A geometric interpretation of vertical structure and eddy-mean flow interaction in the Southern Ocean

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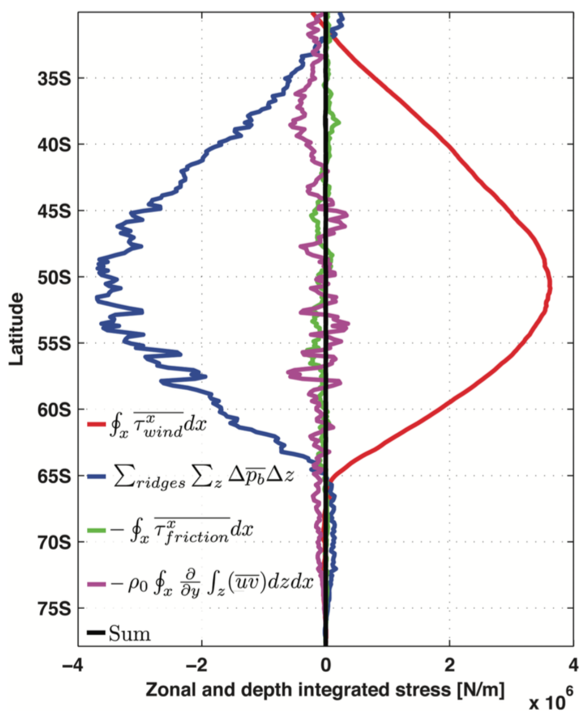
Eddy interfacial form stress



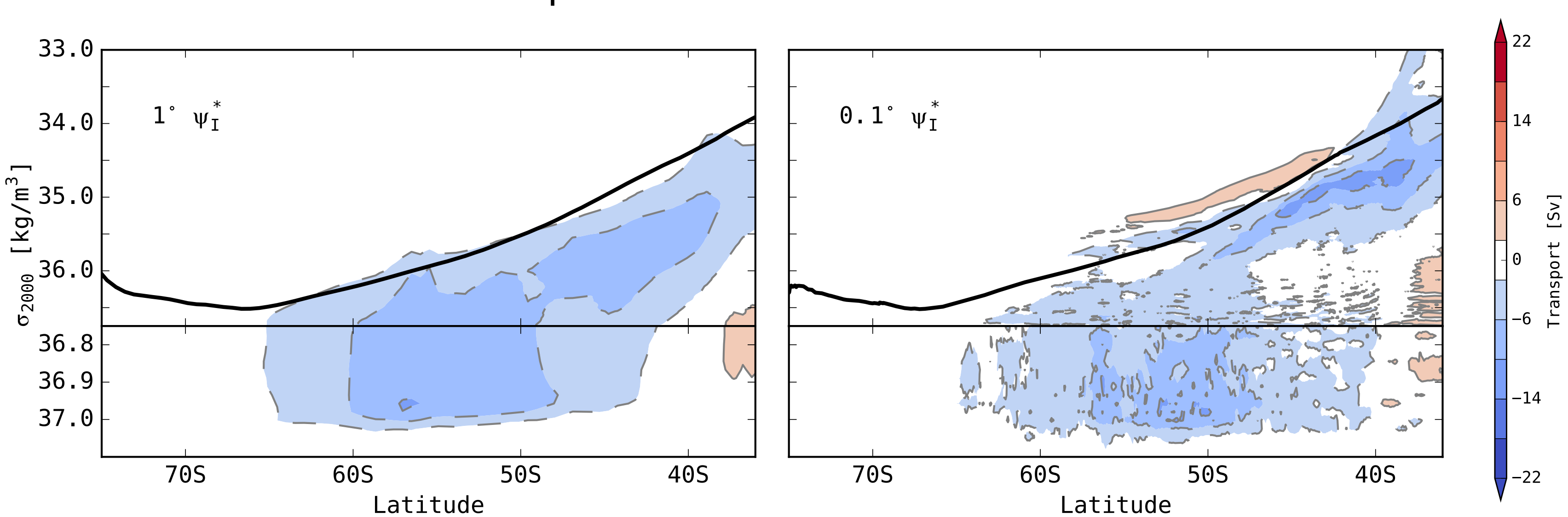
Masich et al. (2015)

1. Surface wind stress balanced by topographic form stress in the Southern Ocean.
2. Vertical momentum transfer through eddy interfacial form stress.
3. What sets the structure of the stress?

$$\frac{f_0}{\mathcal{N}_0^2}\overline{b'\mathbf{u}_g'} = -\frac{1}{\rho_0}\overline{\mathbf{k}\times(\eta'\nabla p')} = \frac{\mathbf{k}\times\tau_i}{\rho_0}$$

Johnson and Bryden (1989)

… and how should we parameterise these fluxes in coarse resolution OGCMs?



