Behaviour of Cloud Models as seen from Asymptotic Analysis

Hamburg 2018

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April 6, 2018

Table of contents

- Clouds described by ordinary differential equations
- Derivation from first principles not possible
- Many different parameterisations
- Contain nonlinear terms
- Not clear if long term behaviour is at least similar

How can we compare the behaviour of different cloud models?

Ideas:

- Asymptotic analysis
- Theory of dynamical systems

Assumption:

- Parcel model
- Constant environmental conditions
  - temperature, pressure, supersaturation

Modeled quantities:

- Distinguish between cloud droplets and rain drops
- Cloud droplets: small, do not fall
- Rain drops: large, fall due to gravity

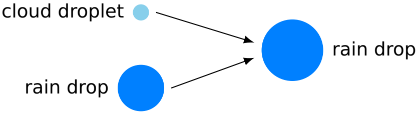
Mass concentrations:

$$q_c = \frac{\text{mass of cloud droplets}}{\text{mass of dry air}}$$
$$q_r = \frac{\text{mass of rain drops}}{\text{mass of dry air}}$$

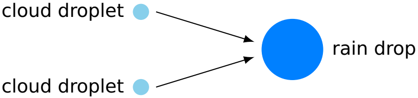
General cloud model II

Modeled cloud processes:

- Accretion: collisional process



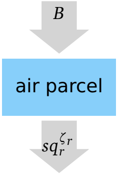
- Autoconversion: collisional process



- Condensation: water vapour $\longleftrightarrow$ cloud droplets
- Evaporation: rain drops $\longleftrightarrow$ water vapour
- Sedimentation: rain drops leave air parcel due to gravity

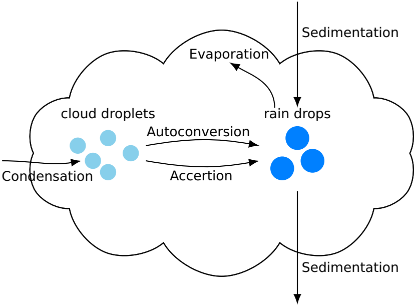
Special treatment:

- Condensation - explicit parameterisation, no saturation adjustment
- Sedimentation



Model:

$$\frac{\mathrm{d}q_c'}{\mathrm{d}t'} = C - A_1 - A_2$$
$$\frac{\mathrm{d}q_r'}{\mathrm{d}t'} = A_1 + A_2 - E + D$$



Cloud Processes:

- Accretion: $A_1 = a_1' q_c'^{\beta_c} q_r'^{\beta_r}$
- Autoconversion: $A_2 = a_2' q_c'^{\gamma_c}$
- Condensation: $C = c' S q_c'$
- Evaporation: $E = e_1' S q_r'^{\delta_{r1}} + e_2' S q_r'^{\delta_{r2}}$
- Sedimentation: $D = -s' q_r'^{\zeta_r} + B'$

→ Cloud models differ by choice of parameters

Three cloud models

- Wacker: idealized model published by U. Wacker (1992)
- COSMO: warm rain scheme incorporated in COSMO model
- IFS: warm rain scheme incorporated in IFS model

Linear coefficients pressure $p = 10^5\,\mathrm{Pa}$, temperature $T = 300\,\mathrm{K}$

	accretion	autoconversion	evaporation		sedimentation	
	a_1'	a_2'	e_1'	e_2'	s'	B'
Wacker	7.5	10^{-4}	0	0	$3.88 \cdot 10^{-3}$	10^{-7}
COSMO	10^{-3}	1.96	$3.16 \cdot 10^{-5}$	$2.96 \cdot 10^{-4}$	$1.29 \cdot 10^{-2}$	10^{-7}
IFS	134	$7.45 \cdot 10^{-2}$	$1.79 \cdot 10^{-2}$	$4.47 \cdot 10^{-4}$	$4 \cdot 10^{-3}$	10^{-7}

Exponents

	accretion		autoconversion	evaporation		sedimentation
	β_c	β_r	γ_c	δ_{r1}	δ_{r2}	ζ_r
Wacker	1	1	1	-	-	1
COSMO	1	$\frac{7}{8}$	1	$\frac{1}{2}$	$\frac{11}{16}$	$\frac{9}{8}$
IFS	1.15	1.15	2.47	2	0.635	1

Can powers and linear parameters compensate each other?

