Lagrangian Transport in an Idealised Meandering Jet

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Imperial College London

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Table of Contents

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Motivation behind the project

Most ocean GCMs lack the necessary spatial resolution to resolve the eddy scales, therefore eddies and their effects have to be parameterised, including that of eddy-induced material transport.

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- isotropic,
- diffusive,
- and for the most part homogenous.

Why are these assumptions made?

Turbulence is traditionally treated as downgradient diffusion, i.e that the flow behaves as a random walk, as suggested by Taylor in 1921.

The aims of the project

The aim of this project is to build a transport model that can be used to advect Lagrangian particles by the velocity field of a realistic but simple meadering jet.

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Statistics of the particle trajectories will then be analysed in order to devise a more suitable method of parameterising eddy-induced transport in the ocean for use in non-eddy resolving GCMs.

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Two routes will be explored in this presentation :

- Divising a new dispersion measure.
- Using EOFs as a motivation behind a kinematic model.

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Table of Contents

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Dynamical Model

We will use a doubly periodic 2-layer Quasi-Geostrophic model using the β-plane approximation and a rigid lid to generate a meandering jet.

Governing Equations

$$\frac{D_1}{Dt}\left(\nabla^2\psi_1 - S_1(\psi_1 - \psi_2)\right) + \beta v_1 = F_1 \quad (1)$$

$$\frac{D_2}{Dt}\left(\nabla^2\psi_2 - S_2(\psi_2 - \psi_1)\right) + \beta v_2 = F_2 \quad (2)$$

where S_1 and S_2 are the stratification parameters : $S_n = f_0^2/g'H_n$ and $g' = \Delta\rho/\rho$ is the reduced gravity. F_n consists of viscosity in both layers and bottom friction in the bottom layer.

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The jet regime

The dynamical model was run with a grid resolution of 512 x 512, a domain size of 520km x 520km and an eastward zonal velocity of 6 cm s^{-1} in the top layer and a viscosity of 1 m^2s^{-1}.

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Two different parameter regimes were generated by varying the bottom friction: a coherent jet with a bottom friction of $1 \times 10^{-8} s^{-1}$ and a latent jet with it set as $2.5 \times 10^{-8} s^{-1}$.

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The jet regime

Figure: The PV in the top layer for the two regimes.

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The jet regime

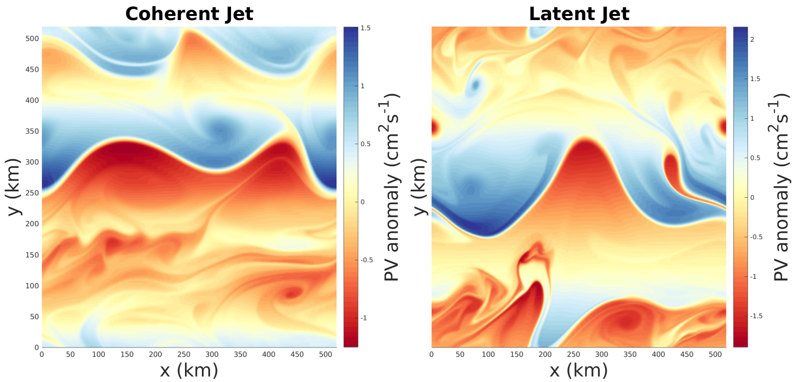

Figure: The PV anomaly in the top layer for the two regimes.

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Table of Contents

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Extracting the 'eddying' part.

Two different eddying trajectories will be examined:

- eddy-only (EO), where the particles are advected using the eddying velocity only: $\mathbf{u}'(t) = \mathbf{u}(t) - \overline{\mathbf{u}}$;

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- full-following-eddy (FFE), which takes into account the mean flow's ability to advect particles between eddies.
  - The displacement due to the mean flow is calculated following the full trajectory.
  - At each time step the difference between the full displacement and mean displacement is calculated.
  - The difference in displacements are then cumulatively added.

Single-Particle Statistics

Single-Particle Dispersion (SPD)

$$D_x(t) = \frac{1}{N}\sum_{n=1}^{N}(x_n(t) - x_n(0))^2, \quad D_y(t) = \frac{1}{N}\sum_{n=1}^{N}(y_n(t) - y_n(0))^2.$$

Fitting the SPD to time using a power law: $D_i \sim t^{\alpha_i}$ allows us to quantify diffusivity.

- if $0.8 < \alpha_i < 1.2$, transport is said to be roughly diffusive,
- if $\alpha_i < 0.8$, transport is said to be sub-diffusive,
- if $\alpha_i > 1.2$, transport is said to be super-diffusive.

Time Scale

The domain is divided into 10 equally sized zonal bins of width W.
The time scale is the time at which $\sqrt{D_y(t)} > W$.

