**Efficient modelling of the gravity-wave interaction with unbalanced resolved flows:**

# Pseudo-momentum-flux convergence vs direct approach


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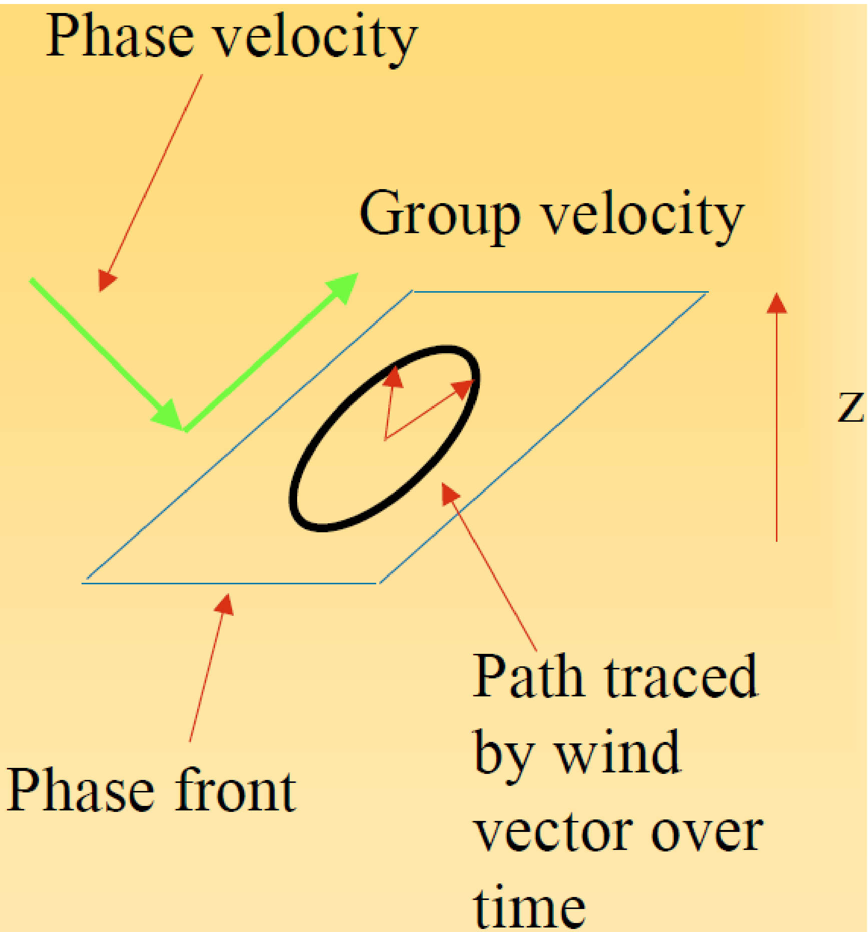
Fourth annual workshop on *“Scales and scaling cascades in geophysical systems”* in Hamburg during April 4-6, 2018

# Inertia-Gravity Waves




- **Long horizontal wavelength**
- **Short vertical wavelength**
- **Long intrinsic wave period**
- **Low vertical group velocity**
- **Affected by earth rotation**

# A Tale of Two Schemes

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## The WKB-Balance Scheme

The first scheme only parameterizes one single term:

1) Pseudomomentum flux convergence in the momentum equation

*Note that this idea is used in many current models*

# The WKB-Balance Scheme

## 3D Case

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\bar{\rho}}\frac{\partial}{\partial x}(AkCg_x) - \frac{1}{\bar{\rho}}\frac{\partial}{\partial y}\left(AkCg_y\right) - \frac{1}{\bar{\rho}}\frac{\partial}{\partial z}(AkCg_z) + B_u \qquad (1)$$

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**Pseudomomentum Flux Convergence**

# The WKB-Balance Scheme

## 2.5D Wave Packet



$$\frac{\partial \bar{u}}{\partial t} = \boxed{-\frac{1}{\bar{\rho}}\frac{\partial}{\partial x}(AkCg_x) - \cancel{\frac{1}{\bar{\rho}}\frac{\partial}{\partial y}(AkCg_y)} - \frac{1}{\bar{\rho}}\frac{\partial}{\partial z}(AkCg_z)} + B_u \qquad (1)$$

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MS-GWaves

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The second scheme parameterizes the following three terms:

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# The WKB-Direct Scheme

## 3D Case

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\bar{\rho}}\frac{\partial}{\partial x}\left(\bar{\rho}\overline{u'u'}\right) - \frac{1}{\bar{\rho}}\frac{\partial}{\partial y}\left(\bar{\rho}\overline{u'v'}\right) - \frac{1}{\bar{\rho}}\frac{\partial}{\partial z}\left(\bar{\rho}\overline{u'w'}\right) - \frac{f}{g}\overline{v'b'} + B_u \qquad (1)$$

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$$\frac{\partial \bar{\theta}}{\partial t} = -\frac{1}{\bar{\rho}}\frac{\partial}{\partial x}\left(\bar{\rho}\overline{u'\theta'}\right) - \frac{1}{\bar{\rho}}\frac{\partial}{\partial y}\left(\bar{\rho}\overline{v'\theta'}\right) + B_\theta \qquad (3)$$

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Momentum Flux Convergence

Elastic Term

Heating Term

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# MF versus PMF

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## MF versus PMF

$$\bar{\rho}\overline{u'u'} = MF_{xx} = AkCg_x \times \left(1 - \frac{1+(l/k)^2}{1-(\Omega/f)^2}\right) = PMF_{xx} \times \frac{1}{\gamma_{xx}} \quad (6)$$

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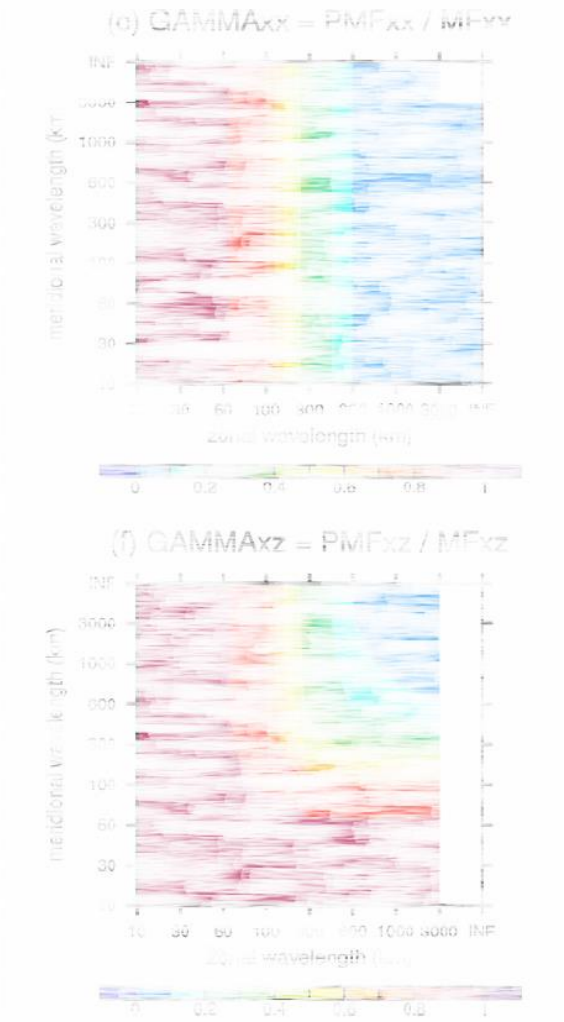


Fig. The horizontal wavelength space distribution of GAMMAs.