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Can powers and linear parameters compensate each other?

Reference time and reference value:

$$q_{\mathrm{ref}} = 1.0 \cdot 10^{-4} \qquad t_{\mathrm{ref}} = 1\,\mathrm{s}$$

Transformation:

$$q_c(t) = q_{ref}\, q_c'\big(t'\big) \qquad q_r(t) = q_{ref}\, q_r'\big(t'\big) \qquad t = t_{ref}\, t'$$

Compute rescaled derivatives and right hand side (RHS)
Reference system with renamed constants:

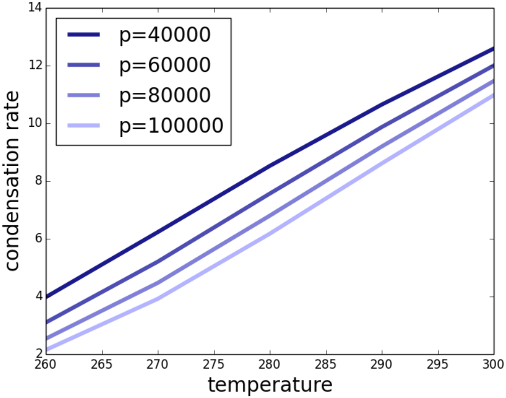
$$\frac{\mathrm{d}q_c}{\mathrm{d}t} = cSq_c - a_1 q_c^{\beta_c} q_r^{\beta_r} - a_2 q_c^{\gamma_c}$$
$$\frac{\mathrm{d}q_r}{\mathrm{d}t} = a_1 q_c^{\beta_c} q_r^{\beta_r} + a_2 q_c^{\gamma_c} - e_1 S q_r^{\delta_{r1}} - e_2 S q_r^{\delta_{r2}} - s q_r^{\zeta} + B$$

Due to scaling, we can expect $q_c \approx 1, q_r \approx 1$.

Asymptotic investigations - Distinguished Limit

1. c, a_1, a_2, e_1, e_2, B expressed in ε powers
2. Supersaturation S expressed as $S = \varepsilon^{\alpha}$

Caution: Some parameters depend on temperature and/or pressure!



Behaviour of models on different time scales

- Choose the time scale by another time transformation

$$\tau = \varepsilon^{\omega} t \qquad q_c^*(\tau) = q_c(t) \qquad q_r^*(\tau) = q_r(t)$$

Get a system of the form

$$\frac{\mathrm{d}q_c^*}{\mathrm{d}\tau} = c^* \varepsilon^{\mu} q_c^* + \dots$$
$$\frac{\mathrm{d}q_r^*}{\mathrm{d}\tau} = a_1^* \varepsilon^{\nu} q_c^{*\beta_c} q_r^{*\beta_r} + \dots$$

with $c^*, a_1^*, \dots \in \mathcal{O}(1)$. We choose the ansatz:

$$q_c^* = q_c^{*(0)} + \varepsilon q_c^{*(1)} + \varepsilon^2 q_r^{*(2)} + \dots, \quad q_r^* = q_r^{*(0)} + \varepsilon q_r^{*(1)} + \varepsilon^2 q_r^{*(2)} + \dots$$

- Ansatz is plugged in RHS of above model
- Its derivative on the left hand side
- Order the ε powers

Asymptotic investigations - Reduced Models II

- Supersaturation corresponding to $\alpha = 3$ ($S = 0.1\%$)
- Time scale of $\tau = \varepsilon^3 t$ (1000 seconds)

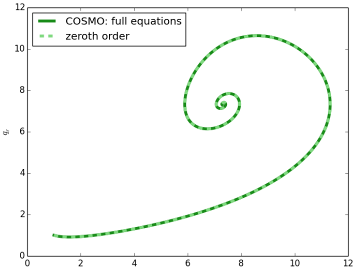
Order of processes

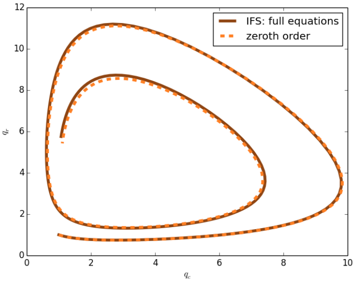
	Wacker	COSMO	IFS
accretion	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
autoconversion	$\mathcal{O}(\varepsilon)$	$\mathcal{O}(1)$	$\mathcal{O}\left(\varepsilon^3\right)$
condensation	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
evaporation	-	$\mathcal{O}\left(\varepsilon^3\right)$	$\mathcal{O}\left(\varepsilon^2\right)$
sedimentation	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$

