Efficient modelling of the gravity-wave interaction with unbalanced resolved flows: Pseudo-momentum-flux convergence vs direct approach

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Inertia-Gravity Waves

- Long horizontal wavelength
- Short vertical wavelength
- Long intrinsic wave period
- Low vertical group velocity
- Affected by earth rotation
A Tale of Two Schemes
A Tale of Two Schemes

The WKB-Balance Scheme

The first scheme only parameterizes one single term:
1) Pseudomomentum flux convergence in the momentum equation

*Note that this idea is used in many current models*
The WKB-Balance Scheme

3D Case

\[
\frac{\partial \tilde{u}}{\partial t} = -\frac{1}{\tilde{\rho}} \frac{\partial}{\partial x} (\tilde{A}k \tilde{C}g_x) - \frac{1}{\tilde{\rho}} \frac{\partial}{\partial y} (\tilde{A}k \tilde{C}g_y) - \frac{1}{\tilde{\rho}} \frac{\partial}{\partial z} (\tilde{A}k \tilde{C}g_z) + B_u \tag{1}
\]

\[
\frac{\partial \tilde{v}}{\partial t} = -\frac{1}{\tilde{\rho}} \frac{\partial}{\partial x} (\tilde{A}l \tilde{C}g_x) - \frac{1}{\tilde{\rho}} \frac{\partial}{\partial y} (\tilde{A}l \tilde{C}g_y) - \frac{1}{\tilde{\rho}} \frac{\partial}{\partial z} (\tilde{A}l \tilde{C}g_z) + B_v \tag{2}
\]
The WKB-Balance Scheme

3D Case

\[
\frac{\partial \bar{\mathbf{u}}}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (A_k C g_x) - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (A_k C g_y) - \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (A_k C g_z) + B_u \tag{1}
\]

\[
\frac{\partial \bar{\mathbf{v}}}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (A_l C g_x) - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (A_l C g_y) - \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (A_l C g_z) + B_v \tag{2}
\]

Pseudomomentum Flux Convergence
The WKB-Balance Scheme

2.5D Wave Packet

\[
\begin{align*}
\frac{\partial \vec{u}}{\partial t} &= -\frac{1}{\rho} \frac{\partial}{\partial x} (A_k C_g x) - \frac{1}{\rho} \frac{\partial}{\partial y} (A_k C_g y) - \frac{1}{\rho} \frac{\partial}{\partial z} (A_k C_g z) + B_u \\
\frac{\partial \vec{v}}{\partial t} &= -\frac{1}{\rho} \frac{\partial}{\partial x} (A_l C_g x) - \frac{1}{\rho} \frac{\partial}{\partial y} (A_l C_g y) - \frac{1}{\rho} \frac{\partial}{\partial z} (A_l C_g z) + B_v
\end{align*}
\]

Pseudomomentum Flux Convergence
The WKB-Balance Scheme

2D Case

\[ \frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho} \frac{\partial}{\partial x} (AkCg_x) - \frac{1}{\rho} \frac{\partial}{\partial y} (AkCg_y) - \frac{1}{\rho} \frac{\partial}{\partial z} (AkCg_z) + B_u \] (1)

\[ \frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho} \frac{\partial}{\partial x} (AICg_x) - \frac{1}{\rho} \frac{\partial}{\partial y} (AICg_y) - \frac{1}{\rho} \frac{\partial}{\partial z} (AICg_z) + B_v \] (2)

Pseudomomentum Flux Convergence
The WKB-Balance Scheme

1D Wave Packet

\[
\frac{\partial \tilde{u}}{\partial t} = -\frac{1}{\rho} \frac{\partial}{\partial x} (A_k C g_x) - \frac{1}{\rho} \frac{\partial}{\partial y} (A_k C g_y) - \frac{1}{\rho} \frac{\partial}{\partial z} (A_k C g_z) + B_u \tag{1}
\]

\[
\frac{\partial \tilde{v}}{\partial t} = -\frac{1}{\rho} \frac{\partial}{\partial x} (A_l C g_x) - \frac{1}{\rho} \frac{\partial}{\partial y} (A_l C g_y) - \frac{1}{\rho} \frac{\partial}{\partial z} (A_l C g_z) + B_v \tag{2}
\]