Momentum Flux (MF) ≥ Pseudomomentum Flux (PMF)

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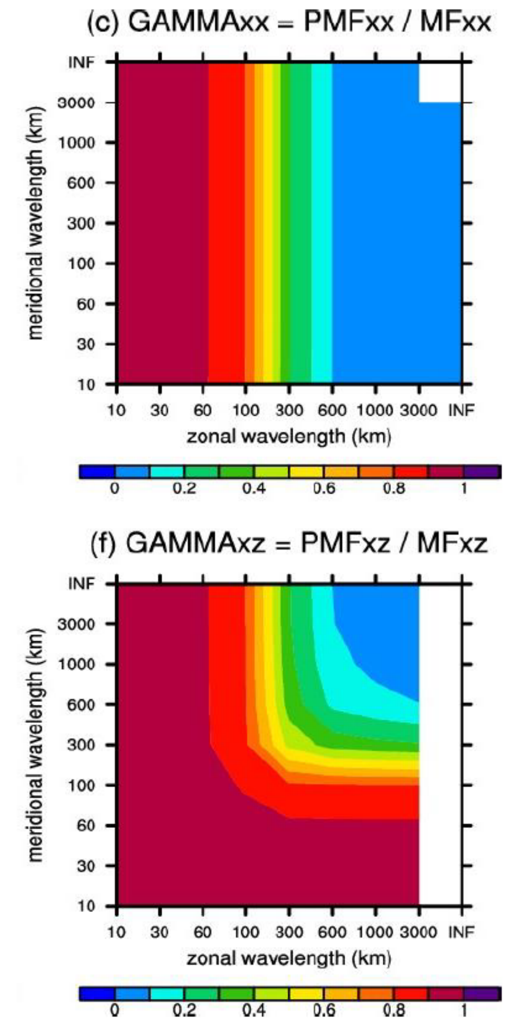


Fig. The horizontal wavelength space distribution of GAMMAs.

# Other Fluxes

$$\bar{\rho}\overline{u'b'} = \frac{AmfN^2}{\Omega(k^2 + l^2 + m^2)} \times l$$

$$\bar{\rho}\overline{v'b'} = \frac{AmfN^2}{\Omega(k^2 + l^2 + m^2)} \times (-k)$$

$$\overline{u'\theta'} = \frac{\bar{\theta}}{g}\overline{u'b'}$$

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- No assumption for the large-scale flow

# Reference for WKB-Balance Scheme

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## Theory

Andrews DG, McIntyre ME. 1976. Planetary waves in horizontal and vertical shear: The generalized Eliassen–Palm relation and the mean zonal acceleration. J. Atmos. Sci. 33: 2031–2048.

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## Application in the Numerical Work

Alexander MJ, Dunkerton TJ. 1999. A spectral parametrization of mean-flow forcing due to breaking gravity waves. J. Atmos. Sci. 56: 4167–4182.

Warner CD, McIntyre ME. 2001. An ultrasimple spectral parametrization for nonorographic gravity waves. J. Atmos. Sci. 58: 1837–1857.

Scinocca JF. 2002. The effect of back-reflection in the parametrization of non-orographic gravity-wave drag. J. Meteorol. Soc. Japan 80: 939–962.

Scinocca JF. 2003. An accurate spectral non-orographic gravity wave drag parameterization for general circulation models. J. Atmos. Sci. 60: 667–682.

Orr A, Bechtold P, Scinocca JF, Ern M, Janiskova M. 2010. Improved middle atmosphere climate and forecasts in the ECMWF model through a non-orographic gravity wave drag parametrization. J. Climate 23: 5905–5926.

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## Application in the Numerical Work

# Two Models Used in the Numerical Investigation

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## A Gravity Wave Parameterization Model

- This model is based on the phase-space Wentzel–Kramers–Brillouin (WKB) ray tracing theory.
- Compared with the standard ray tracers, the advantage of the proposed parameterization model is that it avoids caustic-like situations.

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## Large Eddy Simulation

- The LES is used as a reference for the parameterization.
- In contrast to the WKB simulations, the reference LES is fully nonlinear and enables a relatively realistic description of wave–wave interactions as well as turbulent wave dissipation.

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# Experiment Designs

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**1D Wave Packet**

↓

**2D Case**

↓

**2.5D Wave Packet**

# Numerical Results Based on a 1D Prescribed Wave Packet: Control Run

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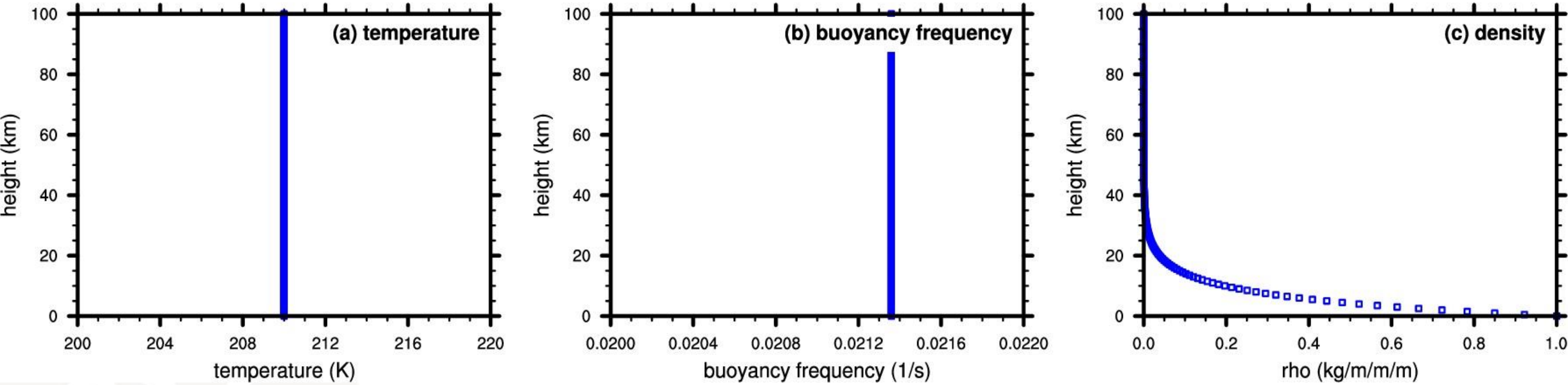
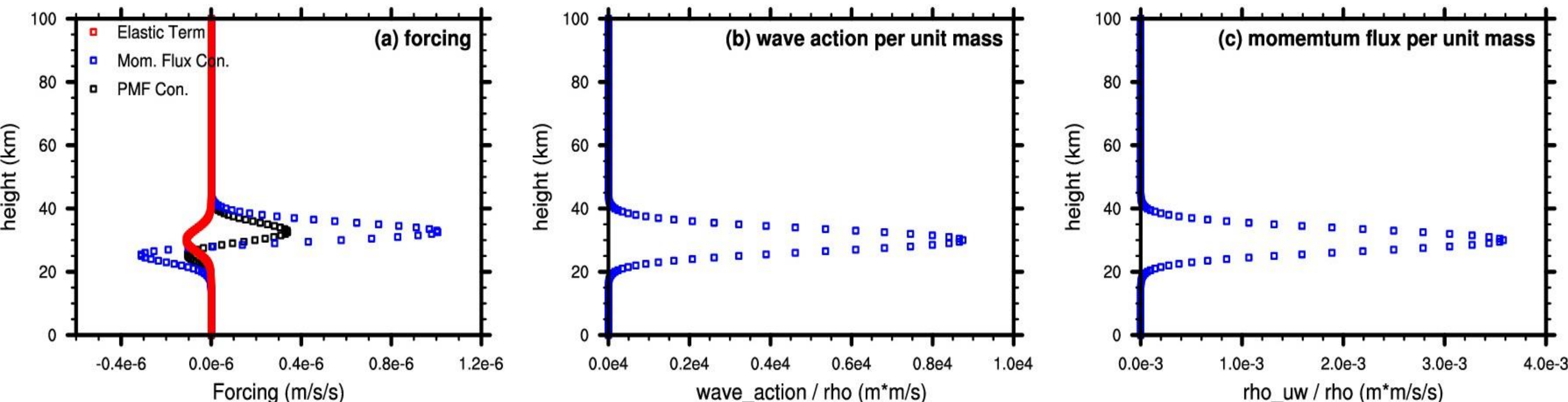
## Idealized thermodynamics profile based on isothermal atmosphere