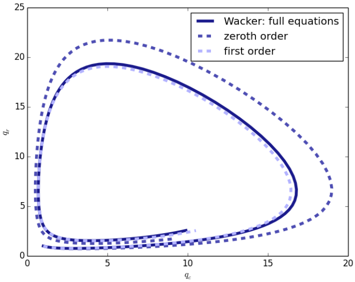
- Accretion, condensation, sedimentation are comparable
- Autoconversion varies with model
- Evaporation of minor importance

	leading order equation
Wacker	$\dot{q}_c^{(0)} = cq_c^{(0)} - a_1 q_c^{(0)} q_r^{(0)}$ $\dot{q}_r^{(0)} = a_1 q_c^{(0)} q_r^{(0)} - sq_r^{(0)} + B$
COSMO	$\dot{q}_c^{(0)} = cq_c^{(0)} - a_1 q_c^{(0)} q_r^{(0)\frac{7}{8}} - a_2 q_c^{(0)}$ $\dot{q}_r^{(0)} = a_1 q_c^{(0)} q_r^{(0)\frac{7}{8}} + a_2 q_c^{(0)} - sq_r^{(0)\frac{9}{8}} + B$
IFS	$\dot{q}_c^{(0)} = cq_c^{(0)} - a_1 q_c^{(0)1.15} q_r^{(0)1.15}$ $\dot{q}_r^{(0)} = a_1 q_c^{(0)1.15} q_r^{(0)1.15} - sq_r^{(0)} + B$

- Condensation, accretion, sedimentation on same time scale
- Discrepancy concerning time scale of autoconversion
- Predator-prey system with constant forcing







- First order approximation:

$$q_c \approx q_c^{(0)} + \varepsilon q_c^{(1)}$$
$$q_r \approx q_r^{(0)} + \varepsilon q_r^{(1)}$$

- First order approximation of Wacker model is bad for large time scales

- Keep supersaturation corresponding to $\alpha = 3$ ($S = 0.1\%$)
- Change time scale and choose $\tau = \varepsilon^2 t$ (100 seconds)

	Wacker	COSMO	IFS
accretion	$\mathcal{O}(\varepsilon)$	$\mathcal{O}(\varepsilon)$	$\mathcal{O}(\varepsilon)$
autoconversion	$\mathcal{O}\left(\varepsilon^2\right)$	$\mathcal{O}(\varepsilon)$	$\mathcal{O}\left(\varepsilon^4\right)$
condensation	$\mathcal{O}(\varepsilon)$	$\mathcal{O}(\varepsilon)$	$\mathcal{O}(\varepsilon)$
evaporation	-	$\mathcal{O}\left(\varepsilon^4\right)$	$\mathcal{O}\left(\varepsilon^3\right)$
sedimentation	$\mathcal{O}(\varepsilon)$	$\mathcal{O}(\varepsilon)$	$\mathcal{O}(\varepsilon)$

- System scales linearly in time

Asymptotic investigations - Reduced Models VI

	zeroth order equation
Wacker	$\dot{q}_c^{(0)} = 0$ $\dot{q}_r^{(0)} = 0$
COSMO	$\dot{q}_c^{(0)} = 0$ $\dot{q}_r^{(0)} = 0$
IFS	$\dot{q}_c^{(0)} = 0$ $\dot{q}_r^{(0)} = 0$

- Constant solutions in leading order

$\rightarrow$ Inital value is reproduced

- Time scale small, cloud microphysics has not yet started
- No valid long term solution

