A geometric decomposition of eddy-mean flow feedbacks

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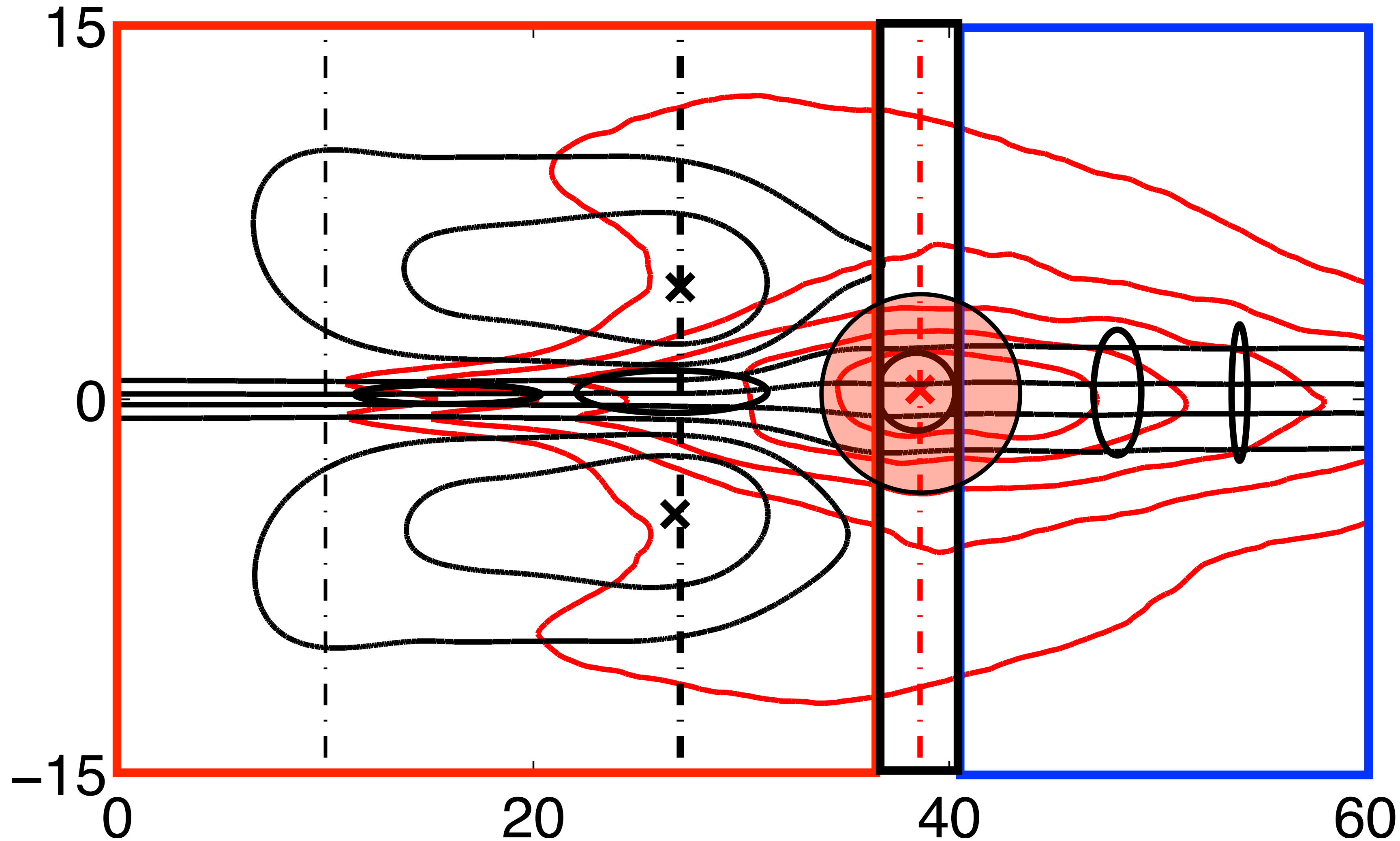


Workshop on Scales and Scaling Cascades in Geophysical Systems
April 4-6 2018
Hamburg, Germany

BACKGROUND & MOTIVATION

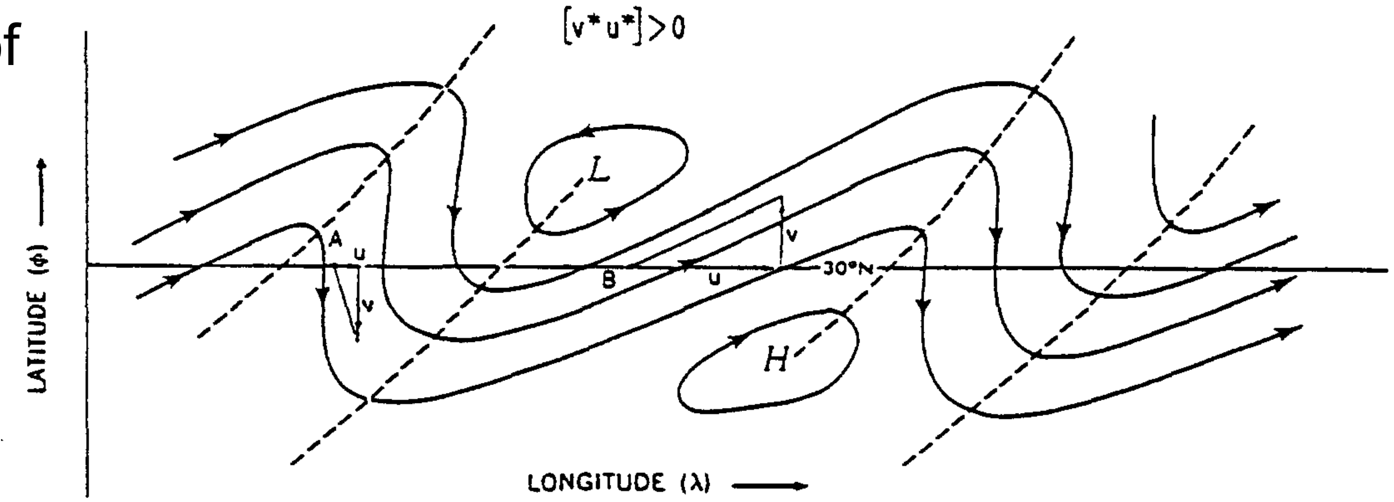
Ways to know & understand eddy feedbacks

- “eddies” play critical roles in the zonally-averaged energy and momentum budgets of the atmospheric general circulation
  - north-south heat transport
  - north-south momentum transport

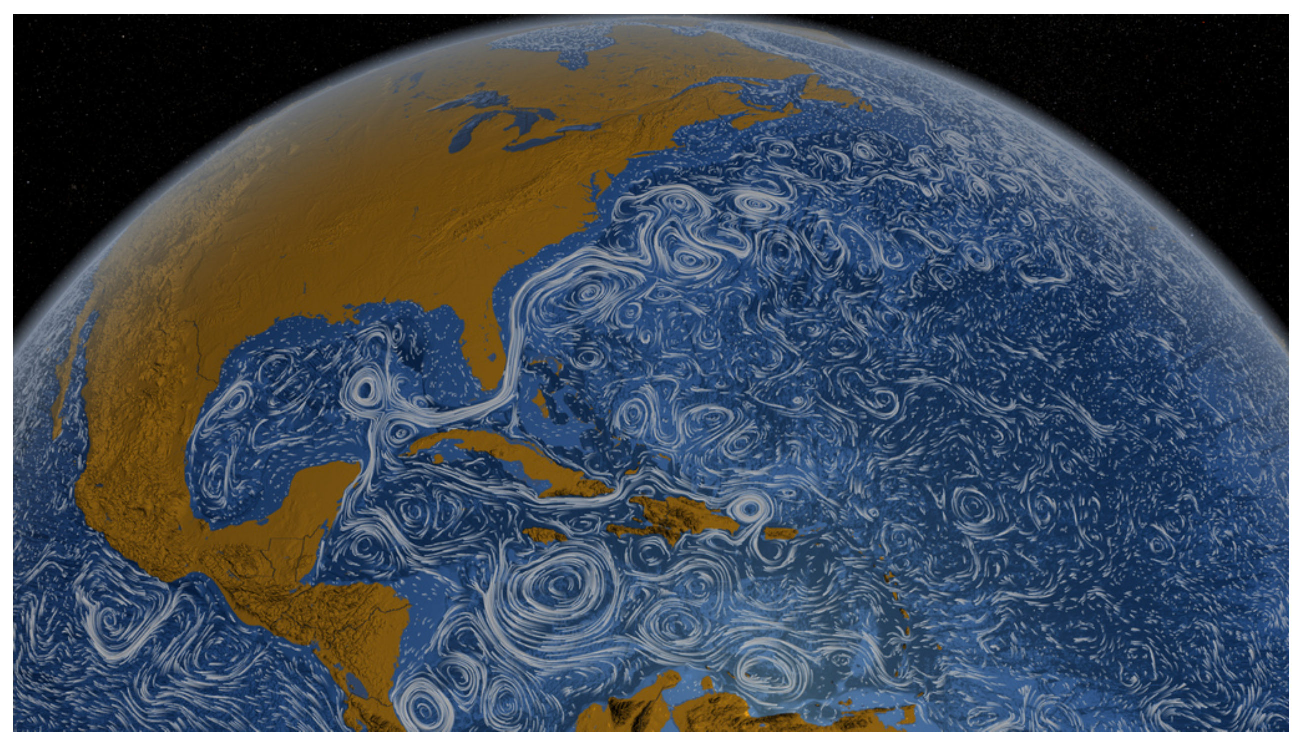
- “eddies” also play critical roles in the oceanic circulation
  - limiting the further acceleration of upper ocean circulation
  - transferring energy and momentum from the upper to the deep ocean
  - establishing mean deep motions

- common to pose the eddy-mean flow interaction problem in a zonal-mean framework but this is not universally relevant nor best for the task of parameterization

- a generalized understanding of role of transient “eddies” in the slowly-evolving large-scale circulation is less well-understood



[ECMWF Meteorological Training Course Lecture Series, 2002]

