Simulating all-scale global weather with the Finite-Volume Module of IFS

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\[
\begin{align*}
\frac{\partial G_\rho}{\partial t} + \nabla \cdot (\mathbf{v} G_\rho) &= 0 \\
\frac{\partial G_\rho u}{\partial t} + \nabla \cdot (\mathbf{v} G_\rho u) &= G_\rho \left( -\Theta_x \tilde{G} \nabla \varphi' - \frac{g}{\Theta_a} (\theta' + \Theta_a (\mathbf{v} \cdot \mathbf{u}) - q_x - q_y) - \nabla \cdot \left( \mathbf{u} + \frac{\nabla}{\Theta_a} \mathbf{u} \right) + M \mathbf{u} \right) \\
\frac{\partial G_\rho \theta'}{\partial t} + \nabla \cdot (\mathbf{v} G_\rho \theta') &= G_\rho \left( -\tilde{G}^T \mathbf{u} \cdot \nabla \theta_a - \frac{L}{c_p \Theta_a} \left( \frac{\Delta T_{vs}}{\Delta t} + E' \right) + \mathbf{u} \right) \\
\frac{\partial G_\rho q_k}{\partial t} + \nabla \cdot (\mathbf{v} G_\rho q_k) &= G_\rho \mathbf{R}^{\text{vs}} \\
\frac{\partial G_\rho \varphi'}{\partial t} + \nabla \cdot (\mathbf{v} G_\rho \varphi') &= G_\rho \sum_{\ell=1}^{3} \left( \frac{a_\ell}{\Theta_\ell} \nabla \cdot \mathbf{z} (\tilde{\nabla} - \tilde{G}^T \nabla \varphi') \right) + \mathbf{b} \varphi' + \mathbf{c}
\end{align*}
\]
**Schematic of spectral-transform method in IFS**

- **Grid-point space**
  - semi-Lagrangian advection
  - physical parametrizations
  - products of terms
  - No grid-staggering of prognostic variables

- **Spectral space**
  - horizontal gradients
  - semi-implicit calculations
  - horizontal diffusion

**Current operational configuration of the Integrated Forecasting System (IFS) at the European Centre for Medium-Range Weather Forecasting:**

- hydrostatic primitive equations (nonhydrostatic option available; see Benard et al. 2014)
- hybrid $\eta - p$ vertical coordinate (Simmons and Burridge, 1982)
- spherical harmonics representation in horizontal (Wedi et al., 2013)
- finite-element discretisation in vertical (Untch and Hortal, 2004)
- semi-implicit semi-Lagrangian (SISL) integration scheme (Temperton et al. 2001, Diamantakis 2014)
- cubic-octahedral (“TCo”) grid (Wedi, 2014, Malardel et al. 2016)
- HRES: TCo1279 (O1280) with $\Delta h \approx 9$ km and 137 vertical levels
- ENS (1+50 perturbed members): TCo639 (O640) with $\Delta h \approx 16$ km and 91 vertical levels

⇒ ECMWF strategy for the year 2025 targets to run ENS with TCo1999 with $\Delta h \approx 5$ km
Quasi-hydrostatic versus nonhydrostatic dynamics

Greyzone evaluations

Daniel Klocke and Nils Wedi

1, 2.5, 5, ~9 km of Tropical Atlantic with ICON & IFS

Total water + ice content

13 km

9 km

2.5 km

ICON oper

IFS oper

tqc 2016081100 +13h

ICON 2.5km

IFS 3999

Christian Kühnein, Sylvie Malardel, Piotr Smolarkiewicz, Nils Wedi
Quasi-hydrostatic versus nonhydrostatic dynamics

Idealized convective storm (Klemp et al. 2015) on a small planet (1/25 reduced) with H and NH formulation of IFS: From what horizontal grid spacing $\Delta_h$ appear significant differences?

$\Delta_h = 5 \text{ km}$

$\Delta_h = 2.5 \text{ km}$

$\Delta_h = 1.25 \text{ km}$

$\Delta_h = 0.625 \text{ km}$

$\rightarrow$ H-IFS and NH-IFS use Forbes et al. 2011 microphysics and similar numerical configurations, in particular TCo grid, FD in vertical, ICI, no explicit diffusion, no convection scheme)
Idealized convective storm (Klemp et al. 2015) on a small planet (1/25 reduced) with H and NH formulation of IFS and NH-FVM:

$\Delta h = 5 \text{ km}$

$\Delta h = 2.5 \text{ km}$

$\Delta h = 1.25 \text{ km}$

$\Delta h = 0.625 \text{ km}$

$\rightarrow$ NH-FVM uses smaller time steps and different microphysics parametrisation!
Finite-Volume Module (FVM) of IFS—key formulation features