Parameterised

Resolved

$$\overline{b'\mathbf{u}_g'} = -\kappa\nabla\overline{b}$$

Poulsen et al. (2018)

Outline

1. Introduce geometric decomposition of the eddy buoyancy flux and its interpretation with respect to eddy-mean flow interaction.
2. Diagnose geometry in an eddy-resolving ocean model.
3. Discuss the possibility to parameterise eddy buoyancy fluxes based on the geometric decomposition.

Eddy stress tensor

Reynolds-averaged quasi-geostrophic momentum equation

$$\frac{\overline{D}\overline{\mathbf{u}_g}}{Dt} + \cdots = -\nabla \cdot \mathbf{E} \qquad \mathbf{E} = \begin{bmatrix} -M+P & N & 0 \\ N & M+P & 0 \\ -S & R & 0 \end{bmatrix}$$

The norm of the Eliassen-Palm flux tensor is bounded by the eddy energy E (kinetic + potential) and allow us to rewrite its components in terms of E and a set of <u>bounded</u> non-dimensional parameters (Marshall et al. 2012):

$$K = \frac{\overline{u_g' u_g'} + \overline{v_g' v_g'}}{2} \qquad P = \frac{\overline{b' b'}}{2\mathcal{N}_0^2}$$

$$M = \frac{\overline{v_g' v_g'} - \overline{u_g' u_g'}}{2} \qquad N = \overline{u_g' v_g'}$$

$$R = \frac{f_0}{\mathcal{N}_0^2}\overline{b' u_g'} \qquad S = \frac{f_0}{\mathcal{N}_0^2}\overline{b' v_g'}$$

$\longrightarrow$

$$E = K + P$$

$$M = -\gamma_m K \cos(2\phi_m)$$

$$N = \gamma_m K \sin(2\phi_m)$$

$$R = \gamma_t \frac{f_0}{\mathcal{N}_0} E \cos(\phi_b) \sin(2\phi_t)$$

$$S = \gamma_t \frac{f_0}{\mathcal{N}_0} E \sin(\phi_b) \sin(2\phi_t)$$

This decomposition is related to the geometry of two ellipses!

Horizontal ellipse geometry

Reynolds stress ellipse

$$\mathbf{E}_{\mathrm{bt}} = \begin{bmatrix} -M+K & N \\ N & M+K \end{bmatrix}$$

Eigenvectors define the orientation of ellipse and eigenvalues the length of the ellipse axes.

<u>Size</u>

$$\Lambda_{\pm} = K\left(1 \pm \gamma_m\right)$$

$$r_{\pm} = \sqrt{\Lambda_{\pm}}$$

<u>Eccentricity</u>

$$\gamma_m = \frac{\Lambda_+ - \Lambda_-}{\Lambda_+ + \Lambda_-} = \frac{\sqrt{M^2+N^2}}{K}$$

$$0 \leq \gamma_m \leq 1$$

<u>Orientation</u>

$$\tan\left(2\phi_m\right) = -\frac{N}{M}$$

It is possible to express M and N through the ellipse geometry (see e.g. Waterman and Lilly (2015)).

$$M = -\gamma_m K \cos\left(2\phi_m\right)$$

$$N = \gamma_m K \sin\left(2\phi_m\right)$$

Vertical ellipse geometry

Interfacial form stress ellipsoid

$$\mathbf{E}_{\mathrm{bc}} = \begin{bmatrix} K & 0 & -\frac{\mathcal{N}_0}{2f_0}S \\ 0 & K & \frac{\mathcal{N}_0}{2f_0}R \\ -\frac{\mathcal{N}_0}{2f_0}S & \frac{\mathcal{N}_0}{2f_0}R & P \end{bmatrix}$$

The intersection between the ellipsoid and a vertical plane along the ellipsoid major axis traces out a vertical ellipse which is perpendicular to the eddy buoyancy flux vector.

Size

$$\Lambda_{\pm} = \frac{E}{2}\left(1 \pm \gamma_t\right)$$

$$r_{\pm} = \sqrt{\Lambda_{\pm}}$$

Eccentricity

$$\gamma_t = \sqrt{\cos^2\left(2\lambda\right) + \sin^2\left(2\lambda\right)\gamma_b^2}$$