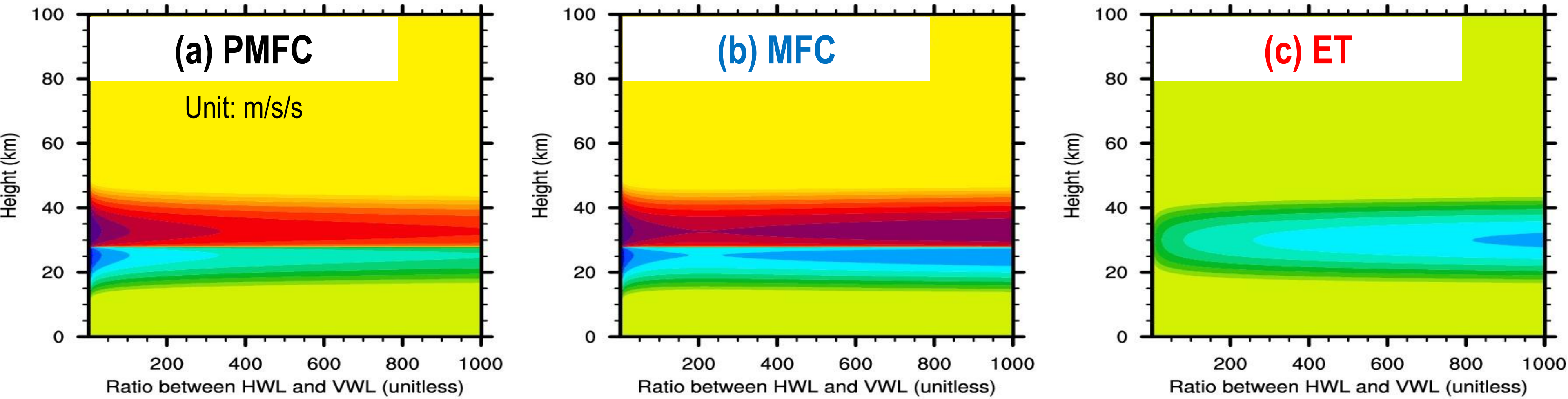

## Comparison of three forcing terms in Control Run

- Horizontal wavelength = 300 km; Vertical wavelength = 1 km
- Sigma = 5 km (vertical wave packet scale)

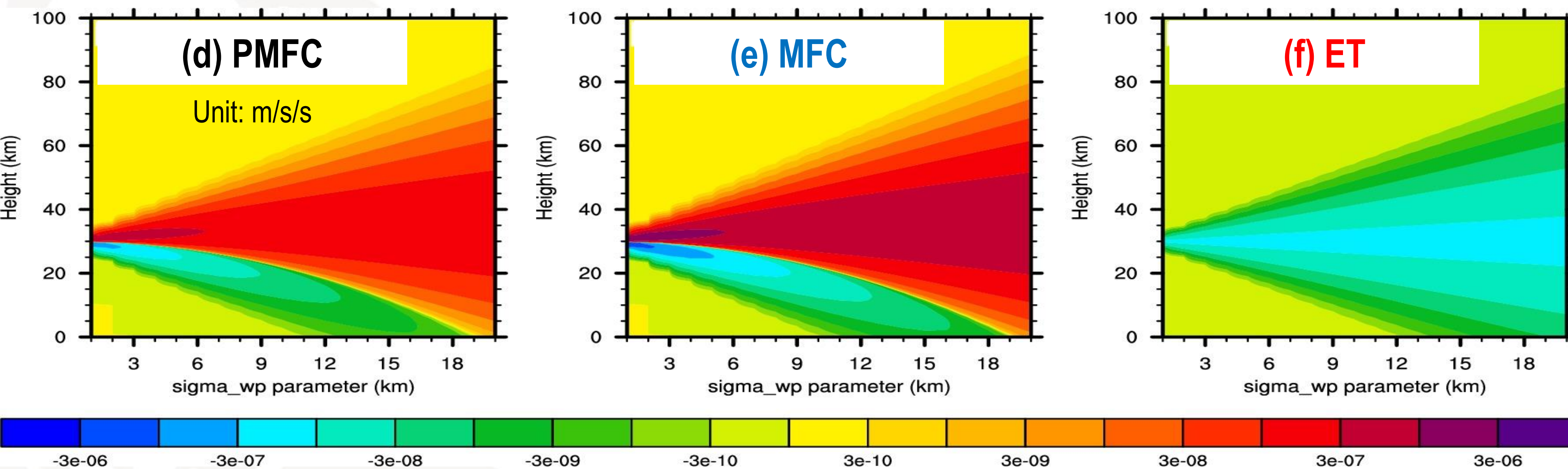


# Numerical Results Based on a 1D Prescribed Wave Packet: Sensitivity Experiment

## Sensitivity EXP to the change of horizontal wavelength




## Sensitivity EXP to the change of vertical wave packet scale



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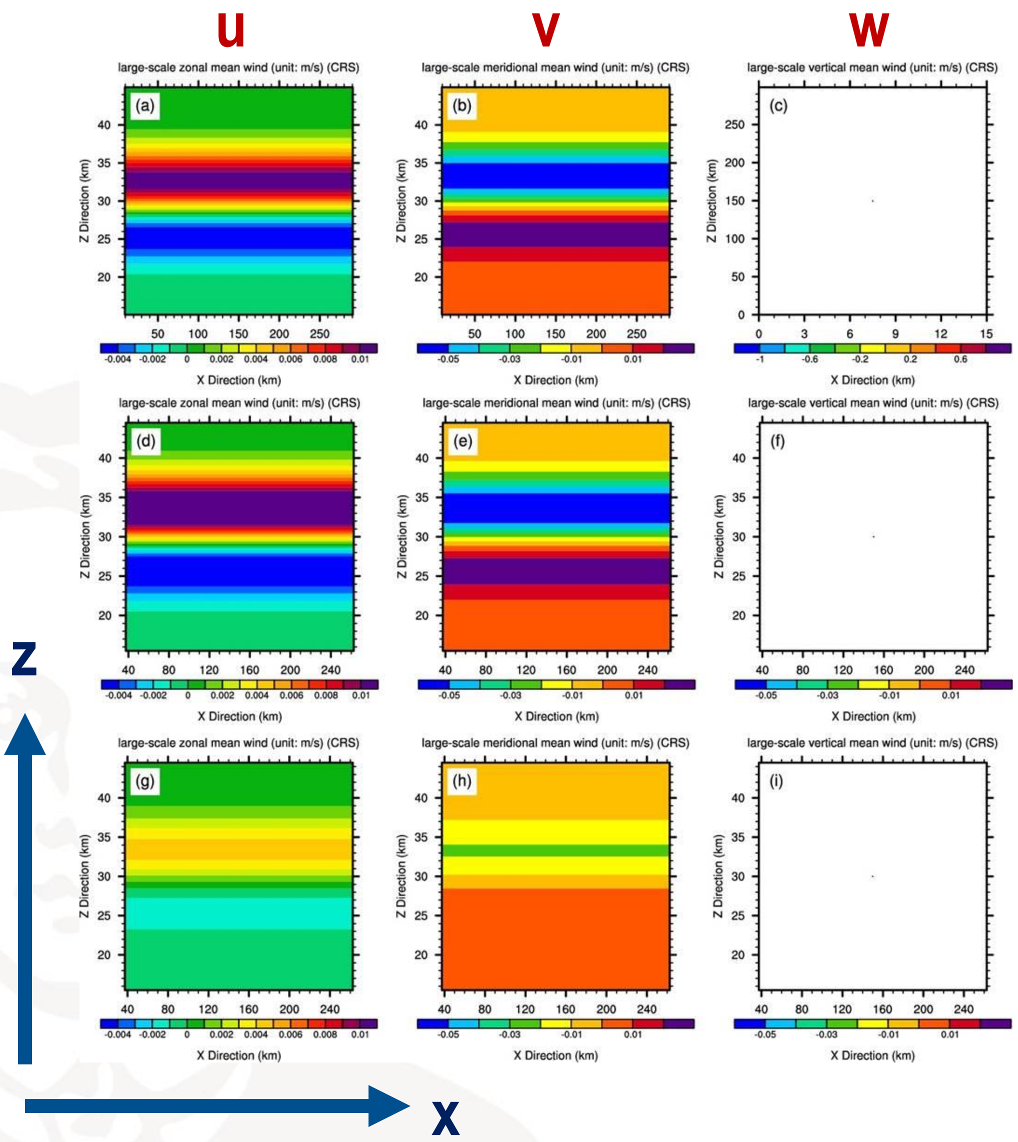
- For 1D wave packet, the differences between the two schemes are sensitive to the following two parameters:

  1) the ratio between the horizontal wavelength and vertical wavelength
  2) vertical wave packet scale

# WKB-Direct vs WKB-Balance

## Test case: one-dimensional wave packet with self-induced mean flow




1500 min after initialization

LES

WKB-Direct

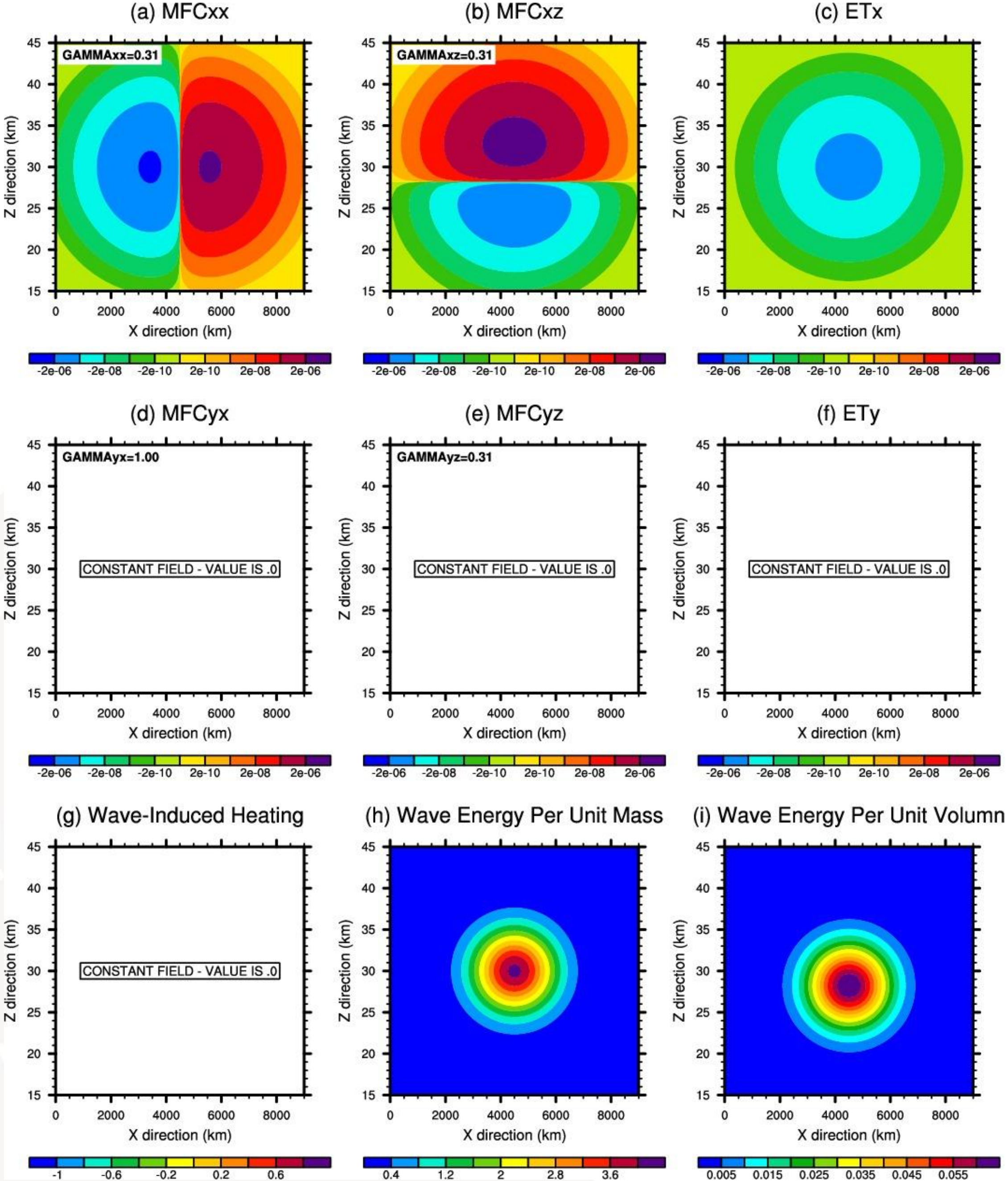
WKB-Balance

# 2D Case

| | LES Code | Phase-Space WKB Ray Tracing Modeling |
|---|---|---|
| **Model Domain** | Domain size in x direction: 9000 km; Domain size in z direction: 100 km; Number of grid points in x direction: 512; Number of grid points in z direction: 1000 | Domain size in x direction: 9000 km; Domain size in z direction: 100 km; Number of grid points in x direction: 32; Number of grid points in z direction: 100 |
| **Gravity Wave Characteristics** | Horizontal wavelength: 300 km; Vertical wavelength: 1 km; Amplitude factor: 0.5; Number of points per one horizontal wavelength: ~17; Number of points per one vertical wavelength: 10 | Horizontal wavelength: 300 km; Vertical wavelength: 1 km; Amplitude factors: 0.5; Number of rays per vertical layer: 10; Number of rays per column: 20; Number of rays within $\Delta m$: 2 |
| **Wave Packet** | The horizontal wave packet scale is 1500 km, and the vertical wave packet scale is 5 km. The wave packet center is at 30 km altitude. | |
| **Background** | Isothermal atmosphere with T=210K; The Coriolis parameter is 0.0001 per second. | |

**Experiments designed for the high-resolution LES and the coarse-resolution model with gravity wave parameterizations based on phase-space WKB ray tracing.**