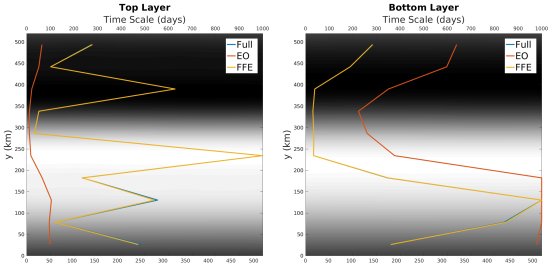

Figure: Time Scale for the coherent jet for 10 bins.

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α_i across the domain

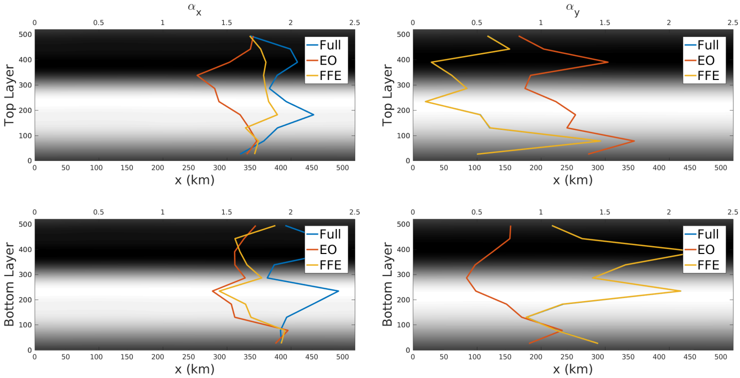

Figure: α_i in 10 zonally averaged bins superimposed on the time-averaged stream function in the coherent jet.

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Table of Contents

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Diffusivity

Definition

$$K_x = \frac{1}{2}\frac{\partial D_x}{\partial t}, \quad K_y = \frac{1}{2}\frac{\partial D_y}{\partial t}. \tag{3}$$

In diffusive regimes, D_x, D_y grow linearly in time, and hence K_x, K_y are constant.

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In diffusive regimes, D_x, D_y grow linearly in time, and hence K_x, K_y are constant.

$\mathbf{K}$ is the parameterised variable in the advection diffusion equation.

Limitations : It only measures a particles 'geographic' diffusivity. In the case of a meandering jet, a particle may remain on the jet and so its meridional dispersion increases even if the particle is not moving relative to the flow.

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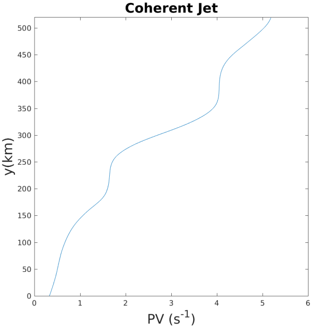
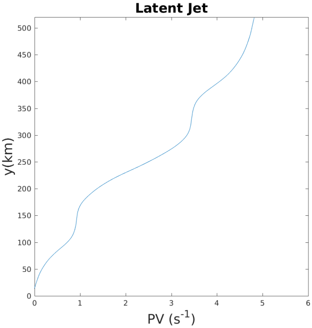
PV based dispersion

- There is a one-to-one map between the zonally and time-averaged full PV, $\tilde{q}$, and y.





PV based dispersion

The method

- Interpolate the instantaneous PV to find the PV at the particle location : $q(x_n(t), y_n(t))$.

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- Interpolate the instantaneous PV to find the PV at the particle location : $q(x_n(t), y_n(t))$.
- Find y such that $\tilde{q}(y) = q(x_n(t), y_n(t))$. Call it $Y(t)$

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The method

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- Find y such that $\tilde{q}(y) = q(x_n(t), y_n(t))$. Call it $Y(t)$
- The PV mapped dispersion is $dY(t) = \langle (Y(t) - Y(0))^2 \rangle$.

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PV based dispersion

- Binning Method : The domain is binned into 10 zonal bins, however unlike for regular single particle dispersion, $\tilde{q}(y)$ is binned uniformly so that regions with high PV gradients have narrower bins.

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PV based dispersion

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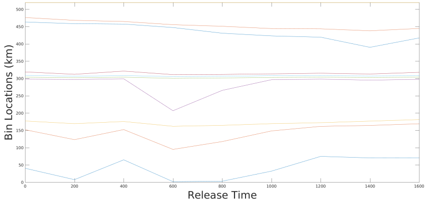
- 5000 particles are released uniformly in each bin in 9 separate releases and run for 1000 days.



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PV based dispersion

We compare the PV mapped dispersion against the meridional component of the SPD for the bin with the smallest averaged bin width and the largest.

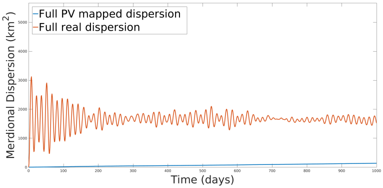

Figure: Bin 6

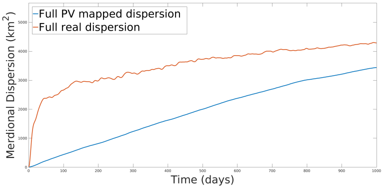

Figure: Bin 8

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Time Scale

PV mapped dispersion and bin widths are averaged over the different releases. The time scale is taken to be the time at which $\sqrt{dY(t)} > W$.