$$\gamma_b = \frac{\mathcal{N}_0}{2f_0}\sqrt{\frac{R^2+S^2}{KP}}$$

$$0 \le \gamma_b \le 1$$

Orientation

$$\tan\left(2\phi_t\right) = \gamma_b \tan\left(2\lambda\right)$$

$$\tan\left(\phi_b\right) = \frac{S}{R}$$

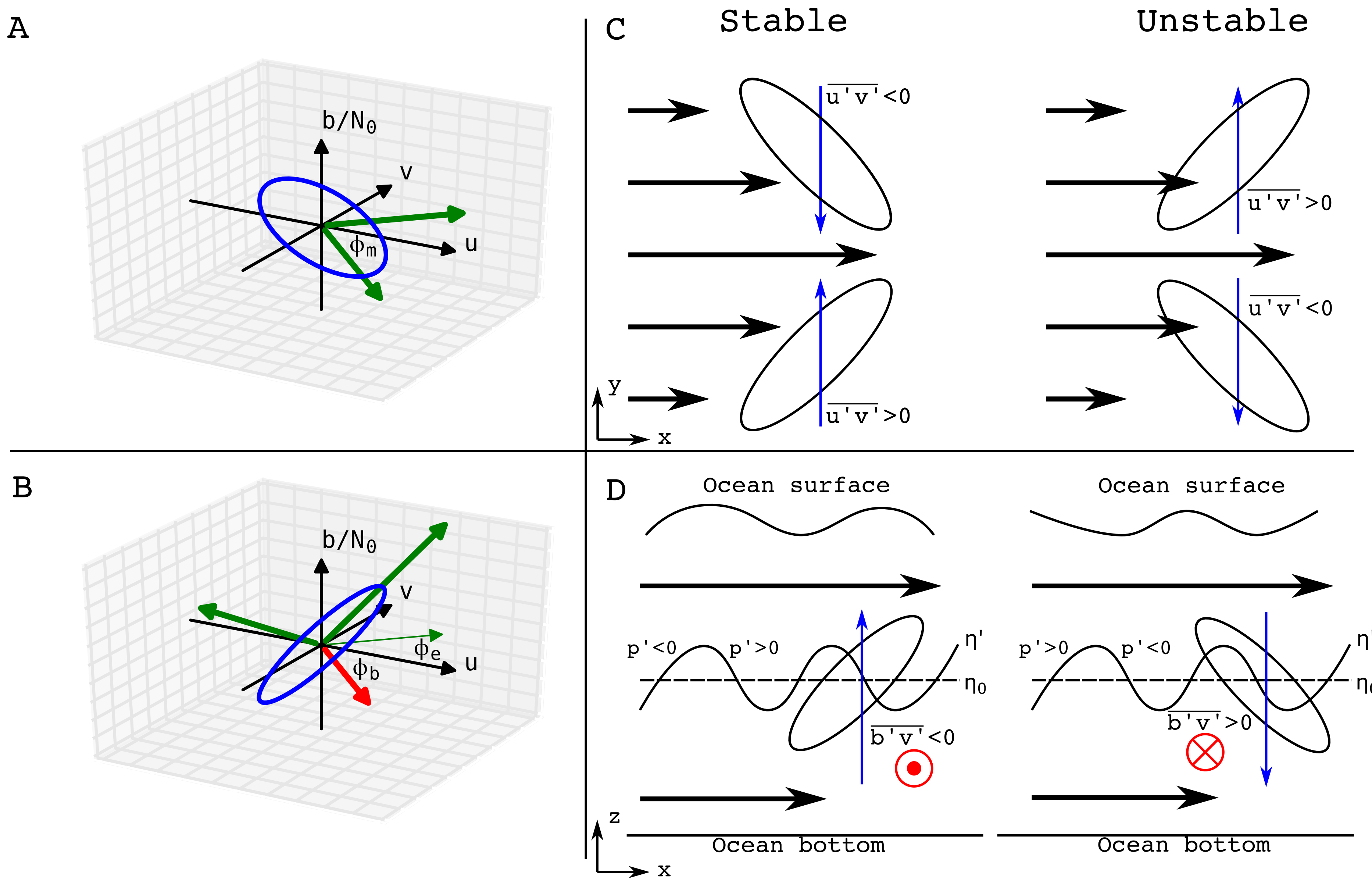
Eddy energy partitioning

$$\frac{P}{E} = \sin^2\left(\lambda\right) \qquad \frac{K}{E} = \cos^2\left(\lambda\right)$$

Decomposition

$$\left(R, S\right) = \gamma_t \frac{f_0}{\mathcal{N}_0} E \sin\left(2\phi_t\right)\left(\cos\left(\phi_b\right), \sin\left(\phi_b\right)\right)$$

Physical interpretation of ellipse geometry



A parameterisation of the eddy geometry is an alternative approach to represent eddy fluxes of buoyancy and momentum in coarse resolution ocean models.