# Wave-Induced Forcing at the Initial Condition

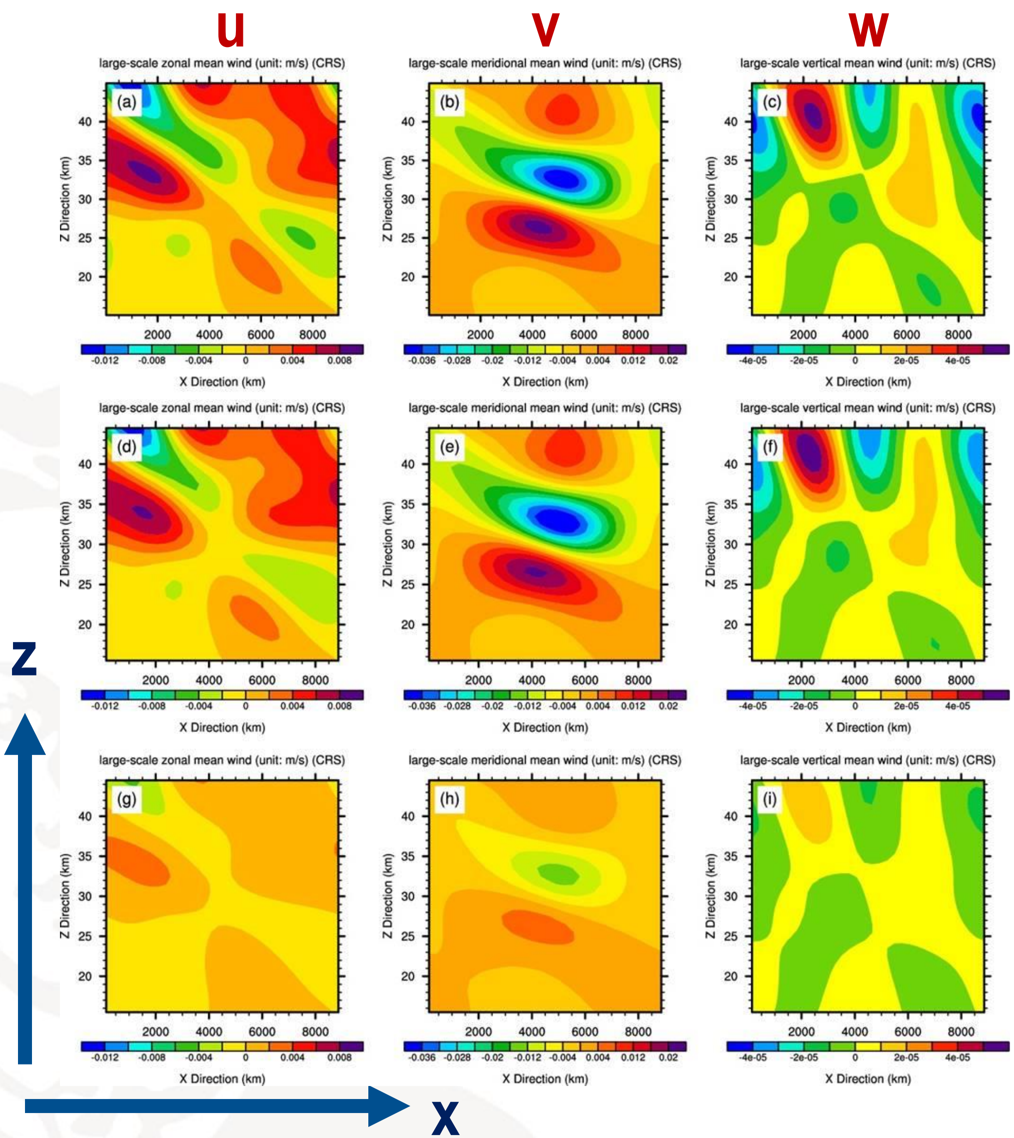



## Budget Analysis for the 2D case

- Same Gamma parameter for both MFCxx and MFCxz
- For the zonal momentum equation, MFCxx is comparable with MFCxz, and it is important in this case. ETx appears to be secondary but not negligible.
- No forcing in the meridional momentum equation.
- No wave-induced heating term

# WKB-Direct vs WKB-Balance

## Test case: two-dimensional case with self-induced mean flow




**1500 min after initialization**

**LES**

**WKB-Direct**

**WKB-Balance**

# 2.5D Wave Packet

| | LES Code | Phase-Space WKB Ray Tracing Modeling |
|---|---|---|
| **Model Domain** | Domain size in x direction: 9000 km;<br>Domain size in y direction: 300 km;<br>Domain size in z direction: 100 km;<br>Npts_Xdir = 512;<br>Npts_Ydir = 16;<br>Npts_Zdir = 1000 | Domain size in x direction: 9000 km;<br>Domain size in z direction: 100 km;<br>Npts_Xdir = 32;<br>Npts_Zdir = 100 |
| **Gravity Wave Characteristics** | Zonal wavelength: 300 km;<br>**Meridional wavelength: 300 km;**<br>Vertical wavelength: 1 km;<br>Amplitude factor: 0.5;<br>Npts per one zonal wavelength: ~17<br>Npts per one meri. wvlength: 16<br>Npts per one vert. wavelength: 10 | Zonal wavelength: 300 km;<br>**Meridional wavelength: 300 km;**<br>Vertical wavelength: 1 km;<br>Amplitude factors: 0.5;<br>Number of rays per vertical layer: 10;<br>Number of rays per column: 20;<br>Number of rays within $\Delta m$: 2 |
| **Wave Packet** | The **zonal** wave packet scale is 1500 km, and the vertical wave packet scale is 5 km. The wave packet center is at 30 km altitude. | |
| **Background** | Isothermal atmosphere with T=210K; The Coriolis parameter is 0.0001 per second. | |

**Experiments designed for the high-resolution LES and the coarse-resolution model with gravity wave parameterizations based on phase-space WKB ray tracing.**