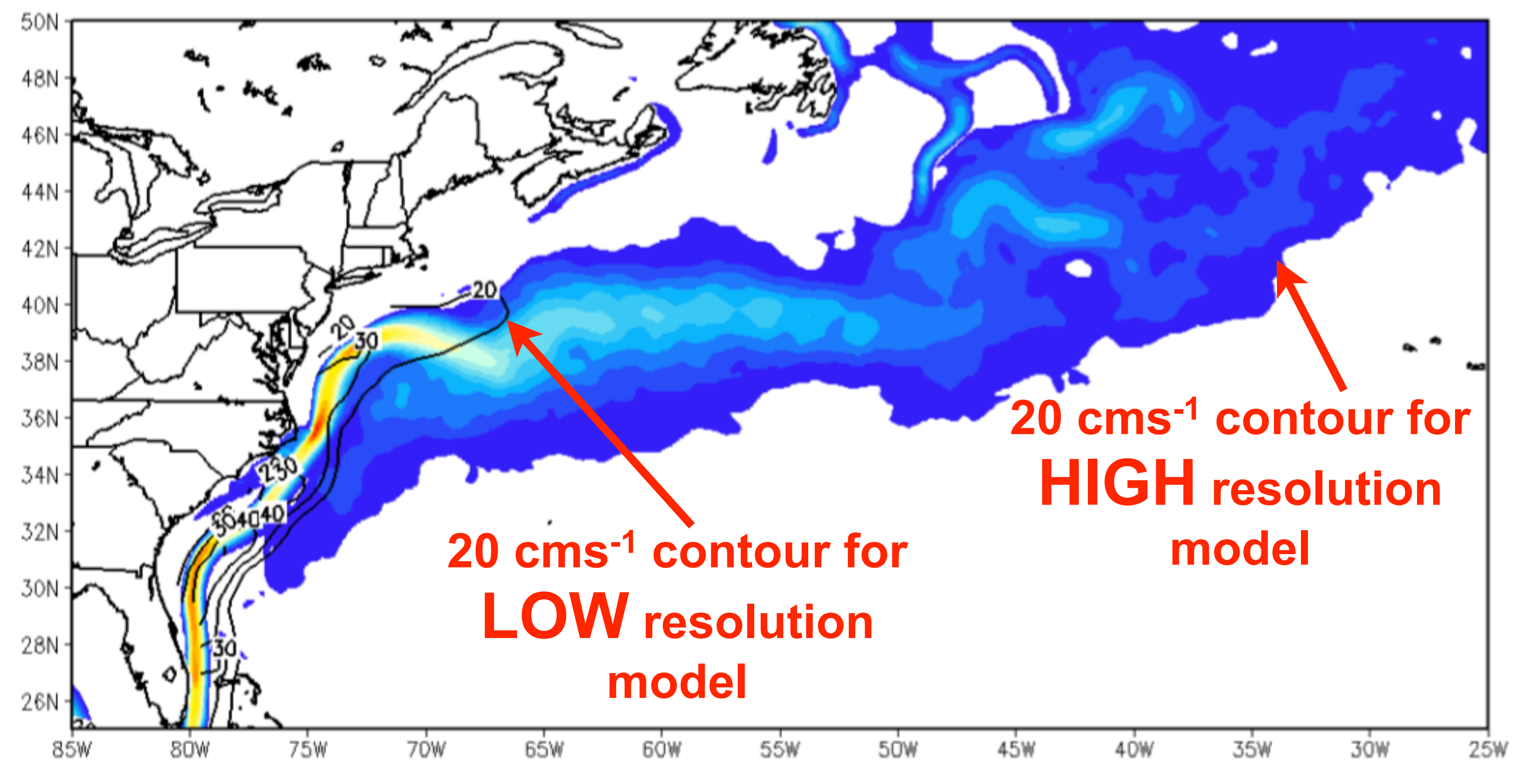


[NASA]

Modern Relevance

= representing eddy effects in coarse resolution models

- resolving mesoscale (O(10 km)) eddy variability in the ocean component model matters for the simulation of large-scale climate
- we need to parameterize both individual processes **and** eddy interactions & feedbacks
- pressing need for parameterizations that scale with resolution and are appropriate for *partially resolved* eddy fields
- the task of parameterization requires identification of the larger-scale (resolved) eddy characteristics that give indication of eddy feedbacks



Annual mean surface current speeds (cms^{-1})

[Kirtman *et al.*, in prep]

1. The Reynolds Decomposition

$$\boxed{U = \underbrace{\bar{U}}_{\text{'mean'}} + \underbrace{U'}_{\text{'eddy'}}}$$

substitution into the governing equations

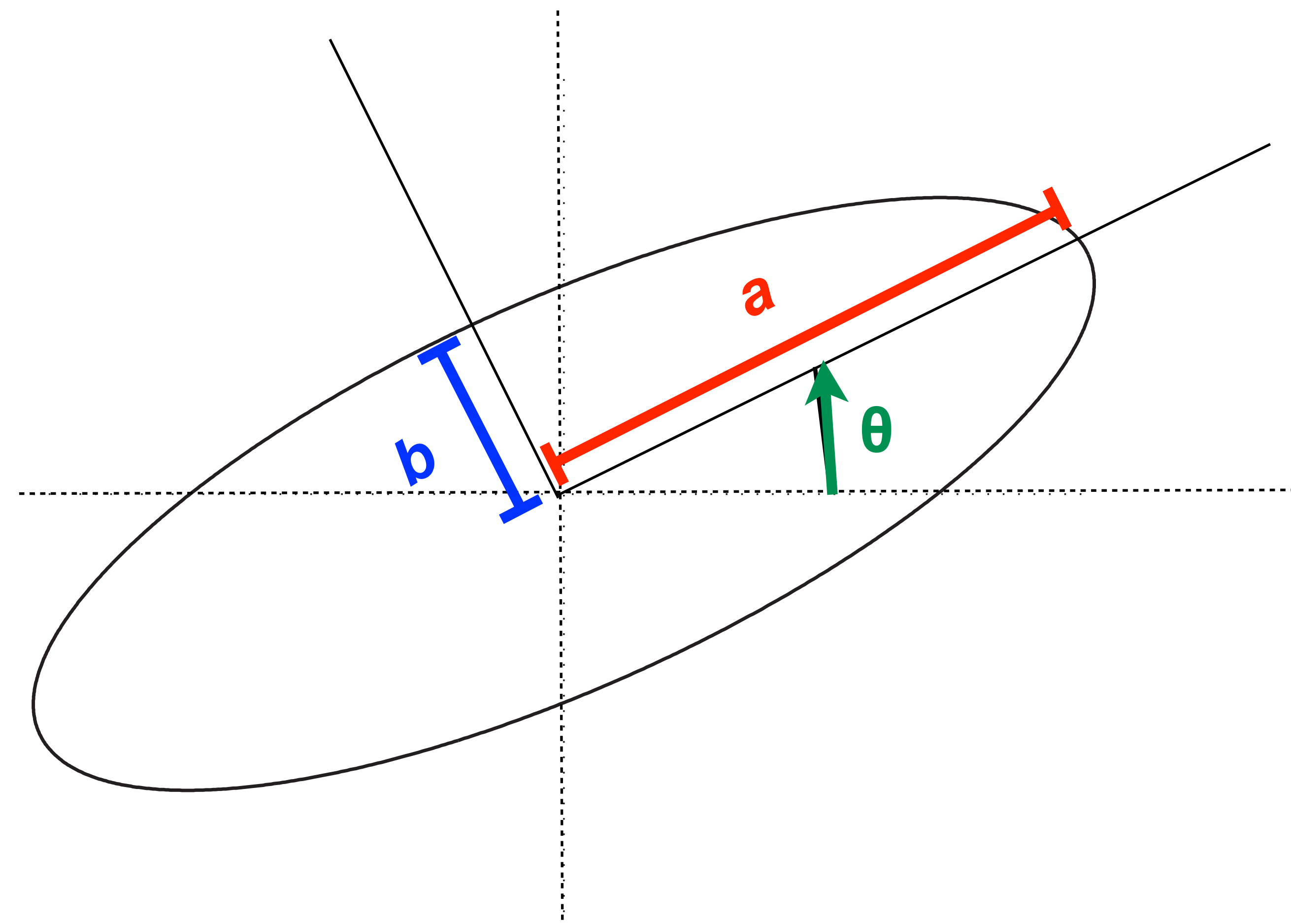
mean field equations = mean eddy flux divergence

'eddy forcing'

- understanding eddy-mean flow feedbacks = understanding mean eddy flux divergences & their impact on the mean flow

2. The Variance Ellipse

- encode info about eddy kinetic energy and the anisotropy & orientation of eddy fluxes
- describe properties of the time-mean eddy motion & eddy forcing of the mean flow



$$a^2 = \overline{u'u'}\cos^2\theta + \overline{u'v'}\sin(2\theta) + \overline{v'v'}\sin^2\theta$$