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	first order equation
Wacker	$\dot{q}_c^{(1)} = c q_c^{(0)} - a_1 q_c^{(0)} q_r^{(0)}$ $\dot{q}_r^{(1)} = a_1 q_c^{(0)} q_r^{(0)} - s q_r^{(0)} + B$
COSMO	$\dot{q}_c^{(1)} = c q_c^{(0)} - a_1 q_c^{(0)} q_r^{(0)\frac{7}{8}} - a_2 q_c^{(0)}$ $\dot{q}_r^{(1)} = a_1 q_c^{(0)} q_r^{(0)\frac{7}{8}} + a_2 q_c^{(0)} - s q_r^{(0)\frac{9}{8}} + B$
IFS	$\dot{q}_c^{(1)} = c q_c^{(0)} - a_1 q_c^{(0)1.15} q_r^{(0)1.15}$ $\dot{q}_r^{(1)} = a_1 q_c^{(0)1.15} q_r^{(0)1.15} - s q_r^{(0)} + B$

- Changes in first order
- Structure of predator-prey system with constant forcing, **but** $q_c^{(0)} \equiv \text{const}, q_r^{(0)} \equiv \text{const}$
- Polynomial of degree 1 as first order approximation

Asymptotic investigations - Reduced Models VIII

- Increase supersaturation to $\alpha = 2$ ($S = 1\%$)
- Keep small time scale of $\tau = \varepsilon^2 t$ (100 seconds)

	Wacker	COSMO	IFS
accretion	$\mathcal{O}(\varepsilon)$	$\mathcal{O}(\varepsilon)$	$\mathcal{O}(\varepsilon)$
autoconversion	$\mathcal{O}\left(\varepsilon^2\right)$	$\mathcal{O}(\varepsilon)$	$\mathcal{O}\left(\varepsilon^4\right)$
condensation	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
evaporation	-	$\mathcal{O}\left(\varepsilon^3\right)$	$\mathcal{O}\left(\varepsilon^2\right)$
sedimentation	$\mathcal{O}(\varepsilon)$	$\mathcal{O}(\varepsilon)$	$\mathcal{O}(\varepsilon)$

- Condensation and evaporation scale linearly in S
- Condensation dominates all processes

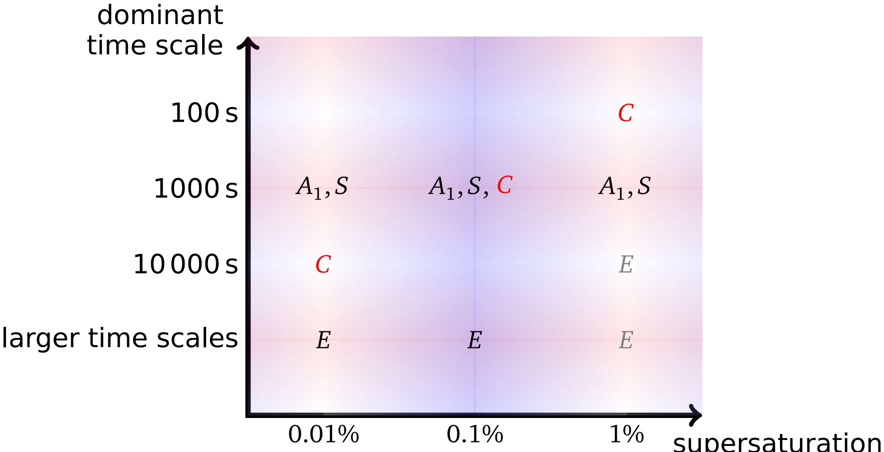
	leading order equation
Wacker	$\dot{q}_c^{(0)} = c q_c^{(0)}$ $\dot{q}_r^{(0)} = 0$
COSMO	$\dot{q}_c^{(0)} = c q_c^{(0)}$ $\dot{q}_r^{(0)} = 0$
IFS	$\dot{q}_c^{(0)} = c q_c^{(0)}$ $\dot{q}_r^{(0)} = 0$

- Condensation is the only process with an impact on the chosen time scale
- Constant solution for q_r
- Exponential solution for q_c

$\longrightarrow$ Not realistic for large time

Asymptotic investigation X

impact of A_2
depends on model



- Assumption: clouds are almost in dynamic equilibirium

$\rightarrow$ Fixed points of cloud models represent cloud

- Usually one fixed point for $q_c = 0$
  - Wacker: $q_c = 0, q_r = 3.88$
  - Unstable if supersaturation is large enough

