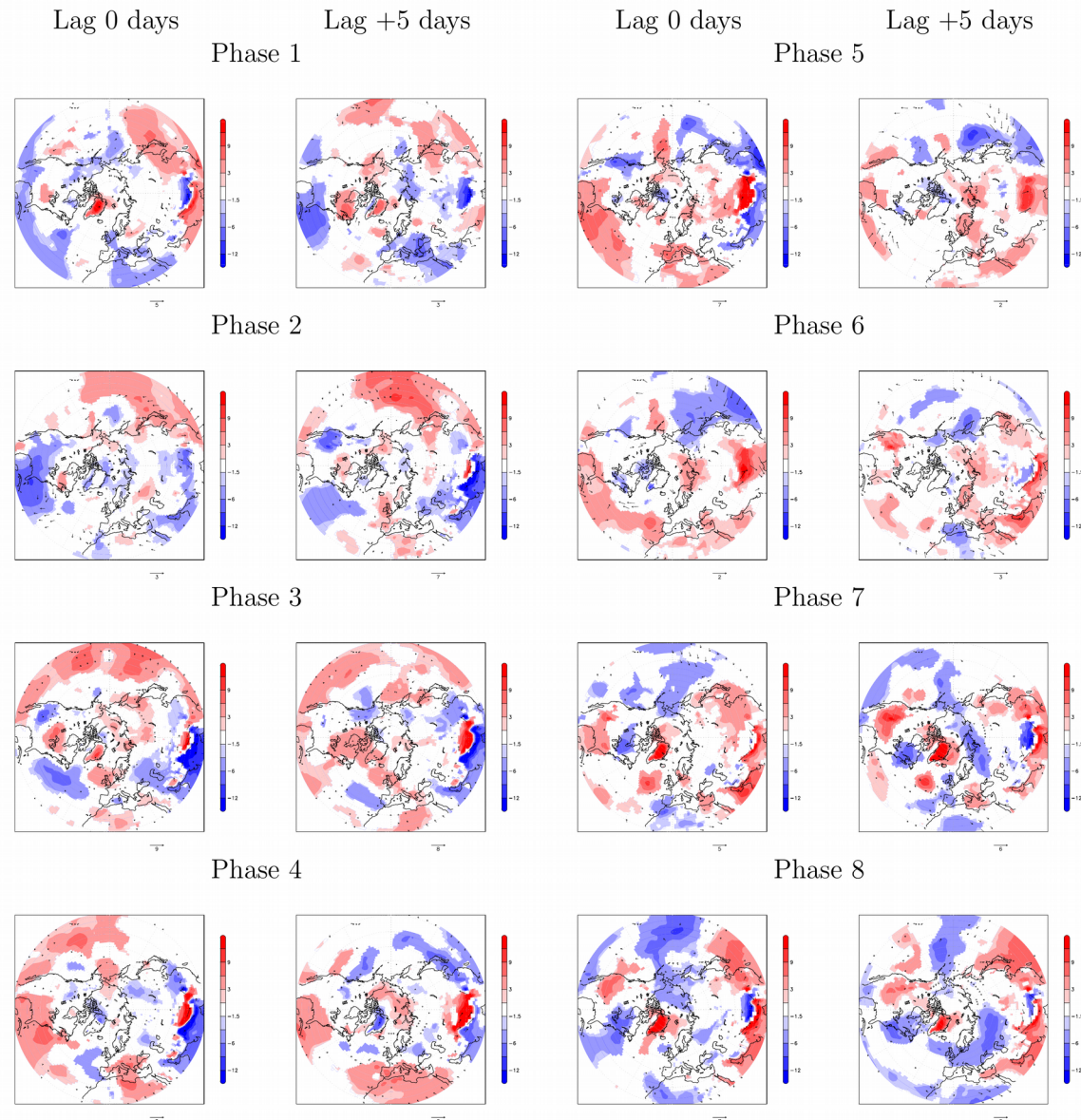
# MJO: Extra-Tropics

## Rossby Flow



# MJO: Extra-Tropics

## Inertio-Gravity





# Summary

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- New tool to decompose flow fields into Rossby and Inertio-Gravity components
- Major IG MJO component is the Kelvin mode
- Rossby flow is more dominant for MJO
  - Rossby components 93% of kinetic energy
  - IG components: 7% of kinetic energy
- IG flow propagates also into the extra-tropics

## Reference:

Franzke, C., D. Jelic, S. Lee and S. Feldstein, 2018: Systematic Decomposition of the MJO and its Northern Hemispheric Extra-Tropical Response into Rossby and Inertio-Gravity Components. Q. J. Roy. Meteorol. Soc., submitted.