# Budget Analysis for the 2D WP with 3D wavenumber vector

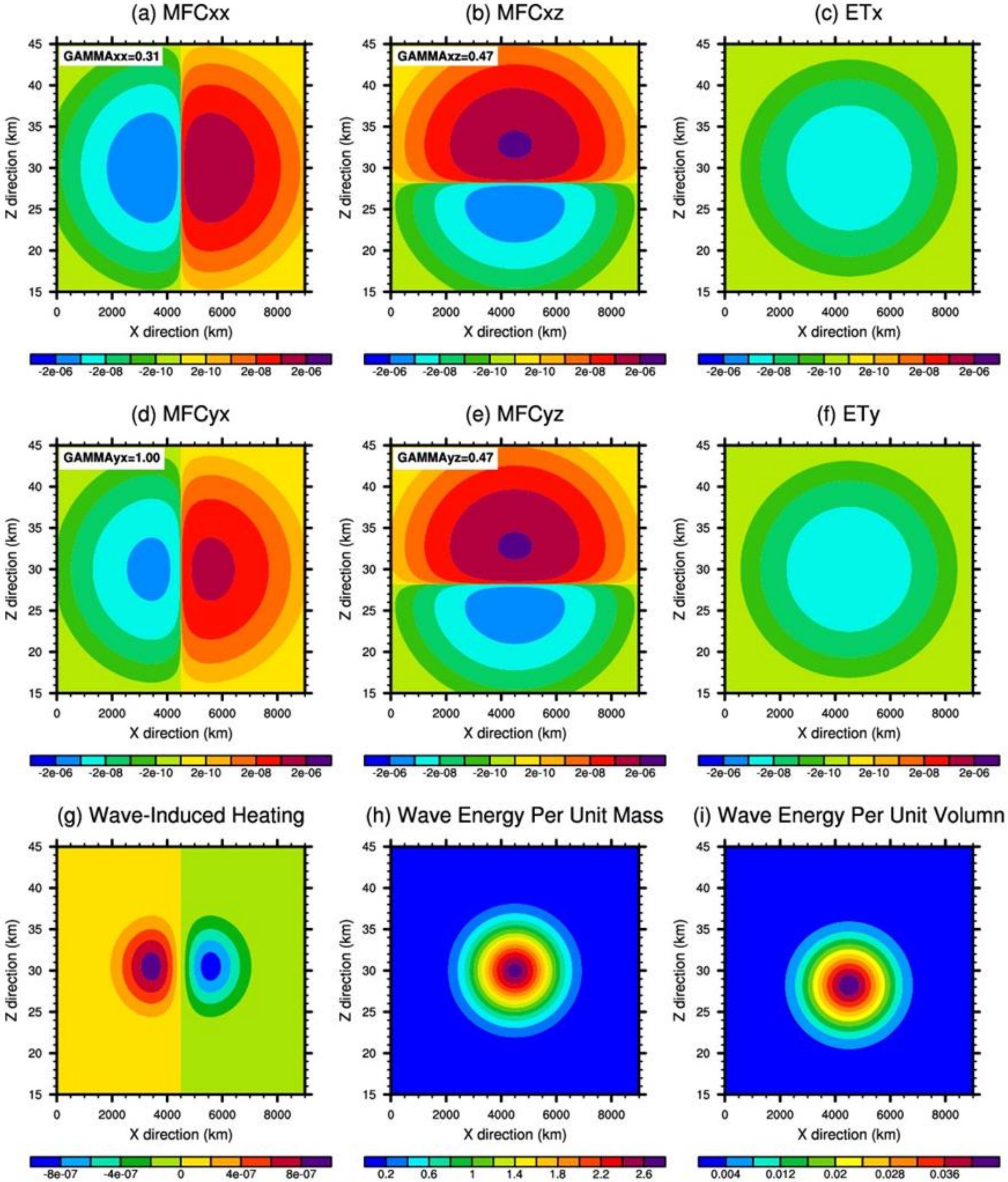



Fig. (a-g) Cross section of each forcing term for the 2D WP case.

- The Gamma parameter, defined as the ratio between each corresponding PMF and MF, is shown in the top left corner if available.
- For the zonal momentum equation, MFCxx is comparable with MFCxz, and it is important in this case. ETx appears to be secondary but not negligible.
- Similar argument still holds in the meridional momentum equation.
- The effect of heating term will be tested by the ray tracing model (not shown).

# WKB-Direct vs WKB-Balance

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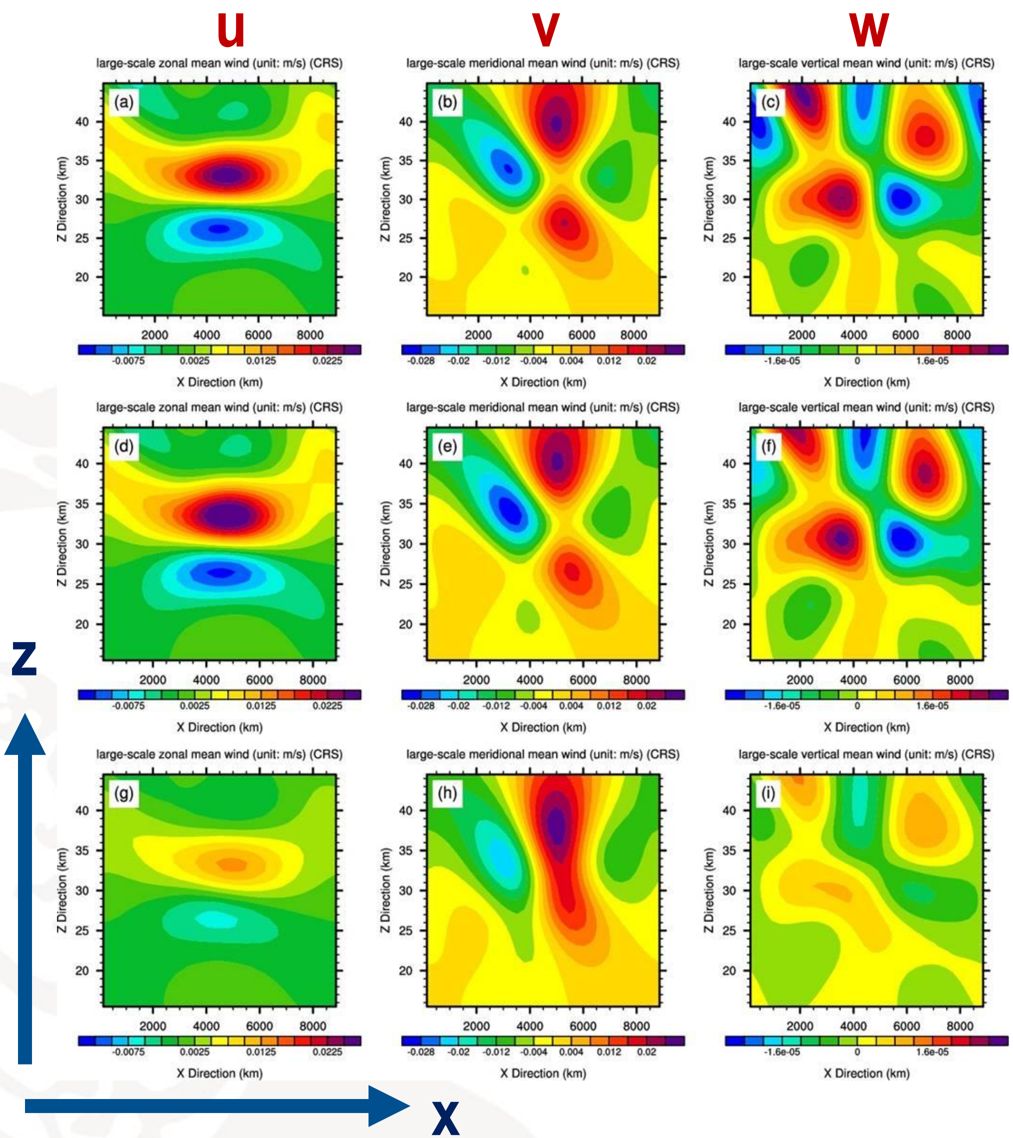
## Test case: 2.5D WP with self-induced mean wind