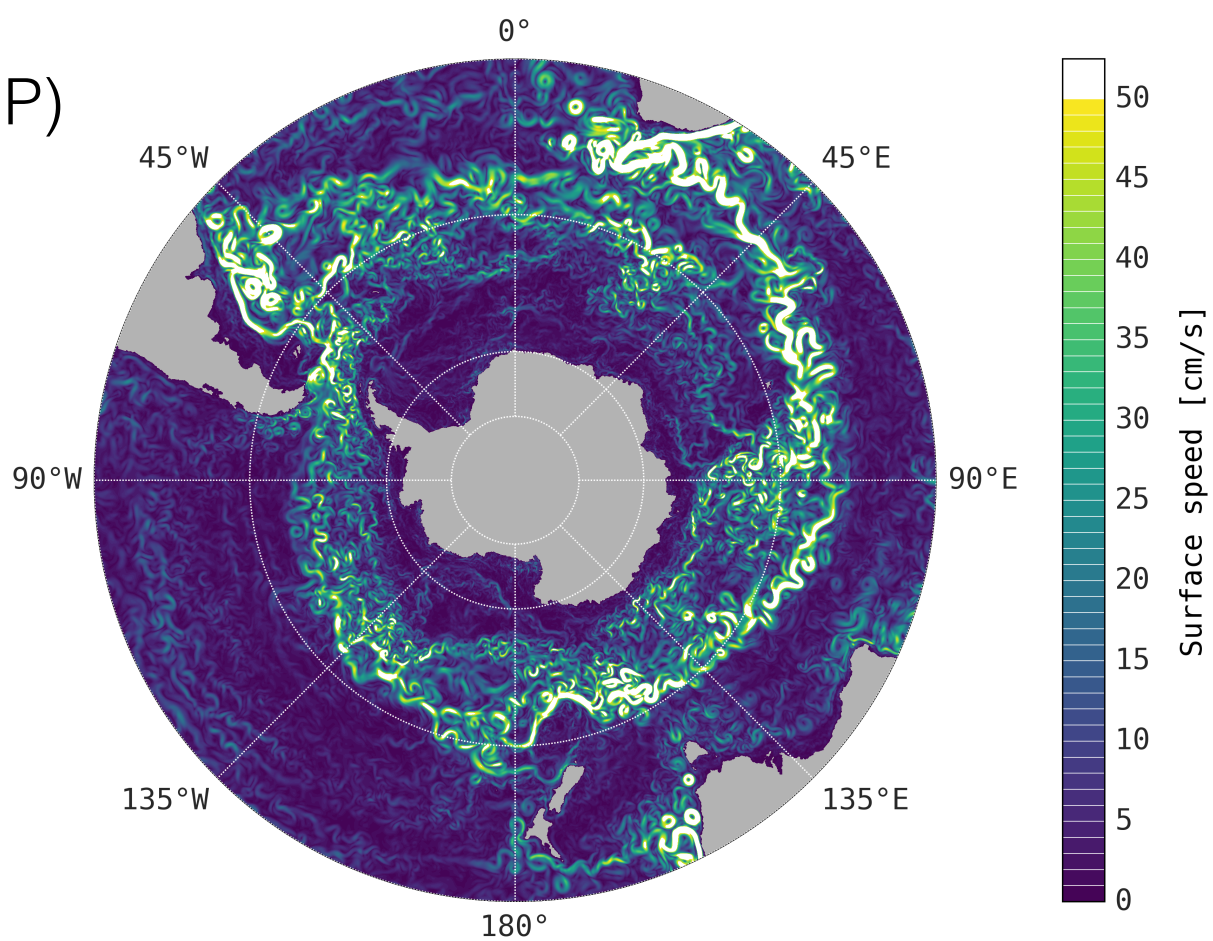
Diagnosing vertical ellipse geometry

The Reynolds stress ellipse geometry in an eddying ocean model has been examined in the recent past (Stewart et al. 2015).

The goal of this study is to diagnose the vertical ellipse geometry in an eddy-resolving ocean GCM and explore the possibility to parameterise it.

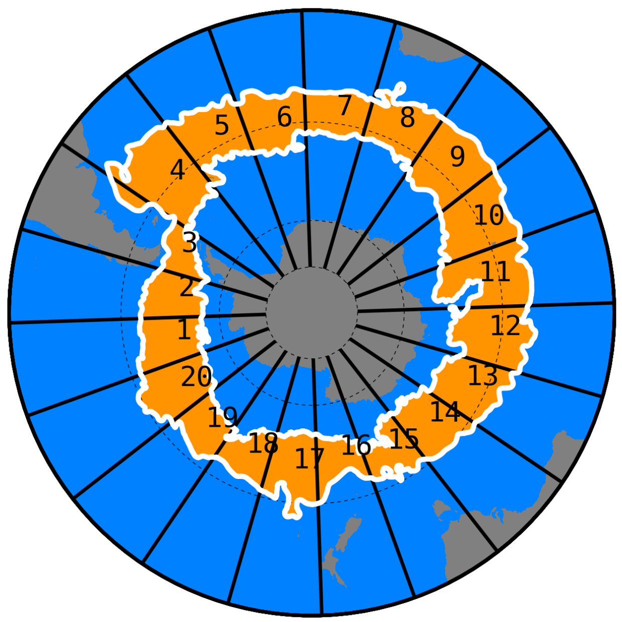
- Global ocean general circulation model (POP)
- CORE.v2 normal year forcing fields
- Active sea ice model
- 0.1 deg. horizontal resolution
- 62 vertical levels
- 15 year spin-up
- ~25 year control simulation
- Three-day mean output
- Offline computation of eddy statistics using ten model years