Pseudomomentum Flux Convergence
A Tale of Two Schemes

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The WKB-Direct Scheme

The second scheme parameterizes the following three terms:
1) Momentum flux convergence in the momentum equation
2) Elastic term in the momentum equation
3) Horizontal entropy-flux convergence in the entropy equation
The WKB-Direct Scheme

3D Case

\[ \frac{\partial \bar{u}}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (\bar{\rho}' u') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (\bar{\rho}' v') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho}' w') - \frac{f}{g} v'b' + B_u \]  \hspace{1cm} (1)

\[ \frac{\partial \bar{v}}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (\bar{\rho}' v'u') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (\bar{\rho}' v'v') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho}' v'w') + \frac{f}{g} u'b' + B_v \]  \hspace{1cm} (2)

\[ \frac{\partial \bar{\theta}}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (\bar{\rho}' u'\theta') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (\bar{\rho}' v'\theta') + B_\theta \]  \hspace{1cm} (3)
The WKB-Direct Scheme

3D Case

\[
\begin{align*}
\frac{\partial \bar{u}}{\partial t} &= -\frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (\bar{\rho} \bar{u}' \bar{u}') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (\bar{\rho} \bar{u}' \bar{v}') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} \bar{u}' \bar{w}') - \frac{f}{g} \bar{v}' \bar{b}' + B_u \\
\frac{\partial \bar{v}}{\partial t} &= -\frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (\bar{\rho} \bar{v}' \bar{u}') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (\bar{\rho} \bar{v}' \bar{v}') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} \bar{v}' \bar{w}') + \frac{f}{g} \bar{u}' \bar{b}' + B_v \\
\frac{\partial \bar{\theta}}{\partial t} &= -\frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (\bar{\rho} \bar{u}' \bar{\theta}') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (\bar{\rho} \bar{v}' \bar{\theta}') + B_\theta
\end{align*}
\]
The WKB-Direct Scheme

2.5D Wave Packet

\[
\frac{\partial \tilde{u}}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (\bar{\rho} \bar{u}' \bar{u}') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (\bar{\rho} \bar{u}' \bar{v}') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} \bar{u}' \bar{w}') - \frac{f}{g} \bar{v}' \bar{b}' + B_u \tag{1}
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\frac{\partial \tilde{v}}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (\bar{\rho} \bar{v}' \bar{u}') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (\bar{\rho} \bar{v}' \bar{v}') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} \bar{v}' \bar{w}') + \frac{f}{g} \bar{u}' \bar{b}' + B_v \tag{2}
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\]

Momentum Flux Convergence

Elastic Term

Heating Term
The WKB-Direct Scheme

2D Case

\[\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (\bar{\rho} u' u') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (\bar{\rho} u' v') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} u' w') - \frac{f}{g} v'b' + B_u \quad (1)\]

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1D Wave Packet

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(1)

\[ \frac{\partial \bar{v}}{\partial t} = \frac{1}{\bar{\rho}} \frac{\partial}{\partial x} (\bar{\rho} v' u') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial y} (\bar{\rho} v' v') - \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} v' w') + \frac{f}{g} u' b' + B_v \]  

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(3)