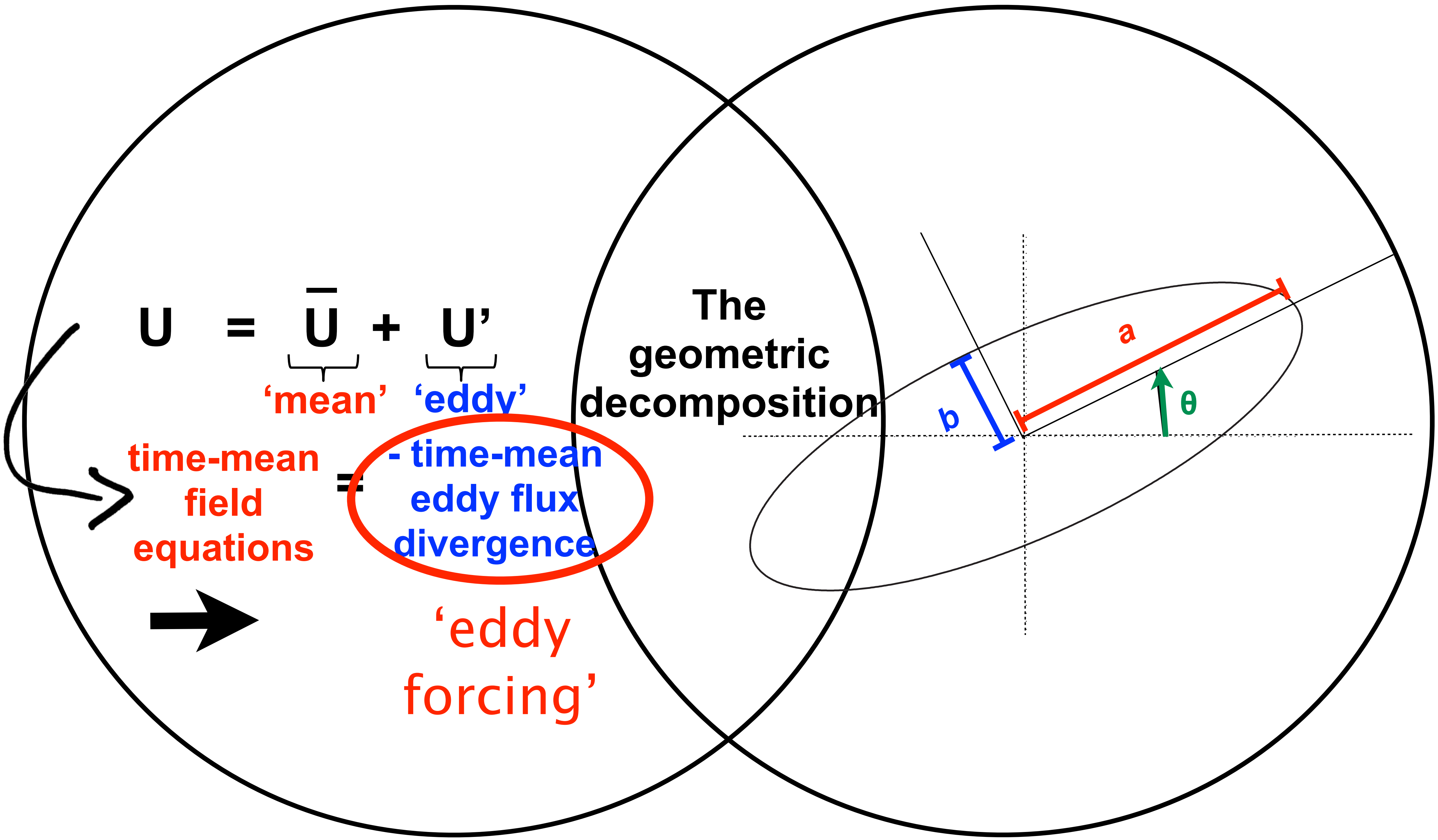
$$b^2 = \overline{u'u'}\cos^2\phi + \overline{u'v'}\sin(2\phi) + \overline{v'v'}\sin^2\phi$$

$$\theta = \tfrac{1}{2}\tan^{-1}\left(\frac{2\,\overline{u'v'}}{\left(\overline{u'u'} - \overline{v'v'}\right)}\right), \quad \phi = \theta + \tfrac{\pi}{2}$$

TODAY'S TALK: BIG PICTURE

The Geometric Decomposition of Eddy Feedbacks

= eddy forcing as the sum of patterns in various aspects of variance ellipse geometry



Talk Overview

1. derivation of the geometric decomposition framework

2. application to an idealized WBC jet

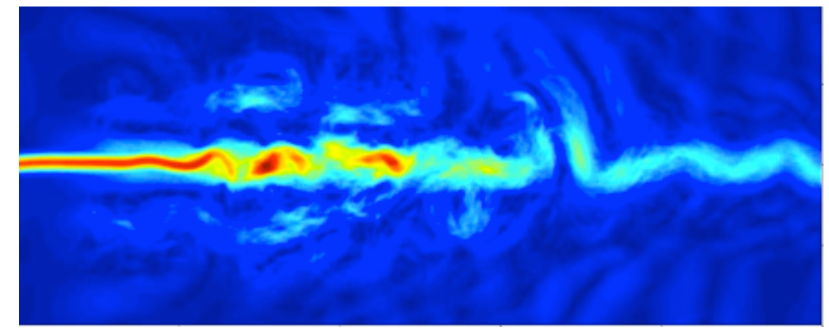
3. new insights into the loss of eddy effects with coarsening model resolution

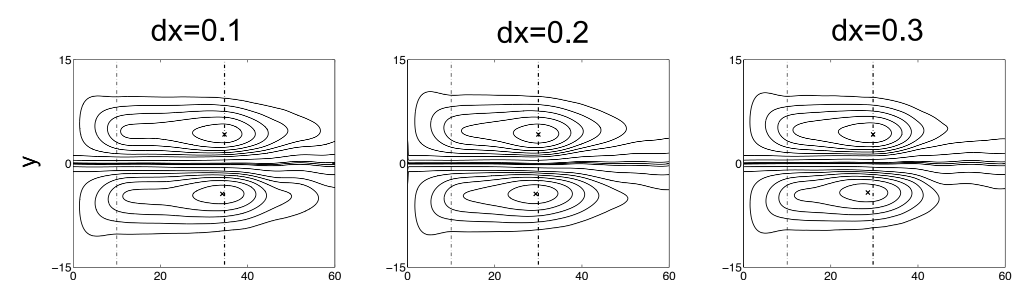
4. application to global observations and model output (work in progress)

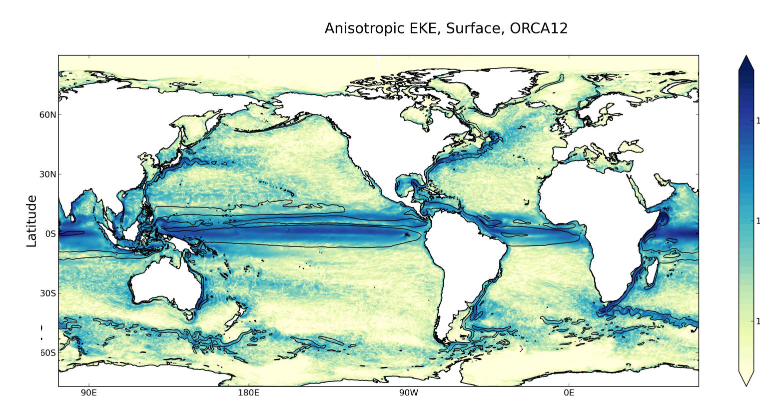
5. extension to 3D dynamics (work in progress)

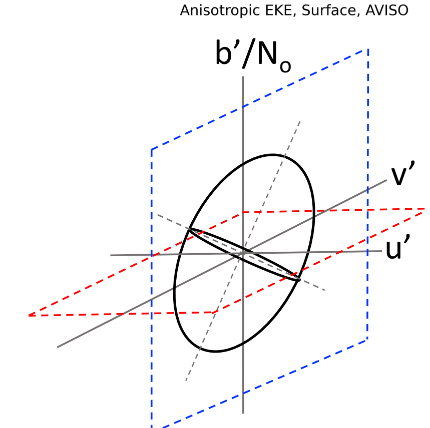
$$\Sigma = \begin{bmatrix} \overline{u'u'} & \overline{u'v'} \\ \overline{u'v'} & \overline{v'v'} \end{bmatrix}$$

$$\mathrm{F} = \mathrm{F}_\theta + F_L = \nabla \cdot \mathbf{f}_\theta + \nabla \cdot \mathbf{f}_L$$









The Geometric Decomposition: Derivation

$$\Sigma = \begin{bmatrix} \overline{u'u'} & \overline{u'v'} \\ \overline{u'v'} & \overline{v'v'} \end{bmatrix}$$

time-mean eddy covariance matrix

time-mean eddy fluxes of horizontal momentum

eddy stress matrix

eddy flux tensor

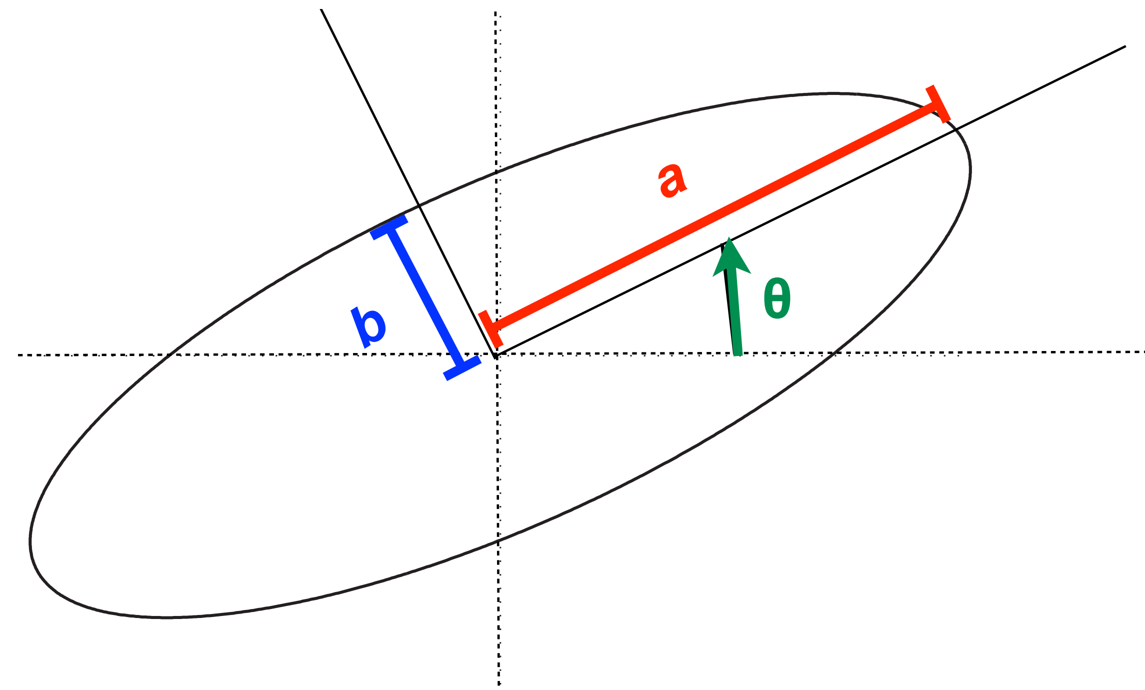
Eliassen-Palm flux tensor

Geometric Interpretation of Σ

$$\Sigma = \begin{bmatrix} \overline{u'u'} & \overline{u'v'} \\ \overline{u'v'} & \overline{v'v'} \end{bmatrix}$$