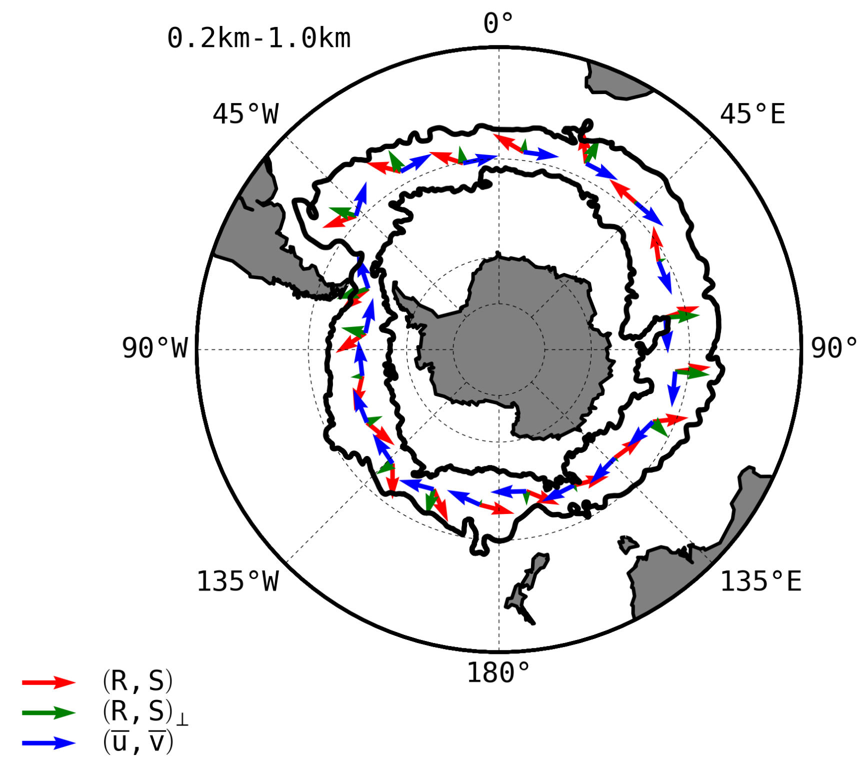
$$(R, S) = \gamma_t \frac{f_0}{N_0} E \sin(2\phi_t) \left(\cos(\phi_b), \sin(\phi_b)\right)$$



Horizontal orientation of vertical ellipse

ACC divided into segments for averaging.