Momentum Flux Convergence  Elastic Term  Heating Term
MF versus PMF

\[
\begin{align*}
\bar{\rho}u'u' &= MF_{xx} = AkCg_x \times \left( 1 - \frac{1 + (l/k)^2}{1 - (\Omega/f)^2} \right) = PMF_{xx} \times \frac{1}{\gamma_{xx}} \quad (6) \\
\bar{\rho}u'v' &= MF_{xy} = AkCg_y = PMF_{xy} \quad (7) \\
\bar{\rho}u'w' &= MF_{xz} = AkCg_z \times \left( \frac{1}{1 - (f/\Omega)^2} \right) = PMF_{xz} \times \frac{1}{\gamma_{xz}} \quad (8) \\
\bar{\rho}v'u' &= MF_{yx} = AlCg_x = PMF_{yx} \quad (9) \\
\bar{\rho}v'v' &= MF_{yy} = AlCg_y \times \left( 1 - \frac{1 + (k/l)^2}{1 - (\Omega/f)^2} \right) = PMF_{yy} \times \frac{1}{\gamma_{yy}} \quad (10) \\
\bar{\rho}v'w' &= MF_{yz} = AlCg_z \times \left( \frac{1}{1 - (f/\Omega)^2} \right) = PMF_{yz} \times \frac{1}{\gamma_{yz}} \quad (11)
\end{align*}
\]

\[\begin{array}{c}
\text{Momentum Flux (MF)} \\
\geq \\
\text{Pseudomomentum Flux (PMF)}
\end{array}\]
MF versus PMF

\[ \overline{\rho u' u'} = MF_{xx} = Ak C g_x \times \left(1 - \frac{1 + (l/k)^2}{1 - (\Omega/f)^2}\right) = PMF_{xx} \times \frac{1}{\gamma_{xx}} \] (6)

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Momentum Flux (MF) ≥ Pseudomomentum Flux (PMF)

Fig. The horizontal wavelength space distribution of GAMMAs.
Other Fluxes

\[ \bar{\rho} u' b' = \frac{Amf N^2}{\Omega(k^2 + l^2 + m^2)} \times l \]

\[ \bar{\rho} v' b' = \frac{Amf N^2}{\Omega(k^2 + l^2 + m^2)} \times (-k) \]

\[ u' \theta' = \frac{\bar{\theta}}{g} u' b' \]
A Tale of Two Schemes

The WKB-Balance Scheme

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Note that this idea is used in many current models

The WKB-Direct Scheme

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• The WKB-Balance Scheme is obtained based on the assumption that the large-scale flow satisfies the geostrophic and hydrostatic balance.

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*No assumption for the large-scale flow*
Reference for WKB-Balance Scheme

Theory

Reference for WKB-Balance Scheme

Theory


Application in the Numerical Work


Reference for WKB-Direct Scheme

Theory


Reference for WKB-Direct Scheme

**Theory**


**Application in the Numerical Work**
Reference for WKB-Direct Scheme

Theory


Application in the Numerical Work
Two Models Used in the Numerical Investigation

A Gravity Wave Parameterization Model

• This model is based on the phase-space Wentzel–Kramers–Brillouin (WKB) ray tracing theory.

• Compared with the standard ray tracers, the advantage of the proposed parameterization model is that it avoids caustic-like situations.
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Large Eddy Simulation

• The LES is used as a reference for the parameterization.

• In contrast to the WKB simulations, the reference LES is fully nonlinear and enables a relatively realistic description of wave–wave interactions as well as turbulent wave dissipation.
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Experiment Designs

1D Wave Packet

2D Case

2.5D Wave Packet
Numerical Results Based on a 1D Prescribed Wave Packet: Control Run

Idealized thermodynamics profile based on isothermal atmosphere

Comparison of three forcing terms in Control Run

- Horizontal wavelength = 300 km; Vertical wavelength = 1 km
- Sigma = 5 km (vertical wave packet scale)
Numerical Results Based on a 1D Prescribed Wave Packet: Sensitivity Experiment

Sensitivity EXP to the change of horizontal wavelength

(a) PMFC
Unit: m/s/s

(b) MFC

(c) ET

Sensitivity EXP to the change of vertical wave packet scale

(d) PMFC
Unit: m/s/s

(e) MFC

(f) ET
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For 1D wave packet, the differences between the two schemes are sensitive to the following two parameters:

1) the ratio between the horizontal wavelength and vertical wavelength
2) vertical wave packet scale
WKB-Direct vs WKB-Balance

Test case: one-dimensional wave packet with self-induced mean flow

1500 min after initialization

LES

WKB-Direct

WKB-Balance
<table>
<thead>
<tr>
<th>2D Case</th>
<th>LES Code</th>
<th>Phase-Space WKB Ray Tracing Modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model Domain</strong></td>
<td>Domain size in x direction: 9000 km; Domain size in z direction: 100 km; Number of grid points in x direction: 512; Number of grid points in z direction: 1000</td>
<td>Domain size in x direction: 9000 km; Domain size in z direction: 100 km; Number of grid points in x direction: 32; Number of grid points in z direction: 100</td>
</tr>
<tr>
<td><strong>Gravity Wave Characteristics</strong></td>
<td>Horizontal wavelength: 300 km; Vertical wavelength: 1 km; Amplitude factor: 0.5; Number of points per one horizontal wavelength: ~17; Number of points per one vertical wavelength: 10</td>
<td>Horizontal wavelength: 300 km; Vertical wavelength: 1 km; Amplitude factors: 0.5; Number of rays per vertical layer: 10; Number of rays per column: 20; Number of rays within $\Delta m$: 2</td>
</tr>
<tr>
<td><strong>Wave Packet</strong></td>
<td>The horizontal wave packet scale is 1500 km, and the vertical wave packet scale is 5 km. The wave packet center is at 30 km altitude.</td>
<td></td>
</tr>
<tr>
<td><strong>Background</strong></td>
<td>Isothermal atmosphere with $T=210K$; The Coriolis parameter is 0.0001 per second.</td>
<td></td>
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</tbody>
</table>

Experiments designed for the high-resolution LES and the coarse-resolution model with gravity wave parameterizations based on phase-space WKB ray tracing.
Wave-Induced Forcing at the Initial Condition

Budget Analysis for the 2D case

- Same Gamma parameter for both MFCxx and MFCxz
- For the zonal momentum equation, MFCxx is comparable with MFCxz, and it is important in this case. ETx appears to be secondary but not negligible.
- No forcing in the meridional momentum equation.
- No wave-induced heating term
WKB-Direct vs WKB-Balance

Test case: two-dimensional case with self-induced mean flow

1500 min after initialization

LES

WKB-Direct

WKB-Balance
# 2.5D Wave Packet

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<tr>
<td>Domain size in x direction: 9000 km; Domain size in y direction: 300 km; Domain size in z direction: 100 km; Npts_Xdir = 512; Npts_Ydir = 16; Npts_Zdir = 1000</td>
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| **Gravity Wave Characteristics** | **Zonal wavelength:** 300 km; **Meridional wavelength:** 300 km; Vertical wavelength: 1 km; Amplitude factor: 0.5; Npts per one zonal wavelength: ~17 Npts per one meri. wvlength: 16 Npts per one vert. wavelength: 10 | **Zonal wavelength:** 300 km; **Meridional wavelength:** 300 km; Vertical wavelength: 1 km; Amplitude factors: 0.5; Number of rays per vertical layer: 10; Number of rays per column: 20; Number of rays within Δm: 2 |

| **Wave Packet** | The **zonal** wave packet scale is 1500 km, and the vertical wave packet scale is 5 km. The wave packet center is at 30 km altitude. |

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Experiments designed for the high-resolution LES and the coarse-resolution model with gravity wave parameterizations based on phase-space WKB ray tracing.
Budget Analysis for the 2D WP with 3D wavenumber vector

- The Gamma parameter, defined as the ratio between each corresponding PMF and MF, is shown in the top left corner if available.

- For the zonal momentum equation, MFCxx is comparable with MFCxz, and it is important in this case. ETx appears to be secondary but not negligible.