**1500 min after initialization**

**LES**

**WKB-Direct**

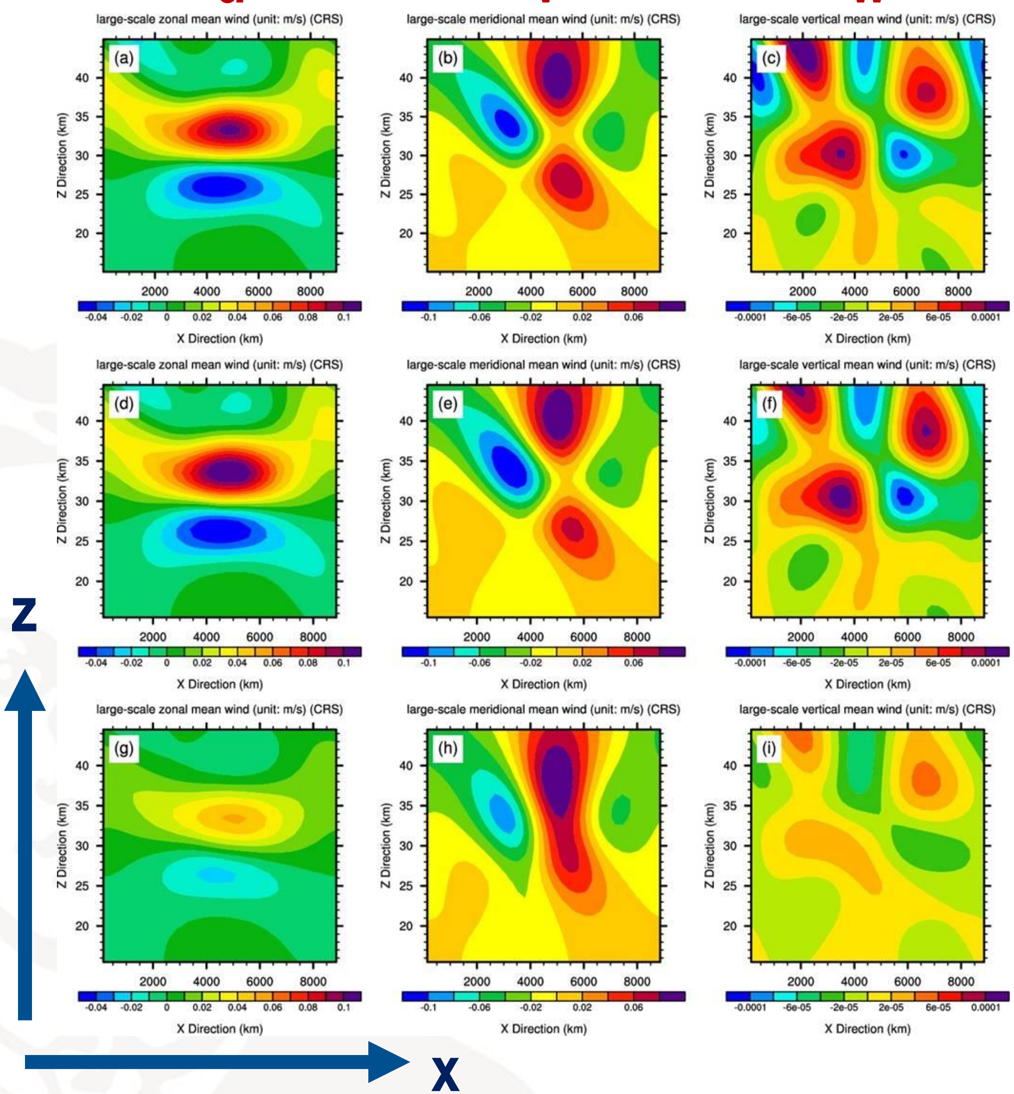
**WKB-Balance**

# WKB-Direct vs WKB-Balance

## Test case: 2.5D WP with initial high wave amplitude

u v w

1500 min after initialization




LES

WKB-Direct

WKB-Balance

# A Tale of Two Schemes

## The WKB-Balance Scheme

The first scheme only parameterizes one single term:

1) Pseudomomentum flux convergence in the momentum equation

*Note that this idea is used in many current models*

## The WKB-Direct Scheme

The second scheme parameterizes the following three terms:

1) Momentum flux convergence in the momentum equation
2) Elastic term in the momentum equation
3) Horizontal entropy-flux convergence in the entropy equation

### Take-Home Notes on IGW Parameterizations

- Theory part: Two schemes for IGW parameterizations are introduced.
- Numerical experiments: With the LES as a reference, WKB-Direct Scheme is generally better than WKB-Balance Scheme.
- Sensitivity experiments

# A Tale of Two Schemes

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MS-GWaves

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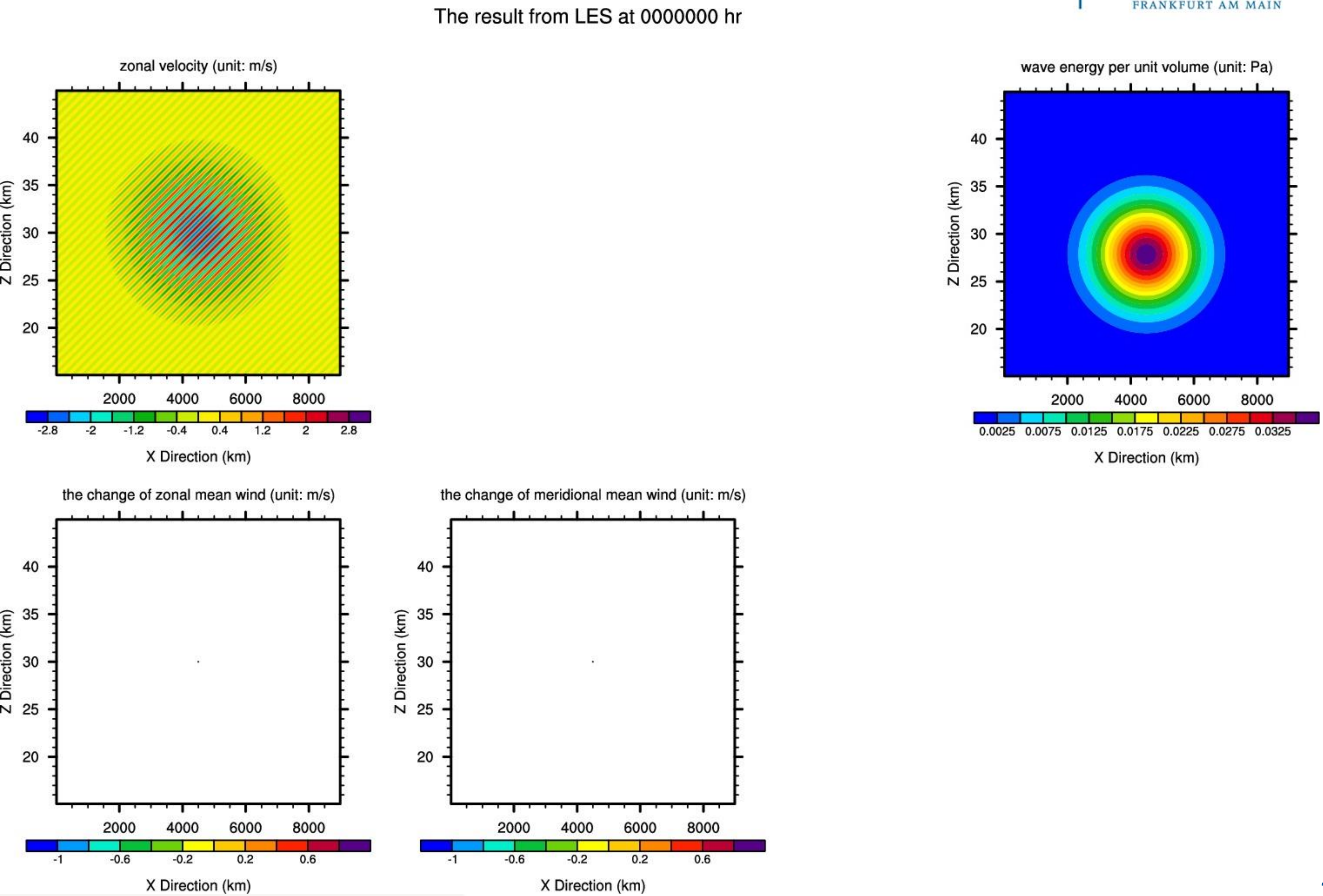
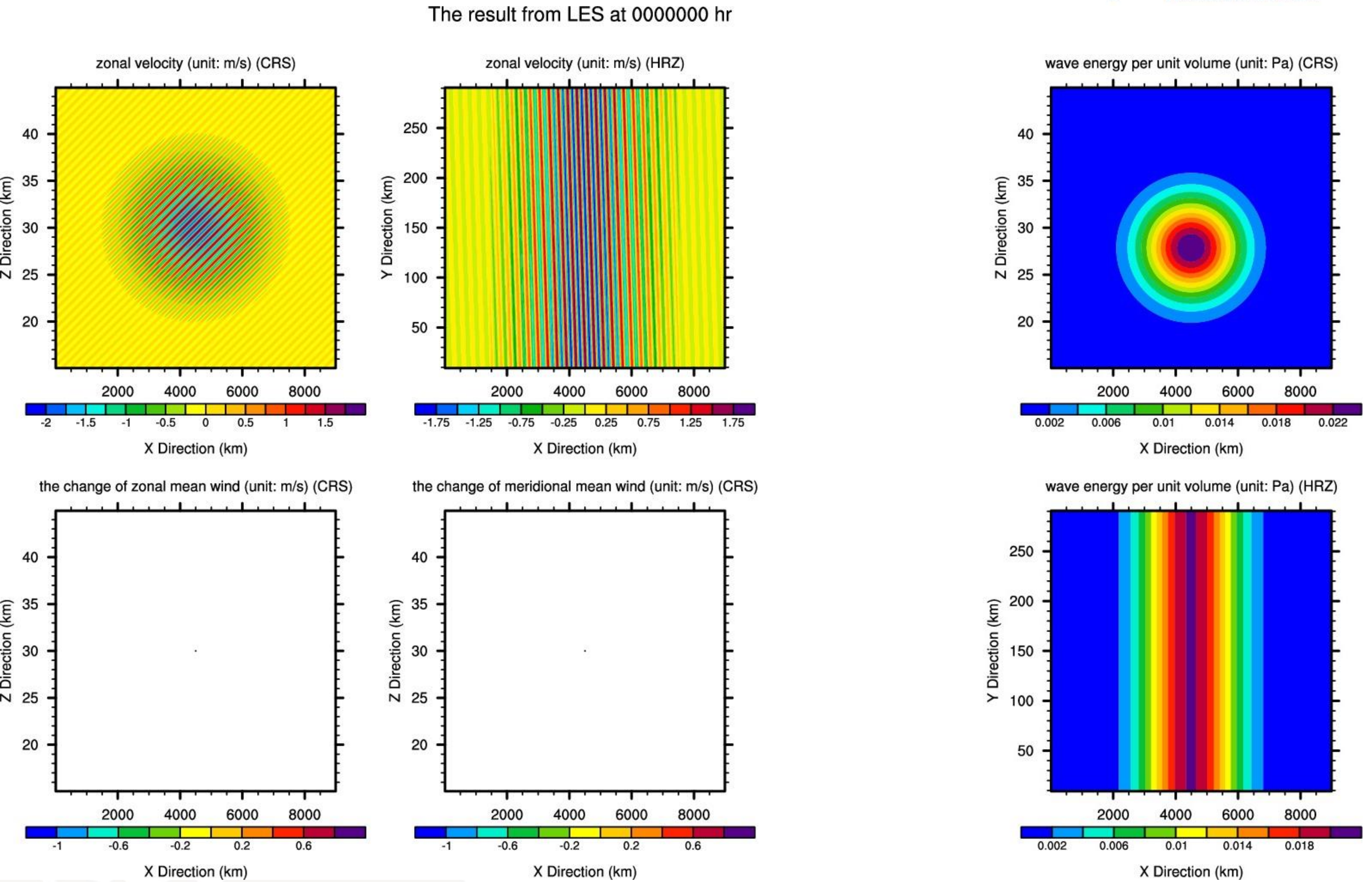
# Zoom-In Plot at Initial Condition