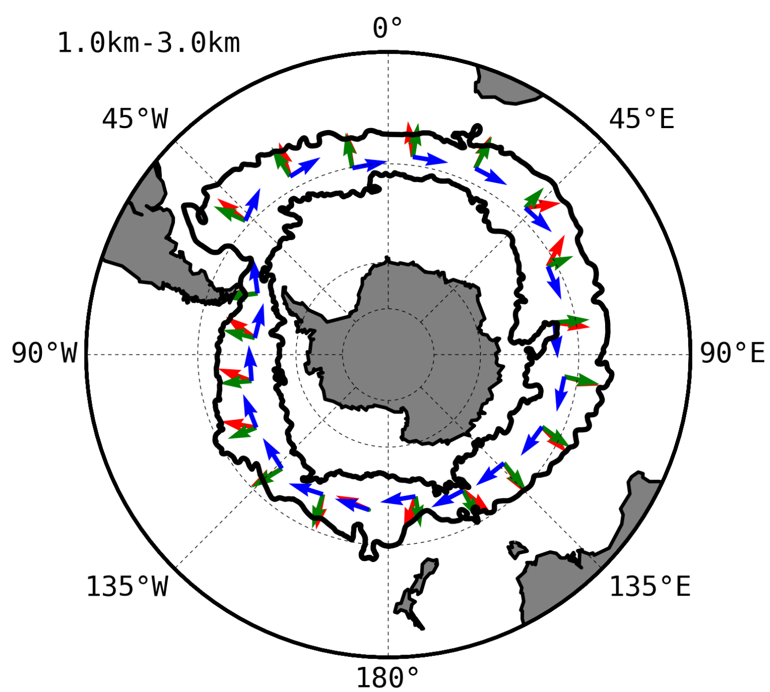


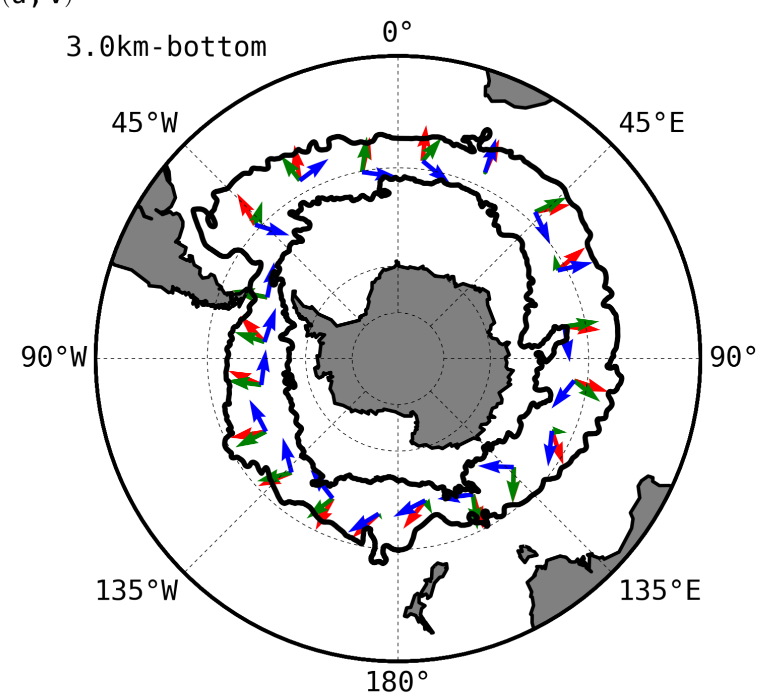


(R,S) tends to be anti-parallel to mean-flow.

$$\overline{b'\mathbf{u}'_g} \stackrel{?}{=} -\kappa\nabla\overline{b}$$

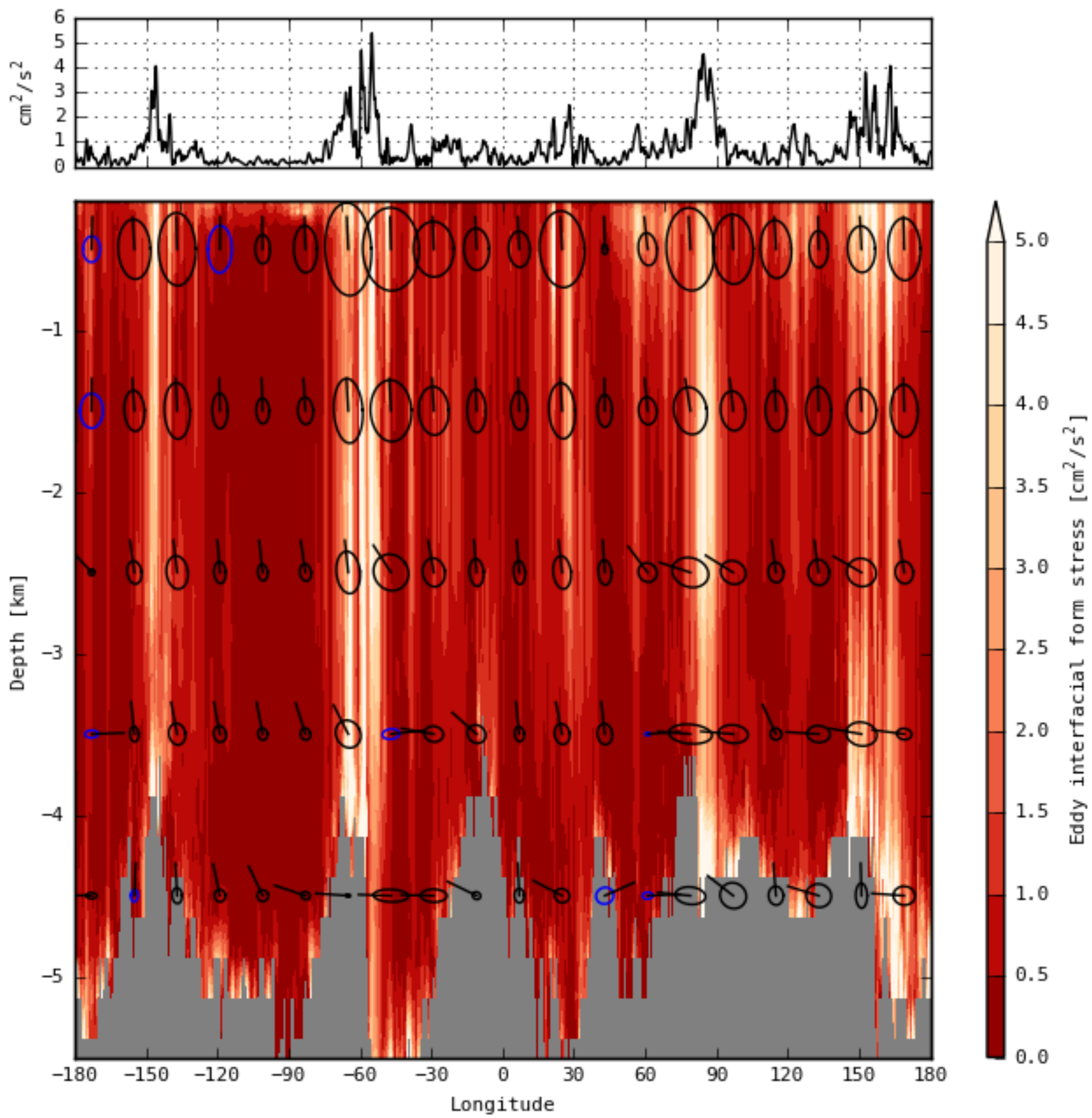
or rotational component?





