Climate-Dependence in Empirical Parameters of Subgrid-Scale Parameterizations using the Fluctuation-Dissipation Theorem

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Motivation

- Due to the complexity of the atmosphere (climate system) and the limited computational power, subgrid-scale (SGS) parameterizations are required.

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- Many SGS parameterizations contain empirical parameters which are tuned against reference data.
  - Works very well for present day climate.

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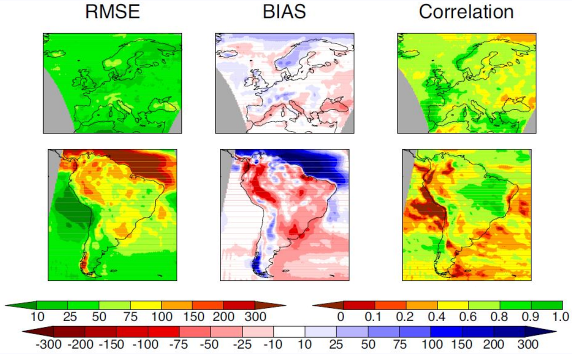
- Empirical parameters are dependent on the reference data.



adapted from Rockel and Geyer Meteor. Z. (2008)

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- Use the Fluctuation-Dissipation theorem (FDT) to introduce climate-dependence in SGS parameterizations (Achatz et al. JAS 2013).

QG3LM

- The Quasi-geostrophic three-layer model (QG3LM) of Marshall and Molteni (JAS 1993) can be written as

$$\frac{d\mathbf{x}}{dt} = G(\mathbf{x})$$

with state vector $\mathbf{x} \in \mathbb{R}^N$ and $N \approx 1500$ degrees of freedom.

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- Change of basis into empirical orthogonal function (EOF) space. The low-order model reads

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where $\mathbf{a} \in \mathbb{R}^M$ contains the principal components and $\tilde{G}(\mathbf{a})$ are the model equations projected onto the leading $M (M \ll N)$ EOFs.

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- SGS tendency error $\dot{\mathbf{s}}(\mathbf{a}, \mathbf{x})$ due to neglected EOFs.

SGS parameterization

- Low-order model with parameterization

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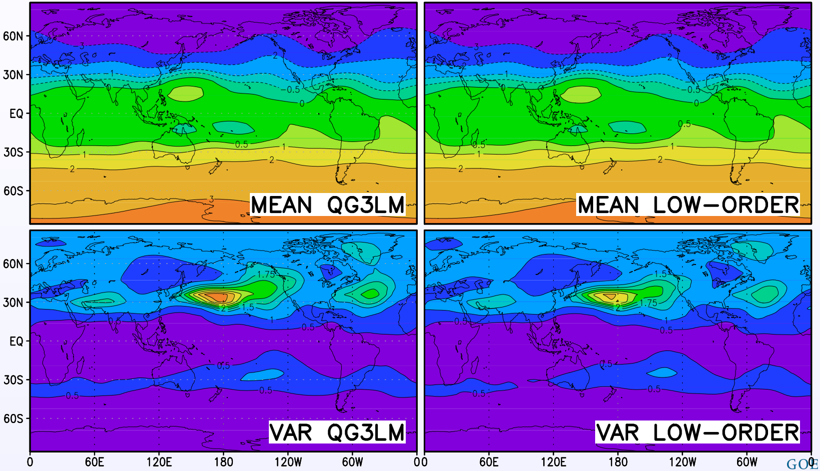
- Minimization over $\langle \| \varepsilon \|^2 \rangle$ yields

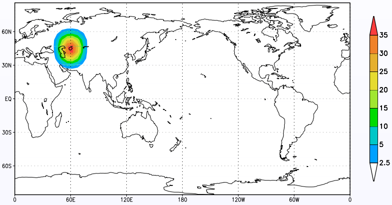
$$\begin{aligned} \mathbf{L} &\propto \langle \mathbf{s}' \mathbf{a}'^{\mathrm{T}} \rangle \langle \mathbf{a}' \mathbf{a}'^{\mathrm{T}} \rangle^{-1} \\ \mathbf{r} &\propto \langle \mathbf{s} \rangle - \mathbf{L} \langle \mathbf{a} \rangle \end{aligned}$$

with the discretized SGS tendency error $\mathbf{s} = \Delta\mathbf{a} - 2\Delta t \tilde{G}(\mathbf{a})$.

Evaluation of low-order model (500 EOF)



Anomalous forcing



- Forcing $\delta\mathbf{f}$ position is fixed at $\varphi = 45^\circ$ with variable $\lambda = \{0^\circ, 30^\circ, \ldots, 330^\circ\}$ (Achatz and Opsteegh JAS 2003; Achatz et al. JAS 2013).