In terms of its **eigenvectors** and **eigenvalues**:

$$= J(\theta) \begin{bmatrix} a^2 & 0 \\ 0 & b^2 \end{bmatrix} J^{T}(\theta)$$



where:

$$\mathrm{J}\,(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

and

$$a^2 = \overline{u'u'}\cos^2\theta + \overline{u'v'}\sin(2\theta) + \overline{v'v'}\sin^2\theta$$ **= ellipse major axis²**

$$b^2 = \overline{u'u'}\cos^2\phi + \overline{u'v'}\sin(2\phi) + \overline{v'v'}\sin^2\phi$$ **= ellipse minor axis²**

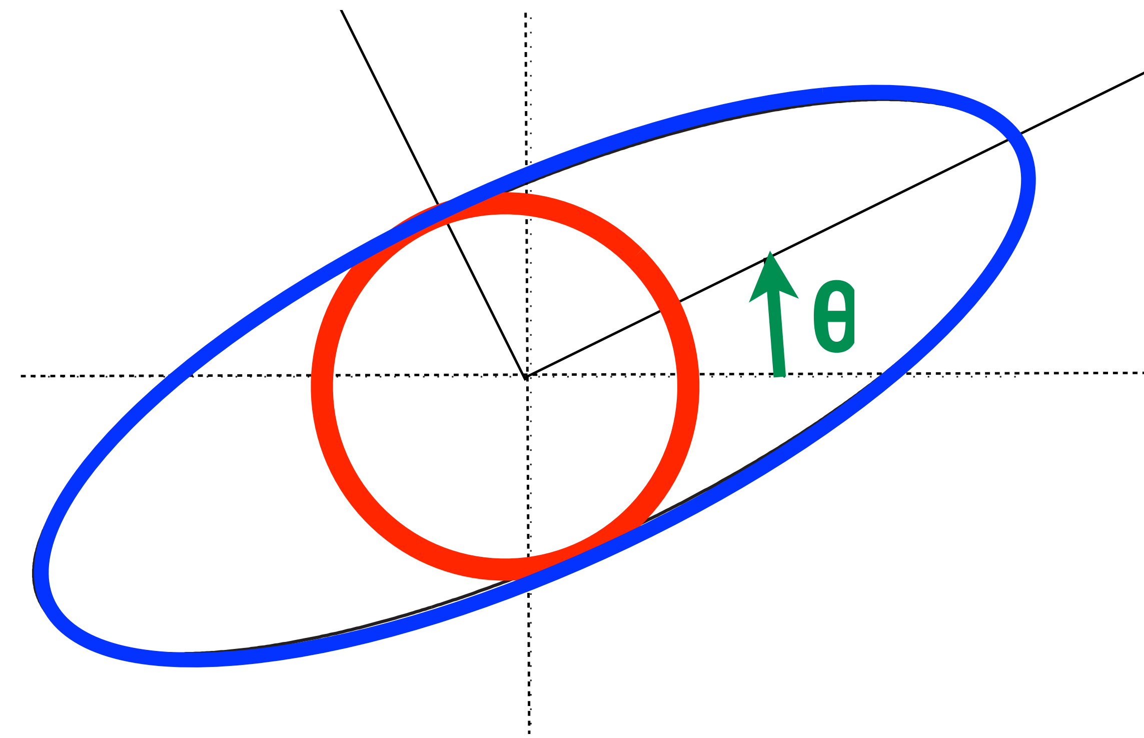
$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\,\overline{u'v'}}{(\overline{u'u'} - \overline{v'v'})}\right) \;, \quad \phi = \theta + \frac{\pi}{2}$$ **= ellipse orientation**

Geometric Interpretation of Σ

$$\Sigma = \begin{bmatrix} \overline{u'u'} & \overline{u'v'} \\ \overline{u'v'} & \overline{v'v'} \end{bmatrix}$$

In terms **ellipse** **size**, **shape** and **orientation:**

$$= K \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + L\,\mathrm{J}(2\theta) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



where:

$$\mathrm{J}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

and

$$K = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2}\right) = \frac{1}{2}\left(a^2 + b^2\right)$$ **= ellipse 'size' (EKE)**

$$L = \frac{1}{2}\left(\overline{u'^2} - \overline{v'^2}\right) = \frac{1}{2}\left(a^2 - b^2\right)$$ **= ellipse 'shape' (energy in anisotropic motions)**

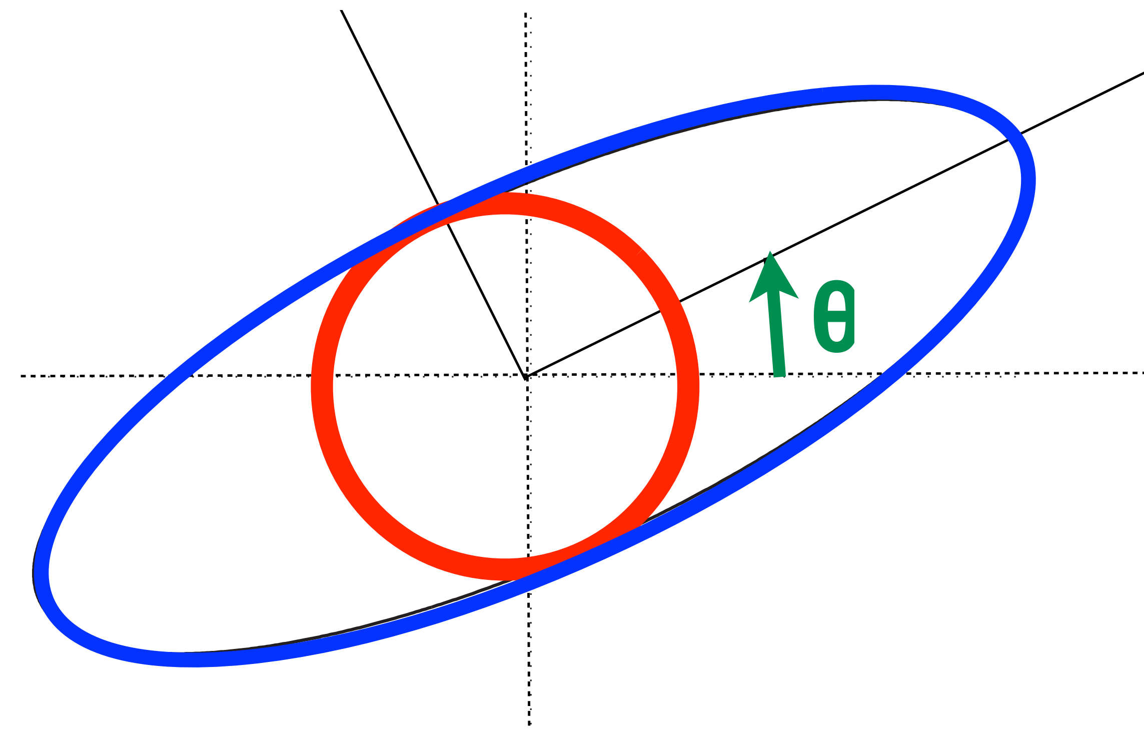
$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\,\overline{u'v'}}{\left(\overline{u'u'} - \overline{v'v'}\right)}\right)$$ **= ellipse orientation**

Geometric Interpretation of Σ

$$\Sigma = \begin{bmatrix} \overline{u'u'} & \overline{u'v'} \\ \overline{u'v'} & \overline{v'v'} \end{bmatrix}$$

In terms **ellipse size**, **shape** and **orientation:**

$$= \underbrace{K \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{isotropic}} + \underbrace{L\,\mathrm{J}(2\theta) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\text{anisotropic}}$$



where:

$$\mathrm{J}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

and

$$K = \tfrac{1}{2}\left(\overline{u'^2} + \overline{v'^2}\right) = \tfrac{1}{2}\left(a^2 + b^2\right)$$ **= ellipse 'size' (EKE)**

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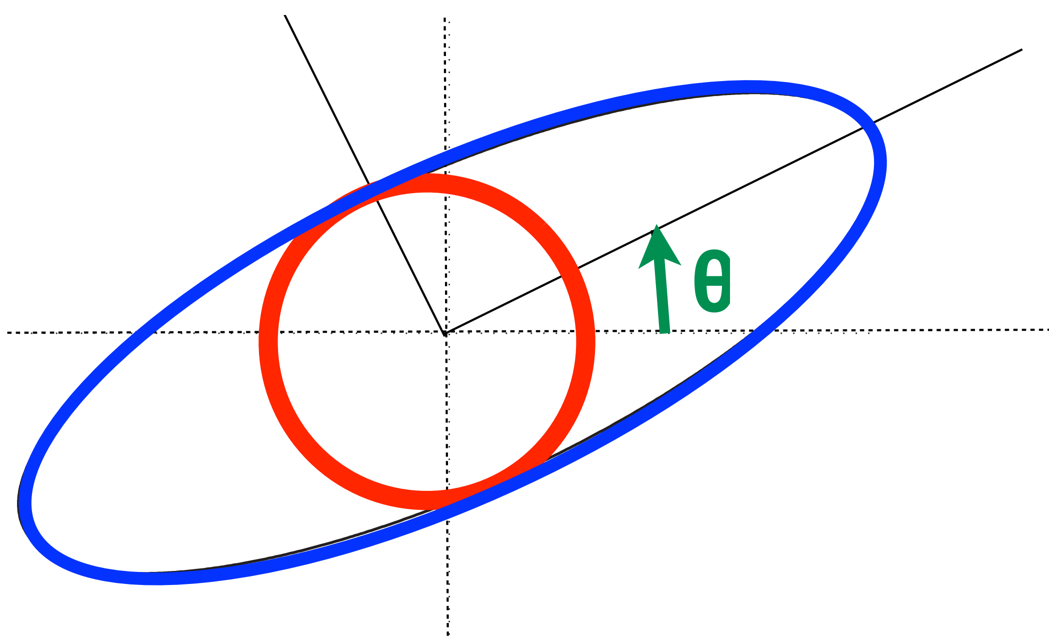
$$\theta = \tfrac{1}{2}\tan^{-1}\left(\frac{2\,\overline{u'v'}}{\left(\overline{u'u'} - \overline{v'v'}\right)}\right)$$ **= ellipse orientation**