(R,S) tends to be perpendicular to mean flow.

Geometric representation of eddy interfacial form stress



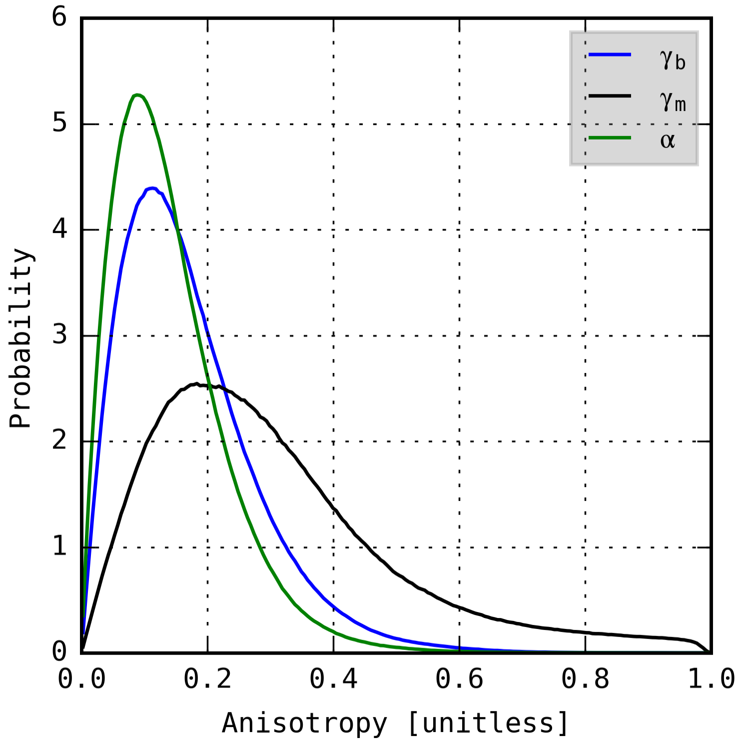
Black ellipse leans into the shear

Blue ellipse leans with the shear

$$\sqrt{R^2 + S^2} = \gamma_t \frac{f_0}{\mathcal{N}_0} E \sin(2\phi_t)$$

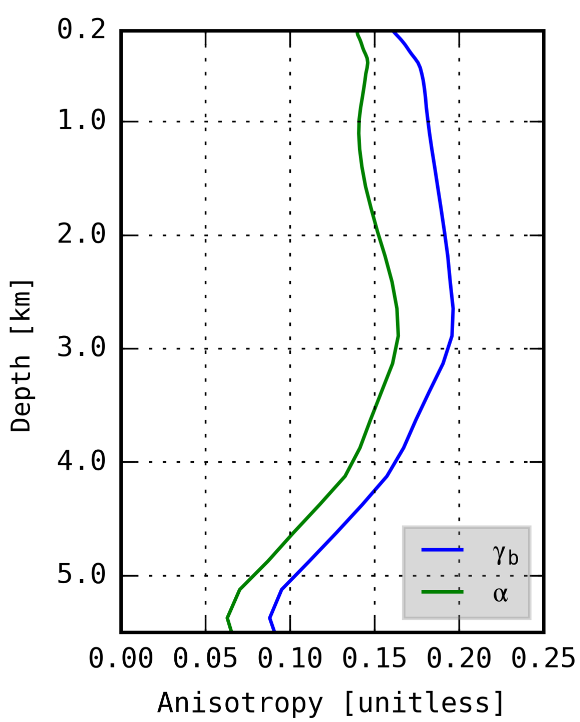
- Majority of ellipses lean into the shear (downward momentum transfer).
- Eddy energy decreases with depth.
- P > K in upper ocean, P < K in abyss.
- A general weak polarisation of eddy buoyancy fluxes.