- deep-atmosphere nonhydrostatic Euler equations in geospherical framework (Szmelter and Smolarkiewicz 2010; Smolarkiewicz et al. 2016; Smolarkiewicz, Kühlein, Grabowski 2017; Kühlein, Malardel, Smolarkiewicz in prep.)
- flexible height-based terrain-following vertical coordinate
- hybrid of horizontally-unstructured median-dual finite-volume with vertically-structured finite-difference/finite-volume discretisation (Szmelter and Smolarkiewicz 2010; Smolarkiewicz et al. 2016)
- all prognostic variables are co-located
- two-time-level semi-implicit integration scheme with 3D implicit acoustic, buoyant and rotational modes (Smolarkiewicz, Kühlein, Wedi JCP 2014)
- preconditioned generalised conjugate residual iterative solver for 3D elliptic problems arising in the semi-implicit integration schemes (Smolarkiewicz and Szmelter 2011 for a more recent review)
- non-oscillatory finite-volume MPDATA scheme (Smolarkiewicz and Szmelter 2005; Kühlein and Smolarkiewicz 2017)
- octahedral reduced Gaussian grid, but the FVM formulation not restricted to this (Szmelter and Smolarkiewicz 2016)
- optional moving mesh capability (as in Kühlein, Smolarkiewicz, Dörnbrack 2012)

\[ \int_{\Omega} \nabla \cdot \mathbf{A} = \int_{\partial \Omega} \mathbf{A} \cdot \mathbf{n} = \frac{1}{V_i} \sum_{j=1}^{I(i)} A_j^\perp S_j \]

**median-dual finite-volume approach**

**terrain-following coordinate**
Octahedral reduced Gaussian grid

- octahedral reduced Gaussian grid (octahedral grid of size OX)
- suitable for spherical harmonics transforms applied in spectral IFS
  - Gaussian latitudes ⇒ Legendre transforms
  - equidistant distribution of nodes along latitudes following octahedral rule ⇒ Fourier transforms
- FVM develops median-dual mesh around nodes of octahedral grid
  ⇒ finite-volume and spectral-transform IFS can operate on same quasi-uniform horizontal grid
- Malardel et al. ECMWF Newsletter 2016, Smolarkiewicz et al. JCP 2016
- operational at ECMWF with HRES and ENS since March 2016
- Mesh generator and parallel data structures for FVM provided by ECMWF’s Atlas framework (Deconinck et al. 2017)
Governing equations in FVM

Flux-form moist compressible Euler equations in generalised curvilinear coordinates (Smolarkiewicz, Kühnlein, Grabowski 2017; Kühnlein, Malardel, Smolarkiewicz in prep.):

\[
\frac{\partial G \rho_d}{\partial t} + \nabla \cdot (vG \rho_d) = 0 ,
\]

\[
\frac{\partial G \rho_d u}{\partial t} + \nabla \cdot (vG \rho_d u) = G \rho_d \left[ -\theta \rho \tilde{G} \nabla \varphi' + g \mathcal{B} - f \times \left( u - \frac{\theta \rho}{\theta_{pa}} u_a \right) + \mathcal{M}' + \mathcal{D} + P^u \right] ,
\]

\[
\frac{\partial G \rho_d \theta'}{\partial t} + \nabla \cdot (vG \rho_d \theta') = G \rho_d \left[ -\tilde{G}^T u \cdot \nabla \theta_a + \mathcal{H} + P^{\theta'} \right] ,
\]

\[
\frac{\partial G \rho_d r_k}{\partial t} + \nabla \cdot (vG \rho_d r_k) = G \rho_d \left[ \mathcal{D}^r_k + P^r_k \right] \quad \text{where} \quad r_k = r_v, r_c, r_r, r_i, r_s ,
\]

\[
\varphi' = c_{pd} \left[ \left( \frac{R_d}{\rho_0} \rho_d \theta \left(1 + r_v/\varepsilon \right) \right) \frac{R_d/c_{vd}}{ - \pi_a} \right] ,
\]

with:

\[
v = \tilde{G}^T u , \quad \theta = \frac{\theta (1 + r_v/\varepsilon)}{1 + r_t} , \quad \varepsilon = \frac{R_d}{R_v} , \quad \theta' = \theta - \theta_a ,
\]

\[
\mathcal{B} = 1 - \frac{\theta}{\theta_{pa}} = 1 - \frac{\eta_{\theta \rho}}{\theta_{pa}} \left( \theta_a + \theta' \right) , \quad \eta_{\theta \rho} \equiv \frac{1 + r_v/\varepsilon}{1 + r_t} , \quad r_t = \sum_k r_k
\]
Dry baroclinic instability (Ullrich et al. 2014) with FVM and spectral-transform IFS (ST):

- Finite-volume solutions achieve accuracy of established spectral-transform IFS for planetary-scale baroclinic instability.
Finite-volume and spectral-transform solutions in IFS