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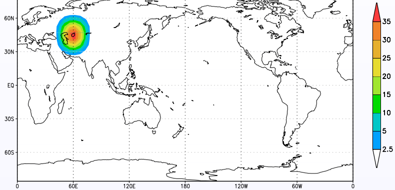

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- Therefore the perturbed QG3LM reads

$$\frac{d\mathbf{x}}{dt} = G(\mathbf{x}) + \delta\mathbf{f}$$

Closure Correction

- Perturbed model due to introduction of anomalous forcing

$$\frac{d\mathbf{a}}{dt} = \tilde{G}(\mathbf{a}) + \mathbf{r}_{\text{pert.}} + \mathbf{L}_{\text{pert.}}\mathbf{a} + \delta\tilde{\mathbf{f}}_{20}$$

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- New closure

$$\mathbf{L}_{\text{pert.}} \propto (\langle \mathbf{s}'\mathbf{a}'^{\mathrm{T}} \rangle + \delta\langle \mathbf{s}'\mathbf{a}'^{\mathrm{T}} \rangle)(\langle \mathbf{a}'\mathbf{a}'^{\mathrm{T}} \rangle + \delta\langle \mathbf{a}'\mathbf{a}'^{\mathrm{T}} \rangle)^{-1}$$
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- Use Fluctuation-Dissipation theorem (FDT, Risken 1984, Majda et al. 2005, Gritsun et al. JAS 2008) to estimate update of empirical closure parameters.

quasi-Gaussian FDT (qG-FDT)

- Response of observable $\mathbf{h}$ to small anomalous forcing $\delta\tilde{\mathbf{f}}$

$$\begin{aligned}\delta\langle\mathbf{h}\rangle &= \int_0^\infty \langle\mathbf{h}(\tau)\mathbf{a}'(0)\rangle d\tau \langle\mathbf{a}'(0)\mathbf{a}'(0)^{\mathrm{T}}\rangle^{-1}\delta\tilde{\mathbf{f}} \\ &= \mathbf{R}(\mathbf{h})\delta\tilde{\mathbf{f}}\end{aligned}$$

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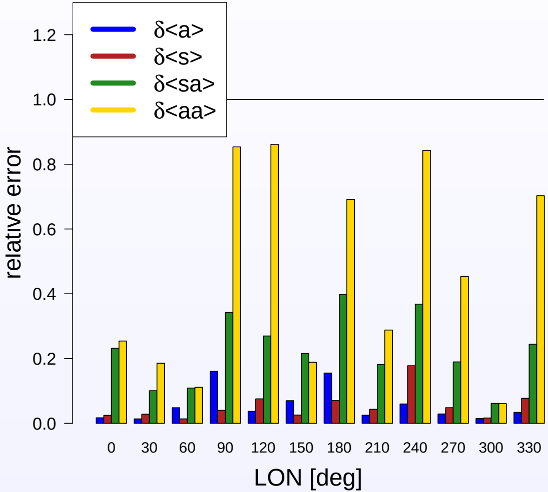
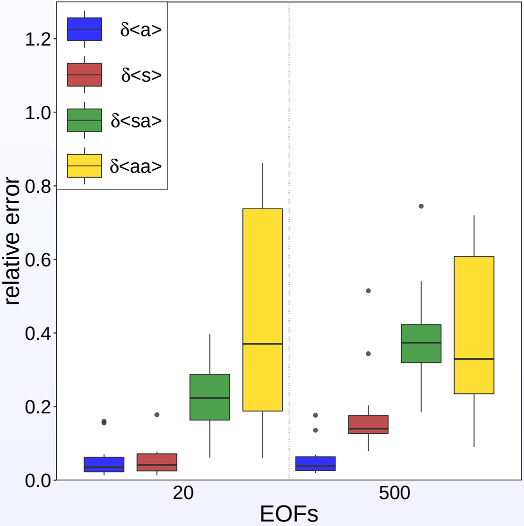
  - independent of $\delta\tilde{\mathbf{f}}$;
  - completely determined by the statistics of the unperturbed model.

FDT prediction statistical moments (20 EOF)



Response of statistical moments



Low-order models with adjusted SGS parameterizations

- Perturbed model due to introduction of anomalous forcing

$$\frac{d\mathbf{a}}{dt} = \tilde{G}(\mathbf{a}) + \mathbf{r}_{\text{pert.}} + \mathbf{L}_{\text{pert.}}\mathbf{a} + \delta\tilde{\mathbf{f}}_{20}$$

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3. qG-FDT correction

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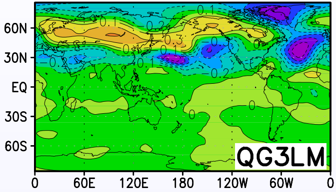
Response variance streamfunction (500EOFs, $\lambda_c = 270°$)



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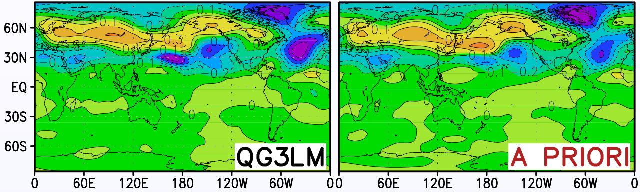