Is it possible to parameterise the anisotropy?

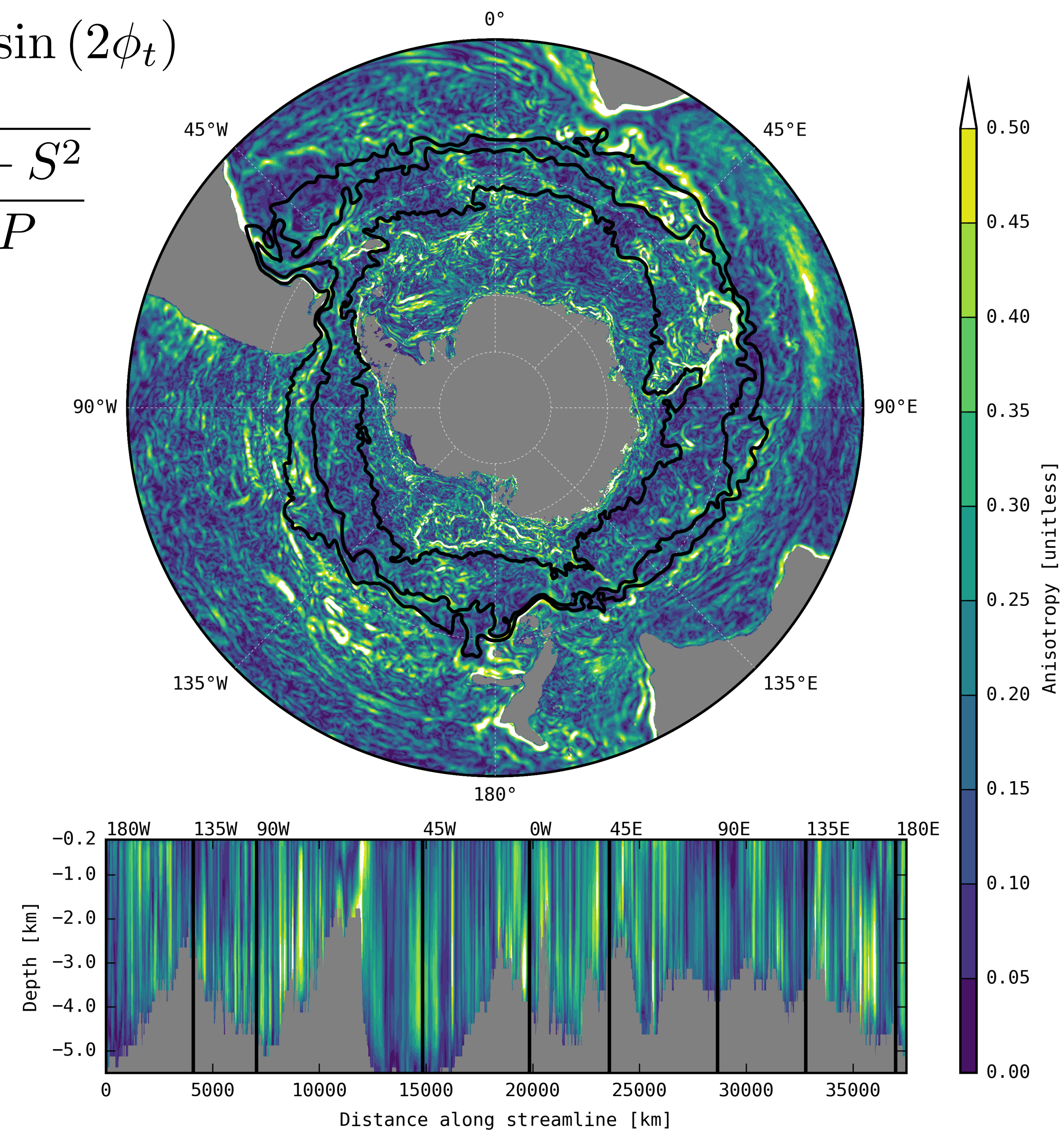


$$\gamma_b \sin(2\lambda) = \gamma_t \sin(2\phi_t)$$

$$\gamma_b = \frac{\mathcal{N}_0}{2f_0}\sqrt{\frac{R^2+S^2}{KP}}$$



The anisotropy is depth-insensitive in upper ~3km. Possible to reduce the problem from 3D to 2D?



Conclusions

We reformulate and interpret eddy buoyancy fluxes in terms of ellipse geometry and explore possible ways to parameterise the geometry in OGCMs.

1. Eddy buoyancy flux vector tends to be perpendicular to mean-flow direction below 1km depth in the Southern Ocean.
2. Spatially-varying downward momentum transfer throughout the circumpolar path.
3. The eddy anisotropy is generally weak, but large where topography steers the flow and depth-insensitive in the interior ocean.