Instantaneous kinetic energy spectra O640/TC0639 ($\Delta h \approx 16$ km) for baroclinic instability at day 15

Near-surface (~ 500 hPa)
Moist baroclinic instability with FVM and spectral-transform IFS (ST) with large-scale condensation and diagnostic precipitation:

- Finite-volume solutions achieve accuracy of established spectral-transform IFS for moist flows

![Image of precipitation and surface pressure comparisons between FVM and ST methods at day 10 and day 15]
Moist baroclinic instability with FVM and spectral-transform IFS (ST) with large-scale condensation and diagnostic precipitation:

**Precipitation (mm/day) at day 10**
- O160/TCo159, $\Delta h \approx 62$ km
- O640/TCo639, $\Delta h \approx 18$ km

**Surface pressure O640/TCo639, $\Delta h \approx 18$ km, day 15**

• Finite-volume solutions achieve accuracy of established spectral-transform IFS for moist flows
Tropical cyclone simulations with FV and ST approaches in IFS

Tropical cyclone simulations with coupling to parametrisations for large-scale condensation with diagnostic rain, surface fluxes and PBL diffusion (Reed and Jablonowski 2011) on O640/L60:

Wind speed (m/s) in horizontal section at $z \approx 1$ km:

Wind speed (m/s) in zonal-height section:

**FV**

**ST**

day 10
**Dynamical Core Model Intercomparison Project (DCMIP)**

**DCMIP2016: A Review of Non-hydrostatic Dynamical Core Design and Intercomparison of Participating Models**


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Mesoscale convective storm on reduced-size planet

Supercell evolution (0.5, 1, 1.5, 2h) with FVM (left) and MPAS (right) at ≈0.5 km grid spacing (cf. Klemp et al. 2015):

\[ Dqv \frac{Dt}{5} = Dqsv \frac{Dt}{1} + Er; \quad (20) \]
\[ Dqc \frac{Dt}{2} = 2 Dqsv \frac{Dt}{2} - Ar - Cr; \quad (21) \]
\[ Dqr \frac{Dt}{2} = 2 Er + Ar + Cr - Vr dq_r dz; \quad (22) \]

Here \( L \) is the latent heat of condensation, \( Ar \) is the autoconversion rate of cloud water to rainwater, \( Cr \) is the collection rate of rainwater, \( Er \) is the rainwater evaporation rate, and \( Vr \) is the rainwater terminal velocity. For each variable \( V / D / 5 / C3 \), where \( C3 \) is the value at the new time level prior to the final microphysics update.
Mesoscale convective storm on reduced-size planet

Supercell for grid spacings (4, 2, 1, 0.5 km) with FVM (left) and MPAS (right) after 2 h of simulation (cf. Klemp et al. 2015):

3.6. MPAS Results for Supercell Simulations
Simulations were integrated over a 2 h time interval, with convection initiated by the warm bubbles described in section 3.3. The overall evolution of the splitting supercells is well illustrated by the horizontal cross sections of vertical velocity and rainwater at 5 km at half hour intervals shown in Figure 11. Here the longitudinal position of the fields are shown based on a ground relative framework (i.e., the $U_{515}$ has been added back into the translation speed of the storm). At 30 min, a single strong updraft is producing significant precipitation that is collocated with the updraft. By 1 h, the initial updraft has split into two distinct updraft cells due to the negative buoyancy associated with rainwater loading along the central axis of symmetry (equator), together with favorable lifting vertical pressure gradients on the flanks of the rotating updrafts [Rotunno and Klemp, 1982, 1985]. By 90 min, the storm splitting has produced two mirror-image supercell storms, one with cyclonic updraft rotation propagating to the right (south) of the mean winds, and the other rotating anticyclonically and propagating to the left (north) of the mean winds. Owing to their transverse propagation, the two supercells continue to move farther apart over the second hour.

The physical relevance of the supercell simulation on an X=120 reduced-radius sphere can be established through comparisons with a corresponding simulation in Cartesian geometry. For this purpose, we have configured an MPAS grid to represent a flat plane of hexagonal grid cells with periodic lateral boundary conditions. Figure 14.

(a) Vertical velocity at 5 km
(b) Rainwater at 5 km

Vertical velocity (m/s) at 5 km
Rainwater (g/kg) at 5 km
Some relevant references

Further reading:

- Waruszewski M., C. Kühnlein, H. Pawlowska, P. K. Smolarkiewicz, MPDATA: Third-order accuracy for arbitrary flows, in press, JCP.
- Kühnlein C., S. Malardel, Smolarkiewicz P.K., et al., Finite-volume and spectral-transform solutions in IFS, in preparation for GMD
- Kühnlein C., R. Klein, Smolarkiewicz P.K., Splitting of advection in an all-scale atmospheric model, in preparation for MWR