# The initial condition in LES



# PMF versus MF

## Three-Dimensional Case

$$\bar{\rho}\overline{u'u'} = AkCg_x \times \left(1 - \frac{1 + (l/k)^2}{1 - (\Omega/f)^2}\right)$$

$$\bar{\rho}\overline{u'v'} = AkCg_y$$

$$\bar{\rho}\overline{u'w'} = AkCg_z \times \left(\frac{1}{1 - (f/\Omega)^2}\right)$$

$$\bar{\rho}\overline{v'v'} = AlCg_y \times \left(1 - \frac{1 + (k/l)^2}{1 - (\Omega/f)^2}\right)$$

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# PMF versus MF

## 3D Case

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Momentum Flux

Pseudomomentum Flux

# PMF versus MF

## 3D Case

$$\boxed{\bar{\rho}\overline{u'u'}} = \boxed{AkCg_x} \times \left(1 - \frac{1+(l/k)^2}{1-(\Omega/f)^2}\right)$$

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$$\boxed{\bar{\rho}\overline{v'w'}} = \boxed{AlCg_z} \times \left(\frac{1}{1-(f/\Omega)^2}\right)$$

$\boxed{\phantom{xxxxxxxxxx}} \geq \boxed{\phantom{xxxxxxxxxx}}$

Momentum Flux — Pseudomomentum Flux

# PMF versus MF

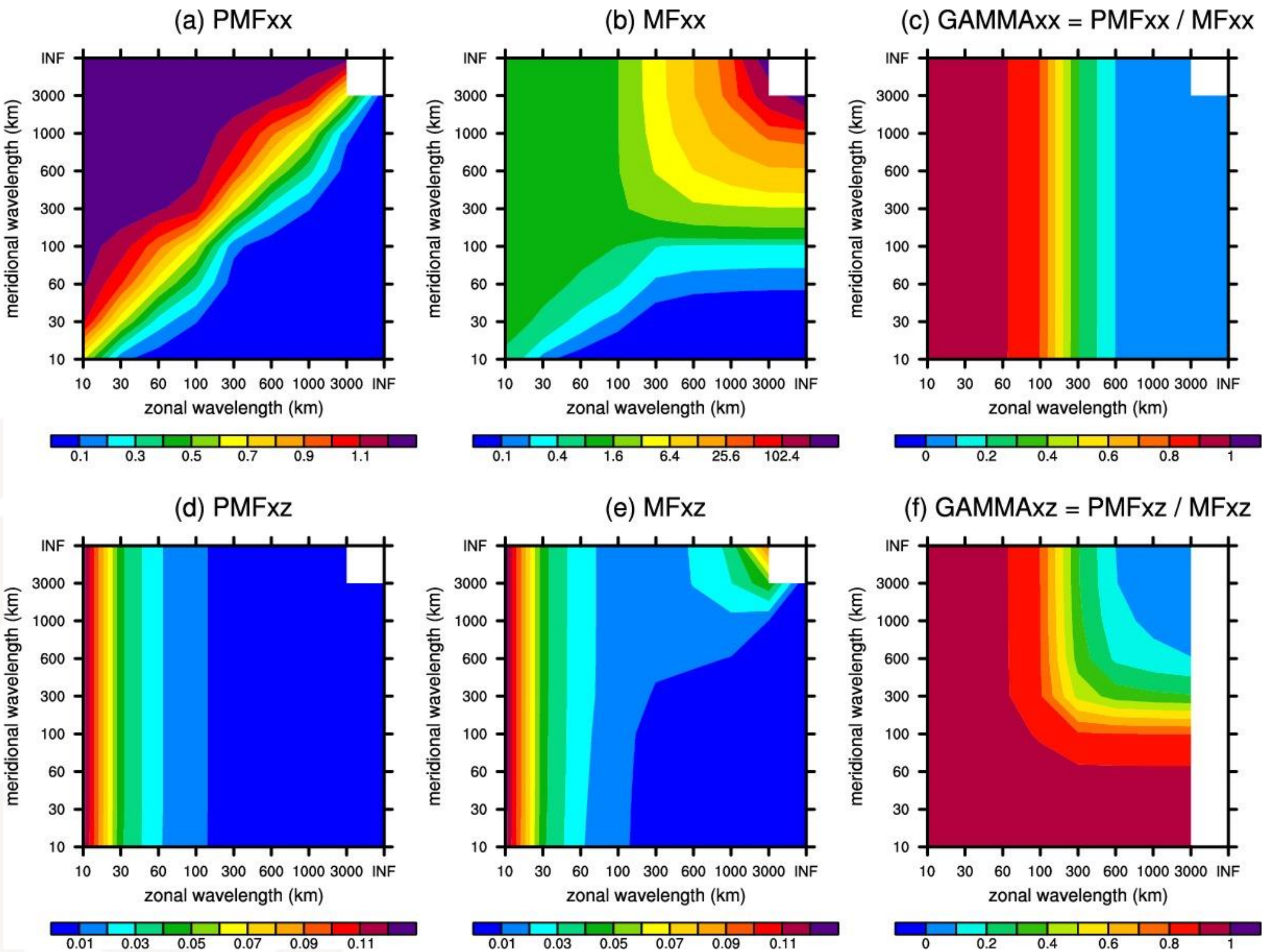


Figure The horizontal wavelength space distribution of (a) PMFxx, (b) MFxx, (c) GAMMAxx, (d) PMFxz, (e) MFxz, and (f) GAMMAxz.