Dynamical Interpretation of Σ

$$\Sigma = \begin{bmatrix} \overline{u'u'} & \overline{u'v'} \\ \overline{u'v'} & \overline{v'v'} \end{bmatrix}$$

eddy momentum flux	Σ
eddy momentum flux divergence	$\nabla \cdot \Sigma$
eddy vorticity flux divergence	$\vec{k} \cdot \nabla \times [\nabla \cdot \Sigma]$

The Geometric Decomposition



$$\Sigma = K \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + L\,\mathrm{J}(2\theta) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Sigma$$

$$\nabla \cdot \Sigma$$

$$\vec{k} \cdot \nabla \times [\nabla \cdot \Sigma]$$

The Geometric Decomposition

- **eddy momentum fluxes**

$$\Sigma = \underbrace{K\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\text{ellipse size / eddy energy}} + \underbrace{L\begin{bmatrix}\cos(2\theta) & -\sin(2\theta)\\ \sin(2\theta) & \cos(2\theta)\end{bmatrix}}_{\text{ellipse shape and orientation}}$$

- **eddy momentum flux divergences**

$$\nabla\cdot\Sigma = \underbrace{\tilde{\nabla}K}_{\text{spatial gradients of ellipse size}} + \left[\underbrace{\tilde{\nabla}L\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}}_{\text{spatial gradients of ellipse shape}} + \underbrace{2L\tilde{\nabla}\theta\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}}_{\text{spatial gradients of ellipse orientation}}\right]$$

- **eddy vorticity flux divergence**

('The eddy forcing')

$$\vec{k}\cdot\nabla\times[\nabla\cdot\Sigma] = A + B + C + D$$

where

$$\underbrace{A = -2\frac{\partial^2}{\partial\tilde{x}\partial\tilde{y}}L}_{\text{curvature in ellipse shape}} \quad \underbrace{B = 2L\left[\frac{\partial^2\theta}{\partial\tilde{x}^2} - \frac{\partial^2\theta}{\partial\tilde{y}^2}\right]}_{\text{curvature in ellipse orientation}} \quad \underbrace{C = 8L\frac{\partial\theta}{\partial\tilde{x}}\frac{\partial\theta}{\partial\tilde{y}}}_{\text{squared gradients in ellipse orientation}} \quad \underbrace{D = 4\left[\frac{\partial L}{\partial\tilde{x}}\frac{\partial\theta}{\partial\tilde{y}} - \frac{\partial L}{\partial\tilde{y}}\frac{\partial\theta}{\partial\tilde{x}}\right]}_{\text{cross-term of gradients of ellipse shape \& ellipse orientation}}$$

NOTE: ~ = in the frame of the local eddy orientation!

Algebraic Simplifications

[algebra...]

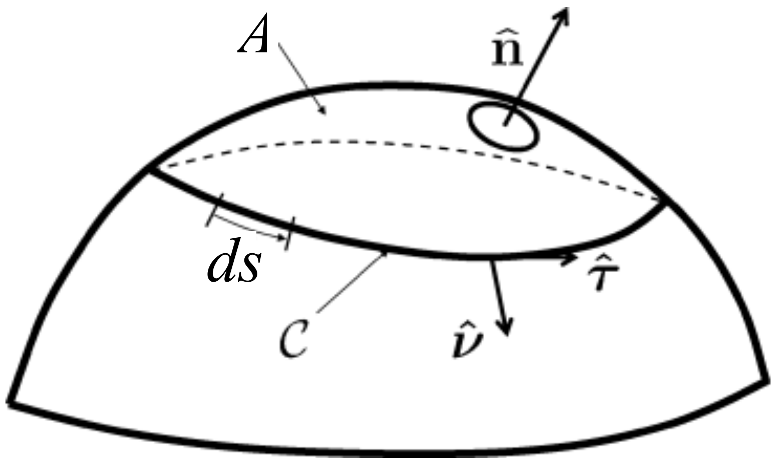
$$\underbrace{\mathrm{F}}_{\substack{\text{eddy vorticity flux}\\ \text{divergence}\\ \text{('The eddy forcing')}}} = \underbrace{\mathrm{F}_\theta}_{\substack{\text{spatial patterns of}\\ \text{ellipse orientation}\\ \text{(+ cross-term)}}} + \underbrace{F_L}_{\substack{\text{spatial patterns of}\\ \text{ellipse shape}\\ \text{(+ cross-term)}}}$$

where

$$\mathrm{F}_\theta = \underbrace{2L\left[\frac{\partial^2\theta}{\partial\tilde{x}^2} - \frac{\partial^2\theta}{\partial\tilde{y}^2}\right]}_{\substack{\text{curvature}\\ \text{in ellipse}\\ \text{orientation}}} + \underbrace{8L\frac{\partial\theta}{\partial\tilde{x}}\frac{\partial\theta}{\partial\tilde{y}}}_{\substack{\text{squared gradients}\\ \text{in ellipse}\\ \text{orientation}}} + \underbrace{2\left[\frac{\partial L}{\partial\tilde{x}}\frac{\partial\theta}{\partial\tilde{y}} - \frac{\partial L}{\partial\tilde{y}}\frac{\partial\theta}{\partial\tilde{x}}\right]}_{\substack{\text{cross-term of gradients of}\\ \text{ellipse shape}\\ \text{\& ellipse orientation}}}$$

$$\mathrm{F}_L = \underbrace{-2\frac{\partial^2}{\partial\tilde{x}\partial\tilde{y}}L}_{\substack{\text{curvature}\\ \text{in ellipse}\\ \text{shape}}} + \underbrace{2\left[\frac{\partial L}{\partial\tilde{x}}\frac{\partial\theta}{\partial\tilde{y}} - \frac{\partial L}{\partial\tilde{y}}\frac{\partial\theta}{\partial\tilde{x}}\right]}_{\substack{\text{cross-term of gradients of}\\ \text{ellipse shape}\\ \text{\& ellipse orientation}}}$$

Algebraic Simplifications



[algebra...]

$$\mathrm{F} = \mathrm{F}_\theta + F_L = \nabla \cdot \mathbf{f}_\theta + \nabla \cdot \mathbf{f}_L$$

divergence theorem

$$\int_A F\, dA = \oint_C \mathbf{f}_\theta \cdot \hat{\mathbf{n}}\, ds + \oint_C \mathbf{f}_L \cdot \hat{\mathbf{n}}\, ds$$

net eddy forcing in the area *A* ('**The eddy forcing**')	linear variations in **ellipse orientation** around the area boundary	linear variations in **ellipse shape** around the area boundary

where

$$\mathbf{f}_\theta = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} 2L\nabla\theta, \quad \mathbf{f}_L = \begin{bmatrix} \sin 2\theta & -\cos 2\theta \\ -\cos 2\theta & -\sin 2\theta \end{bmatrix} \nabla L$$

THE GEOMETRIC DECOMPOSITION

Advantages

- identifies important elements of the eddy variability that have a mean flow forcing effect
- links physical mechanisms underpinning eddy feedbacks to physical mechanisms setting the spatial patterns of eddy geometry
- describes eddy feedbacks in terms of a lower order description of the eddy motion
- suggests ingredients of a parameterization based on resolved eddy geometry

$$\underbrace{\mathrm{F}}_{\text{eddy vorticity flux divergence}} = \underbrace{\mathrm{F}_\theta}_{\text{spatial patterns of ellipse orientation}} + \underbrace{F_L}_{\text{spatial patterns of ellipse shape}}$$

eddy vorticity flux divergence (**'the eddy forcing'**)

spatial patterns of **ellipse orientation** (+ cross-term)

spatial patterns of **ellipse shape** (+ cross-term)

$$\mathbf{f}\cdot\hat{\mathrm{n}}\,ds = \mathbf{f}_\theta\cdot\hat{\mathrm{n}}\,ds + \mathbf{f}_L\cdot\hat{\mathrm{n}}\,ds$$

eddy flux across the boundary

linear variations in **ellipse orientation** along the boundary

linear variations in **ellipse shape** along the boundary

$$\int_A F\,dA = \oint_C \mathbf{f}_\theta\cdot\hat{\mathbf{n}}\,ds + \oint_C \mathbf{f}_L\cdot\hat{\mathbf{n}}\,ds$$