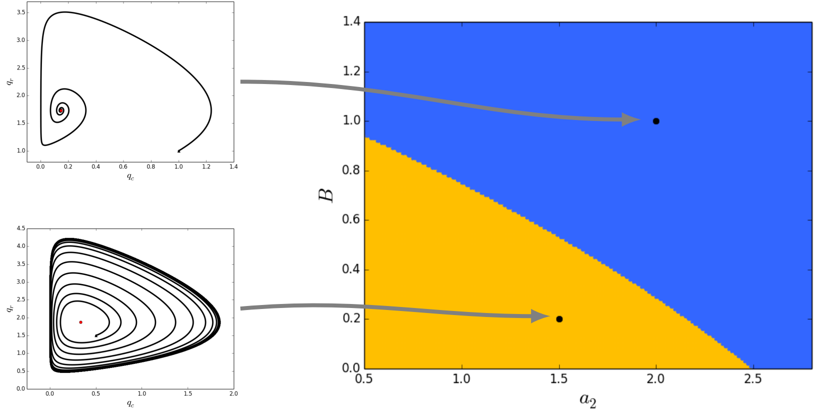
  $\rightarrow$ Interpretation: rain falls through, cloud develops (unstable)
- Usually one further fixed point relevant for describing clouds
  - Wacker: $q_c = 4.87, q_r = 6.53$
- Stability behaviour depends on parameters
- Possibly many fixed points in general cloud model (due to modeling with fractional powers)

Stable fixed points represent the cloud as $t \rightarrow +\infty$.
Asymptotic analysis enables to look at specific time scales.

Bifurcations

- Bifurcations can be detected in the general cloud model
- Example below: all parameters except B and a_2 are fixed

$\longrightarrow$ Autoconversion important for bifurcating behaviour



Fixed point analysis

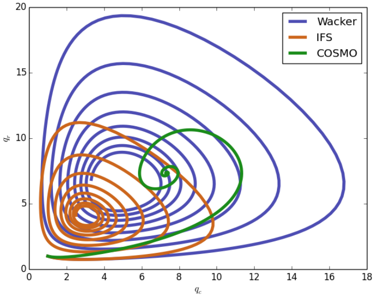
- Consider relevant fixed point $q_c \neq 0$, $S = 0.1\%$, time scale 1000 s
- For nondimensionalized Wacker, COSMO, IFS:

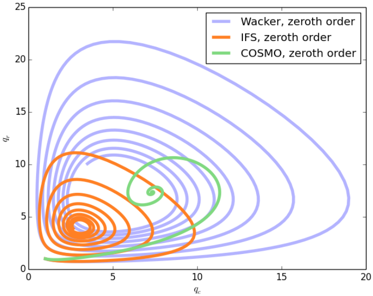
$\rightarrow$ stable fixed points

- Different time scales for approaching fixed point

$\rightarrow$ Wacker, IFS close to bifurcation

$\rightarrow$ Possible periodic feedback to coupled systems





- Models used operationally
- Both models have a fixed points of order 10^{-4}
- COSMO reaches fixed point on time scale of $1000\,\mathrm{s}$
- IFS reaches fixed point on larger time scale

$\rightarrow$ Many cycles around fixed point

$\rightarrow$ Possible periodic feedback to coupled system

- Zeroth order is good approximation for considered case

Conclusion - Asymptotic investigations

- Two options for adjusting the leading order system:
  - time
  - supersaturation
- All processes scale linearly in time
- Condensation, evaporation scale scale linearly with supersaturation
- Time scale of minutes for realistic supersaturations
- Similar models with regard to time scale of cloud processes
- However: Impact of autoconversion depends on model

Conclusion - Fixed point analysis

- One fixed point for $q_c = 0$
  - Unstable due to supersaturation
- Usually one other relevant fixed point describing clouds
- Stable for considered cloud models
- In general stability behaviour depends on parameters
  - Bifurcations possible
  - Also determined by autoconversion
  - Periodic and almost periodic solutions possible
  - In operationally used cloud models: non periodic and almost periodic cloud models
- Can resonances occur?

The End

Thank you for your attention!

References

Mitgedacht

Hier stehen die Antworten auf die Fragen.