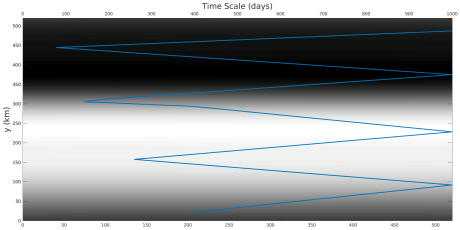

Figure: Time scale for PV mapped dispersion in 10 zonal PV bins for the coherent jet in the top layer.

α across the domain

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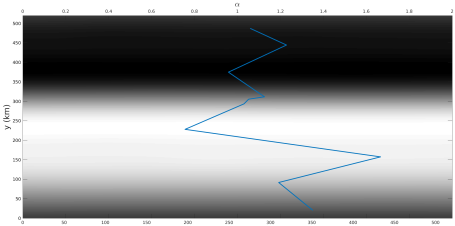
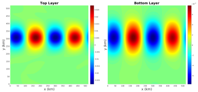


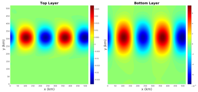
Figure: α for PV mapped dispersion in 10 zonal PV bins for the coherent jet in the top layer.

Table of Contents

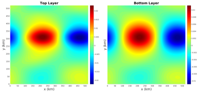
EOFs



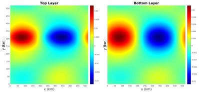
(a) Mode 1



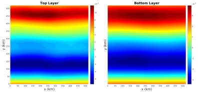
(b) Mode 2



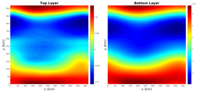
(c) Mode 3



(d) Mode 4



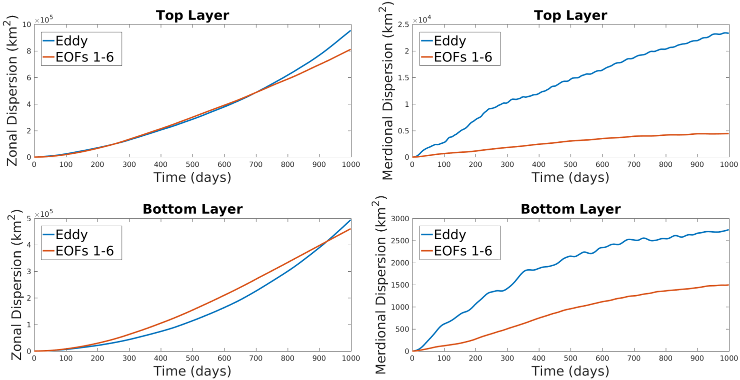
(e) Mode 5



(f) Mode 6

Figure: The first 6 stream function EOFs in the top and bottom layer for the coherent jet.

Comparison of SPDs



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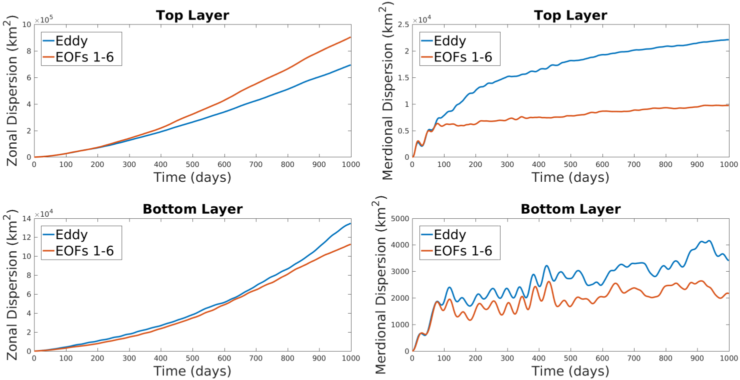

(b) Bin 5

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Table of Contents

Further work and Limitations: PV Mapped Dispersion

Further Work

- Could this method produce a more accurate approximation for a diffusivity?

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- The method relies on finding a one-to-one map between y and some flow dependent variable.

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- The PV is averaged both temporally and zonally - could a sharper map be found?
- This method only applies in the meridional direction of a zonally symmetric flow.

Summary and Further work: EOFs

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- The first 6 EOFs seem to do a pretty good job and capturing Eddy-Only zonal dispersion.

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Thank you for listening.

Further Reading

- Pavel Berloff, James McWilliams and Annalisa Bracco.
  Material Transport in Oceaniv Gyres. Part I: Phenomenology
  Journal of Physical Oceanography vol.32, no.3
- Irina Rypina, Igor Kamenkovich, Pavel Berlofff and Lawrence Pratt
  Eddy-Induced Particle Dispersion in the Near-Surface North Atlantic
  Journal of Physical Oceanography vol.42, no.12
- J.H LaCasce
  Statistics from Lagrangian observations
  Progress in Oceanography, vol.77, no.1