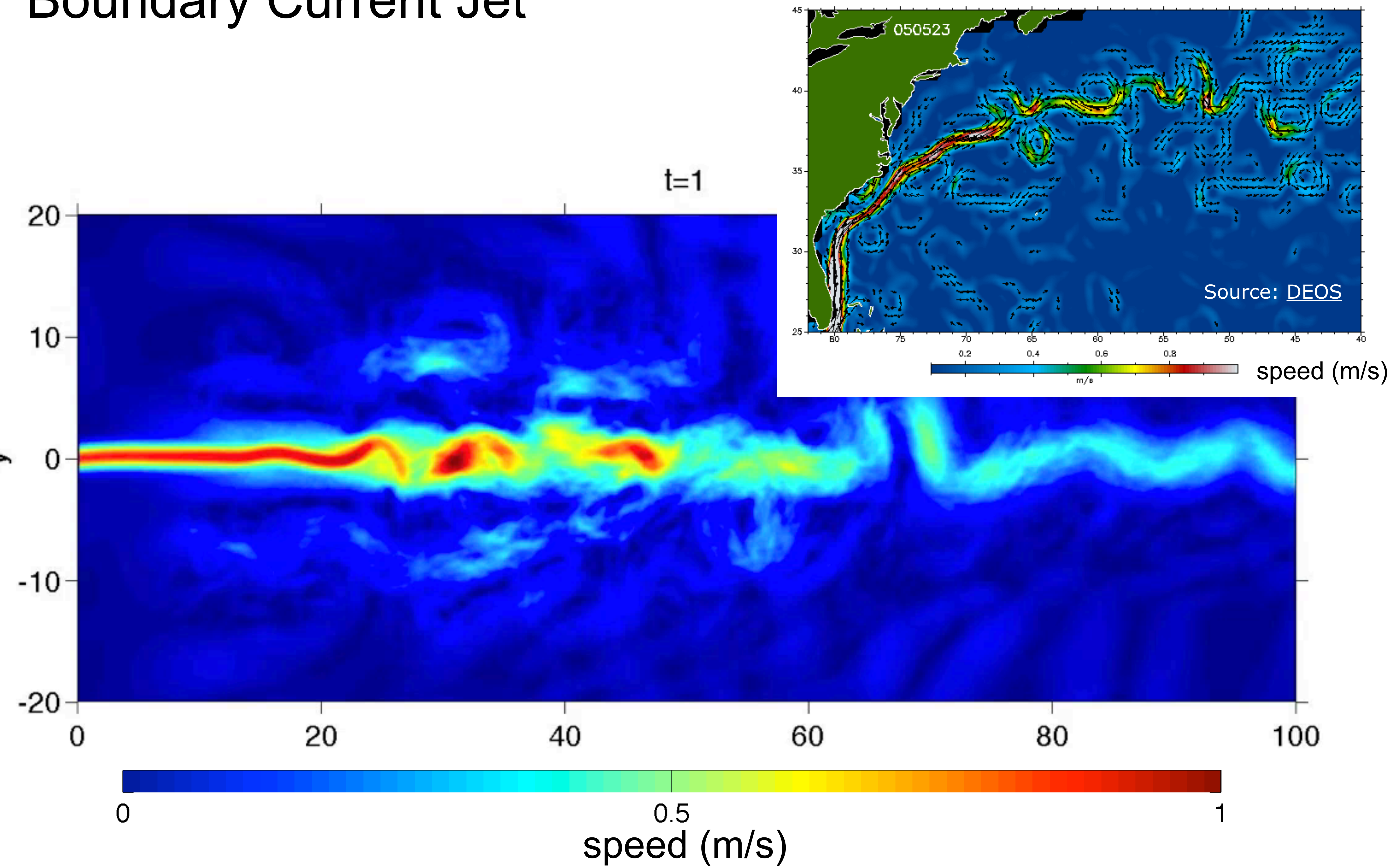
net eddy forcing in the area A (**'the eddy forcing'**)

linear variations in **ellipse orientation** around the area boundary

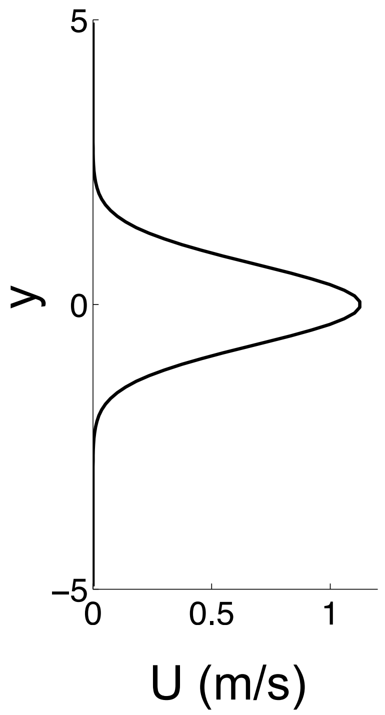
linear variations in **ellipse shape** around the area boundary

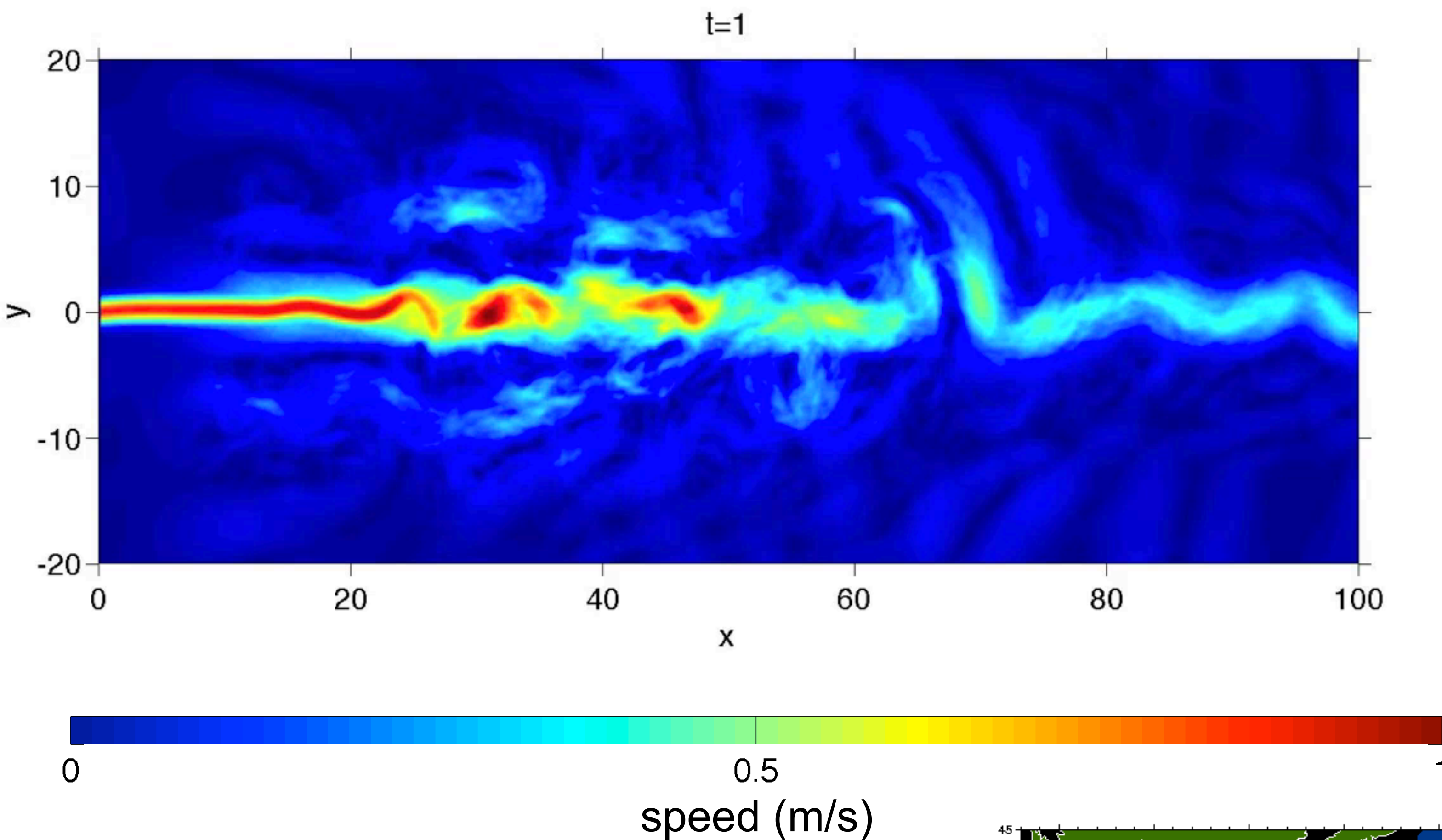
The Geometric Decomposition: Application

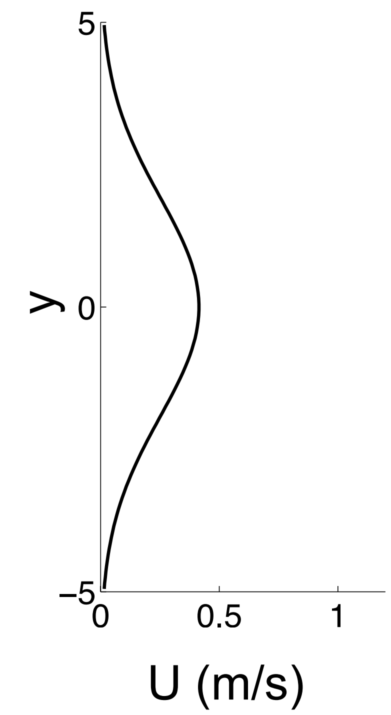
Eddy-mean flow feedbacks in a toy model Western Boundary Current Jet



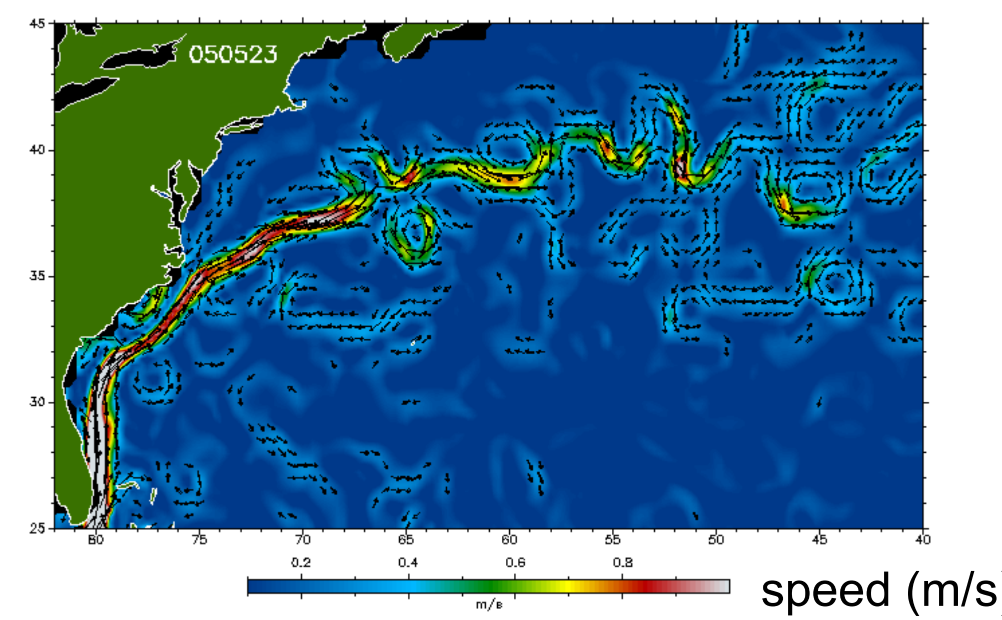
Model set-up







- quasi-geostrophic
- mid-latitude β plane
- barotropic
- forced by an unstable jet inflow at x=0 scaled to look like the Gulf Stream or Kuroshio at the coast at the point of separation
- insensitive to the the outflow condition
- sponge layers on all lateral boundaries model “open ocean”
- negligible dissipation in the interior