- Similar argument still holds in the meridional momentum equation.

- The effect of heating term will be tested by the ray tracing model (not shown).

Fig. (a-g) Cross section of each forcing term for the 2D WP case.
WKB-Direct vs WKB-Balance

Test case: 2.5D WP with self-induced mean wind

1500 min after initialization

LES

WKB-Direct

WKB-Balance
WKB-Direct vs WKB-Balance

Test case: 2.5D WP with initial high wave amplitude

1500 min after initialization

LES

WKB-Direct

WKB-Balance

\[ U \quad V \quad W \]
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Take-Home Notes on IGW Parameterizations

- Theory part: Two schemes for IGW parameterizations are introduced.
- Numerical experiments: With the LES as a reference, WKB-Direct Scheme is generally better than WKB-Balance Scheme.
- Sensitivity experiments
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- Sensitivity experiments
Zoom-In Plot at Initial Condition

The result from LES at 000000 hr

- Zonal velocity (unit: m/s)
- Wave energy per unit volume (unit: Pa)
- Change of zonal mean wind (unit: m/s)
- Change of meridional mean wind (unit: m/s)
The initial condition in LES

The result from LES at 0000000 hr

- Zonal velocity (unit: m/s) (CRS)
- Zonal velocity (unit: m/s) (HRZ)
- Wave energy per unit volume (unit: Pa) (CRS)
- Wave energy per unit volume (unit: Pa) (HRZ)

The change of zonal mean wind (unit: m/s) (CRS)
The change of meridional mean wind (unit: m/s) (CRS)
PMF versus MF
Three-Dimensional Case

\[
\bar{\rho}u'u' = AkCg_x \times \left(1 - \frac{1 + (l/k)^2}{1 - (\Omega/f)^2}\right)
\]

\[
\bar{\rho}u'v' = AkCg_y
\]

\[
\bar{\rho}u'w' = AkCg_z \times \left(\frac{1}{1 - (f/\Omega)^2}\right)
\]

\[
\bar{\rho}v'v' = AlCg_y \times \left(1 - \frac{1 + (k/l)^2}{1 - (\Omega/f)^2}\right)
\]

\[
\bar{\rho}v'w' = AlCg_z \times \left(\frac{1}{1 - (f/\Omega)^2}\right)
\]
PMF versus MF

3D Case

\[
\begin{align*}
\bar{\rho}u'u' &= AkCg_x \times \left(1 - \frac{1 + (l/k)^2}{1 - (\Omega/f)^2}\right) \\
\bar{\rho}u'v' &= AkCg_y \\
\bar{\rho}u'w' &= AkCg_z \times \left(\frac{1}{1 - (f/\Omega)^2}\right) \\
\bar{\rho}v'v' &= AlCg_y \times \left(1 - \frac{1 + (k/l)^2}{1 - (\Omega/f)^2}\right) \\
\bar{\rho}v'w' &= AlCg_z \times \left(\frac{1}{1 - (f/\Omega)^2}\right)
\end{align*}
\]
PMF versus MF

3D Case

\[
\bar{\rho}u'u' = AkCg_x \times \left(1 - \frac{1 + (l/k)^2}{1 - (\Omega/f)^2}\right)
\]

\[
\bar{\rho}u'v' = AkCg_y
\]

\[
\bar{\rho}u'w' = AkCg_z \times \left(1 - \frac{1}{1 - (f/\Omega)^2}\right)
\]

\[
\bar{\rho}v'v' = AlCg_y \times \left(1 - \frac{1 + (k/l)^2}{1 - (\Omega/f)^2}\right)
\]

\[
\bar{\rho}v'w' = AlCg_z \times \left(\frac{1}{1 - (f/\Omega)^2}\right)
\]
Figure The horizontal wavelength space distribution of (a) PMFxx, (b) MFxx, (c) GAMMAxx, (d) PMFxz, (e) MFxz, and (f) GAMMAxz.