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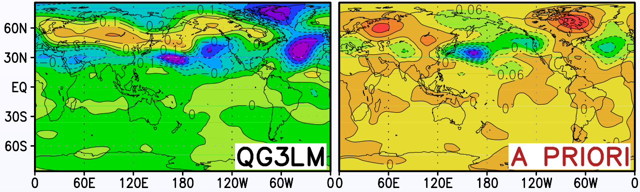

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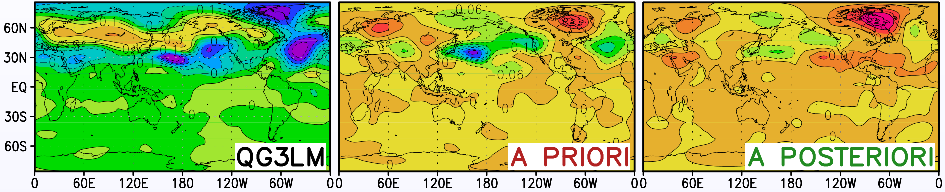

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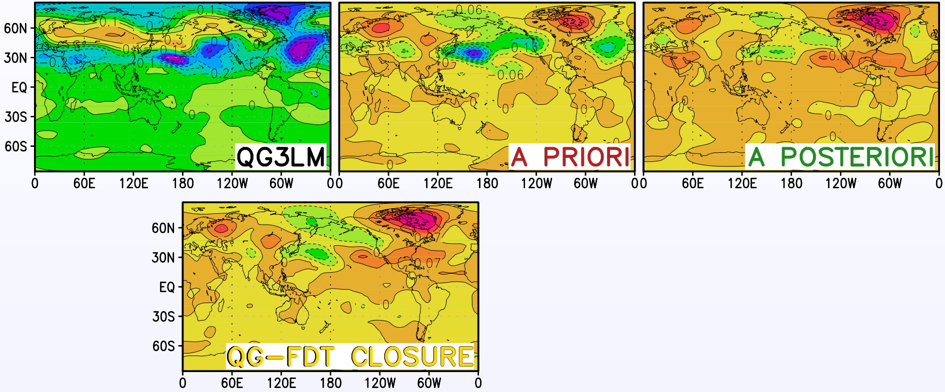

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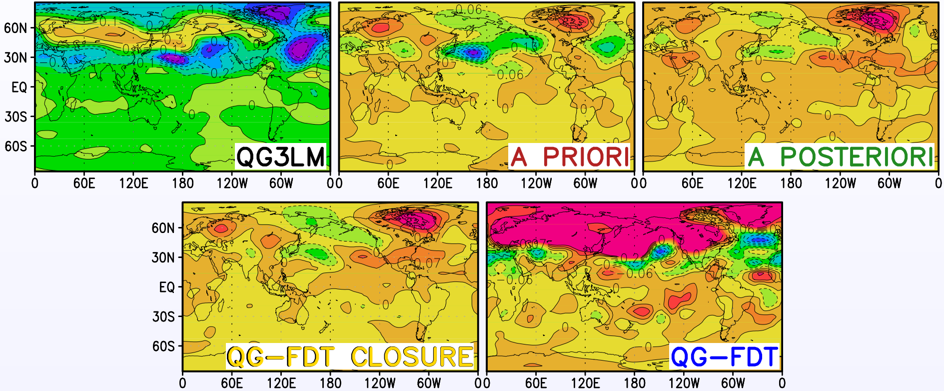

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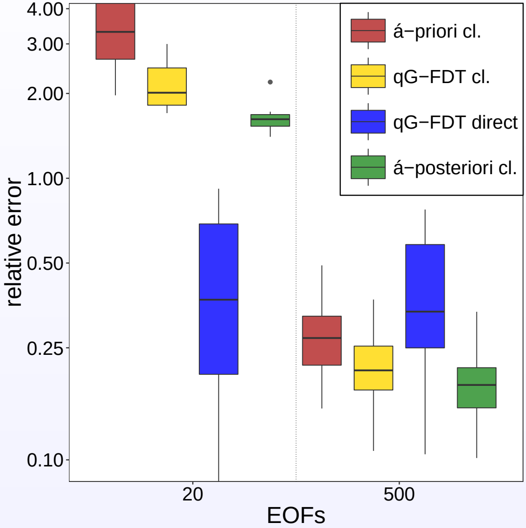

Conclusions

- The qG-FDT estimations of the statistical moments result in useful updates of the tuning parameters.
- The reduced model with the qG-FDT-adjusted SGS parameterization preforms systematically better than the a priori model, in case of an anomalous forcing.
- For 500 EOFs, reduced models with a qG-FDT-adjusted closure outperform the direct qG-FDT estimation.

M. Pieroth, S.I. Dolaptchiev, M. Zacharuk, T. Heppelmann, A. Gritsun, and U. Achatz. **Climate-Dependence in Empirical Parameters of Subgrid-Scale Parameterizations using the Fluctuation-Dissipation Theorem**. *J. Atmos. Sci.*, 2018. Submitted.