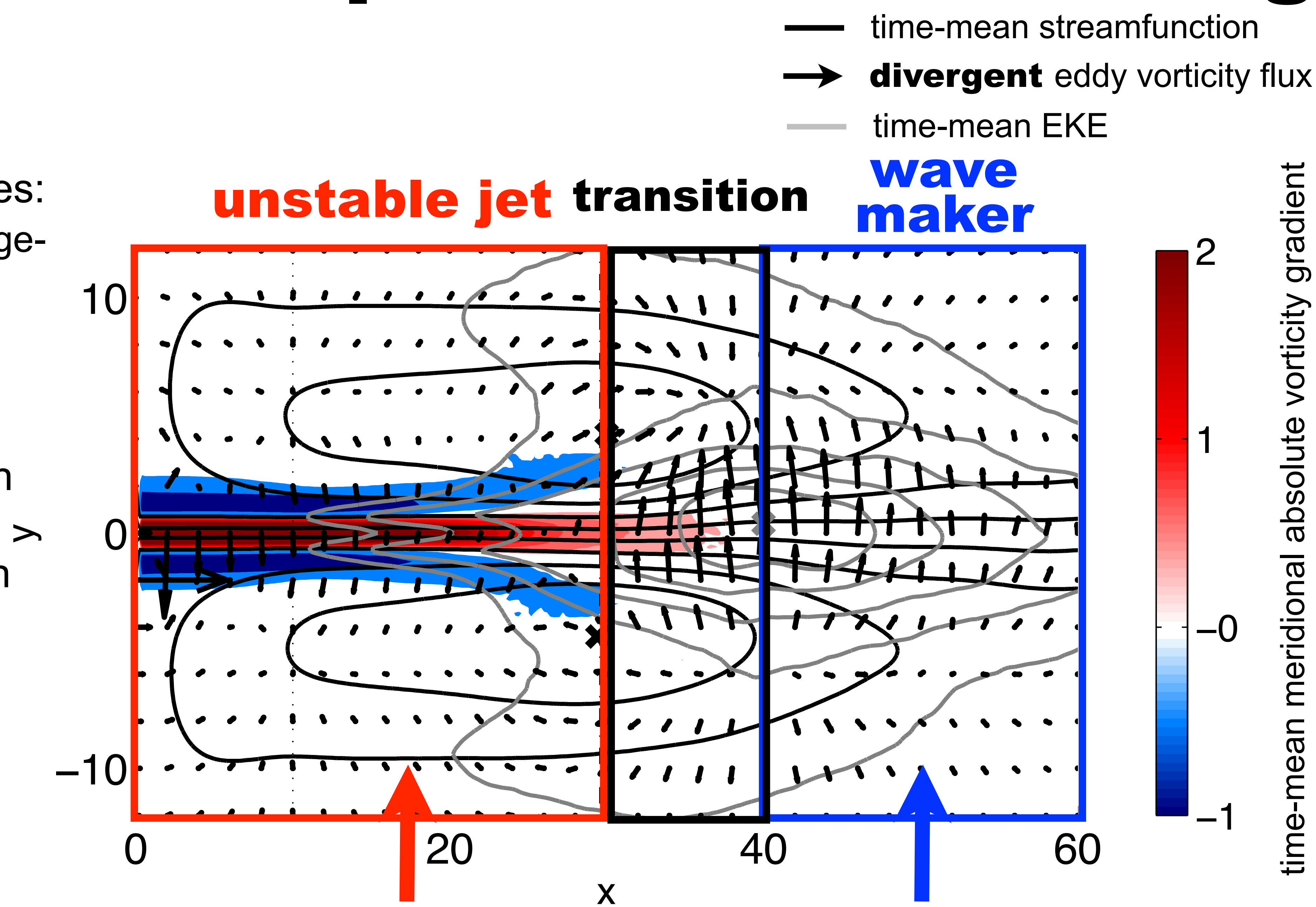
Snapshot of Gulf Stream speed 10 Apr 2013 derived from near-real-time radar altimeter data of the European Environmental Satellite Envisat. Source: DEOS

See Waterman and Jayne 2011

THE GEOMETRIC DECOMPOSITION: APPLICATION

Eddy feedbacks a priori understanding

- eddies play 2 important roles:
  - stabilizing the jet to its large-scale shear
  - driving mean flows
- each effect is localized to a distinct along-stream region defined by the stability properties of the time-mean jet:
  - upstream eddies act to stabilize the jet via a down-gradient vorticity flux
  - downstream eddies strengthen the jet & drive recirculations via an up-gradient vorticity flux permitted by an eddy enstrophy convergence



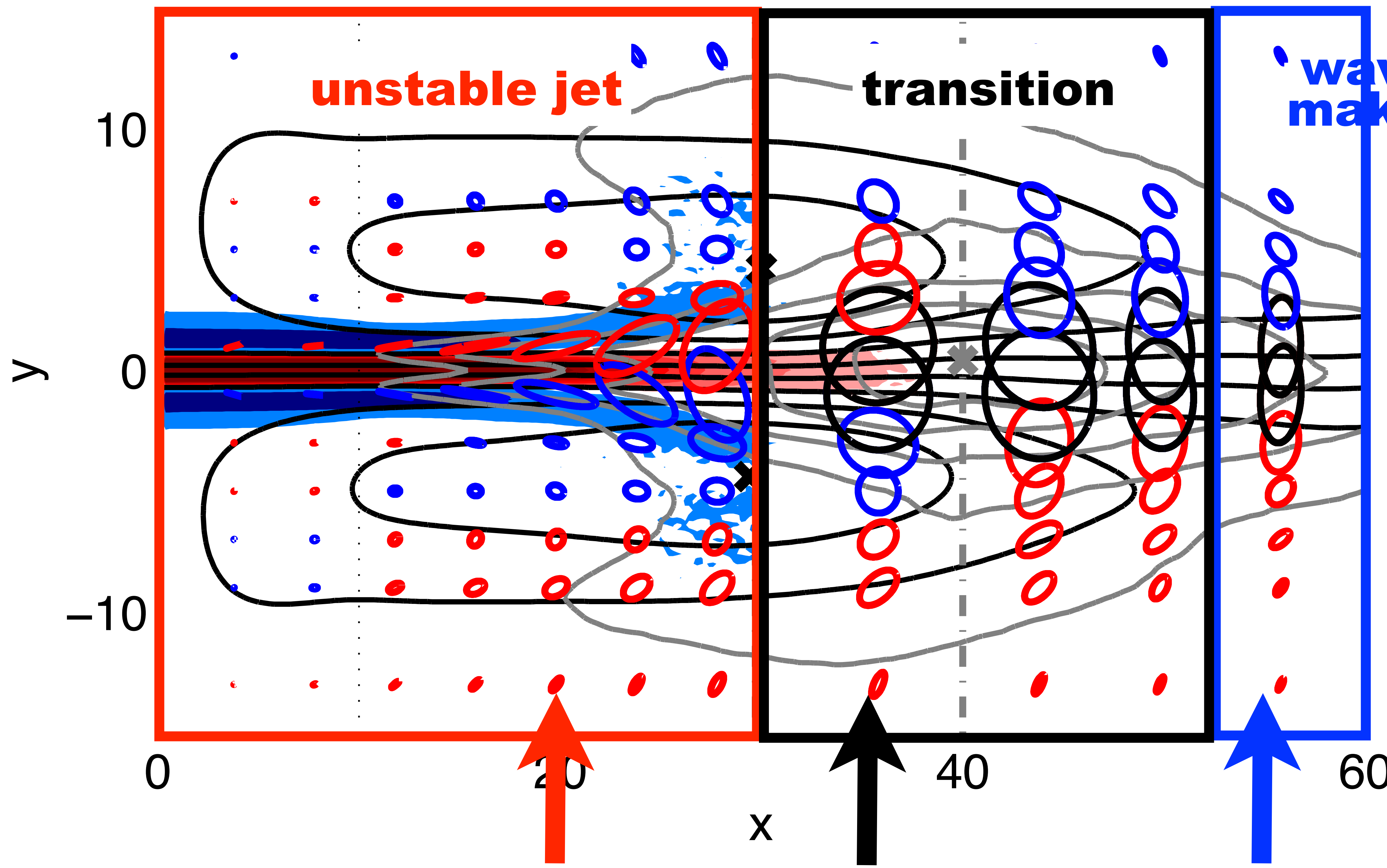
time-mean jet **UNSTABLE**
eddy energy is **GROWING**
eddy enstrophy is **DIVERGENT**
eddies act to **STABILIZE** the jet
via a **DOWN-GRADIENT** eddy vorticity flux

time-mean jet **STABLE**
eddy energy is **DECAYING**
eddy enstrophy is **CONVERGENT**
eddies act to **STRENGTHEN** the jet
via a **UP-GRADIENT** eddy vorticity flux

See Waterman and Jayne 2011

THE GEOMETRIC DECOMPOSITION: APPLICATION

Ellipse Geometry



highly **ANISOTROPIC**
eddies elongated **ALONG** the jet
eddies titled **INTO THE SHEAR**
eddies energy **GROWING**

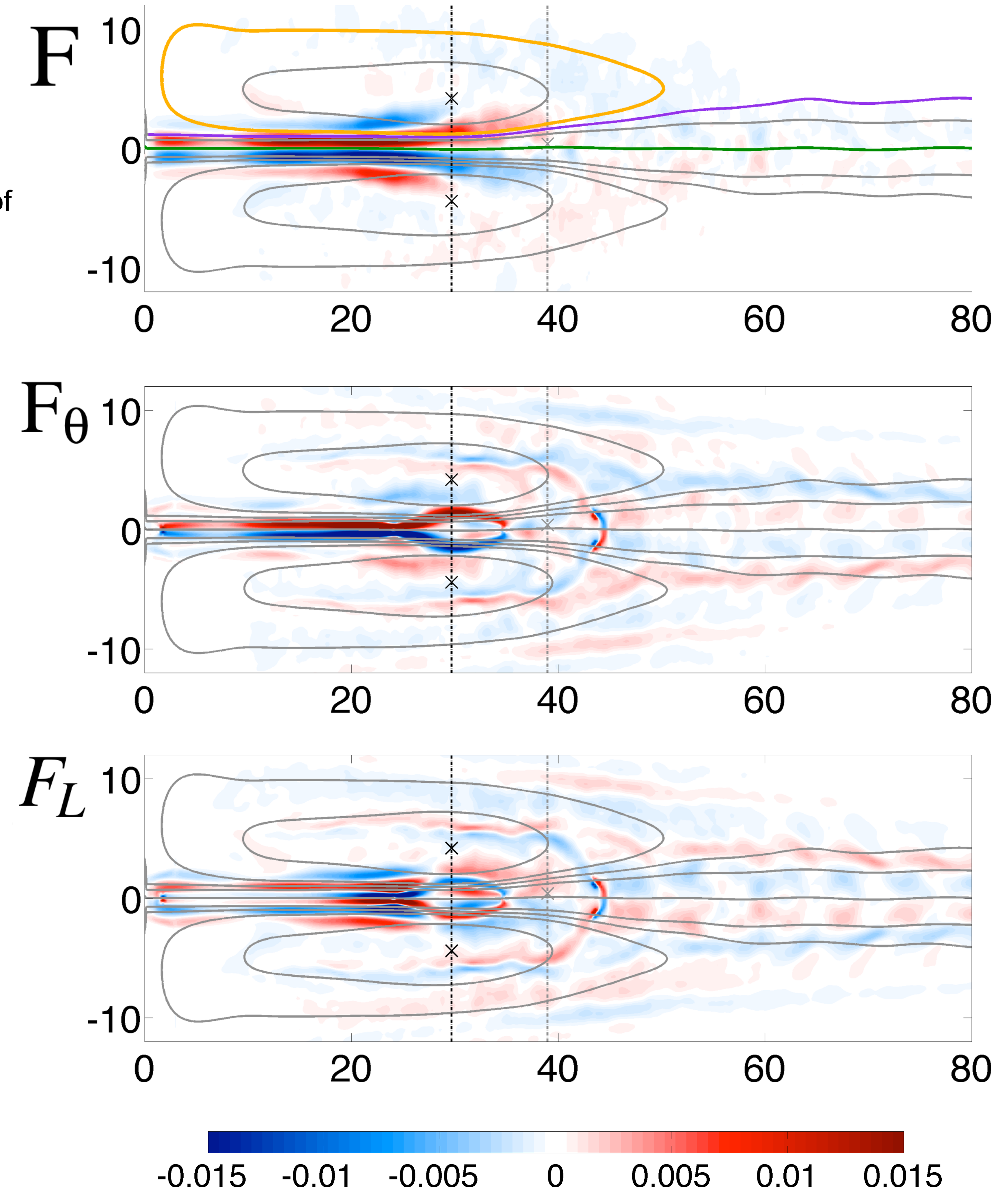
eddies are
ISOTROPIC
eddies energy
MAXIMIZED

increasingly **ANISOTROPIC**
eddies elongated **ACROSS** the jet
eddies titled **WITH THE SHEAR**
eddies energy **DECAYING**

1. via 2nd order spatial patterns

$$\underbrace{\mathrm{F}}_{\text{eddy vorticity flux divergence}} = \underbrace{\mathrm{F}_\theta}_{\text{spatial patterns of ellipse orientation (+ cross-term)}} + \underbrace{F_L}_{\text{spatial patterns of ellipse shape (+ cross-term)}}$$

(**'the eddy forcing'**)



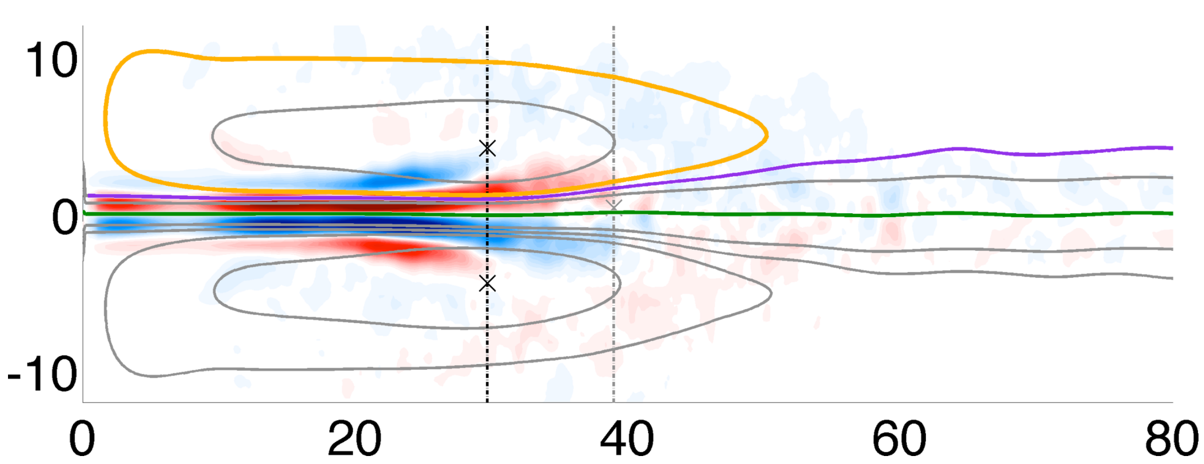
2. via linear variations along a boundary

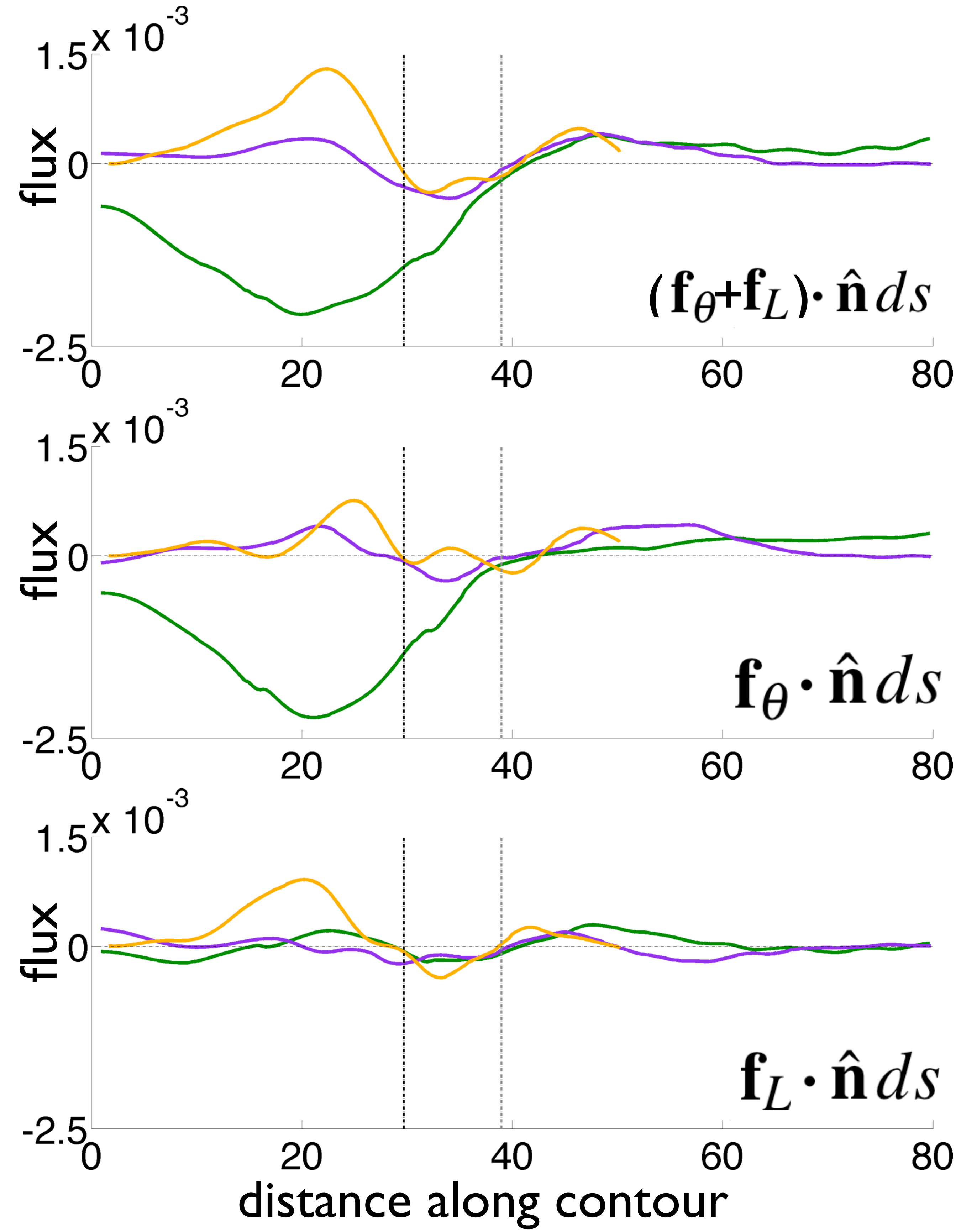
$$\mathbf{f}\cdot\hat{\mathbf{n}}\,ds = \mathbf{f}_\theta\cdot\hat{\mathbf{n}}\,ds + \mathbf{f}_L\cdot\hat{\mathbf{n}}\,ds$$

eddy flux across the boundary

linear variations in **ellipse orientation** along the boundary

linear variations in **ellipse shape** along the boundary





THE GEOMETRIC DECOMPOSITION: APPLICATION

3. via the integral form

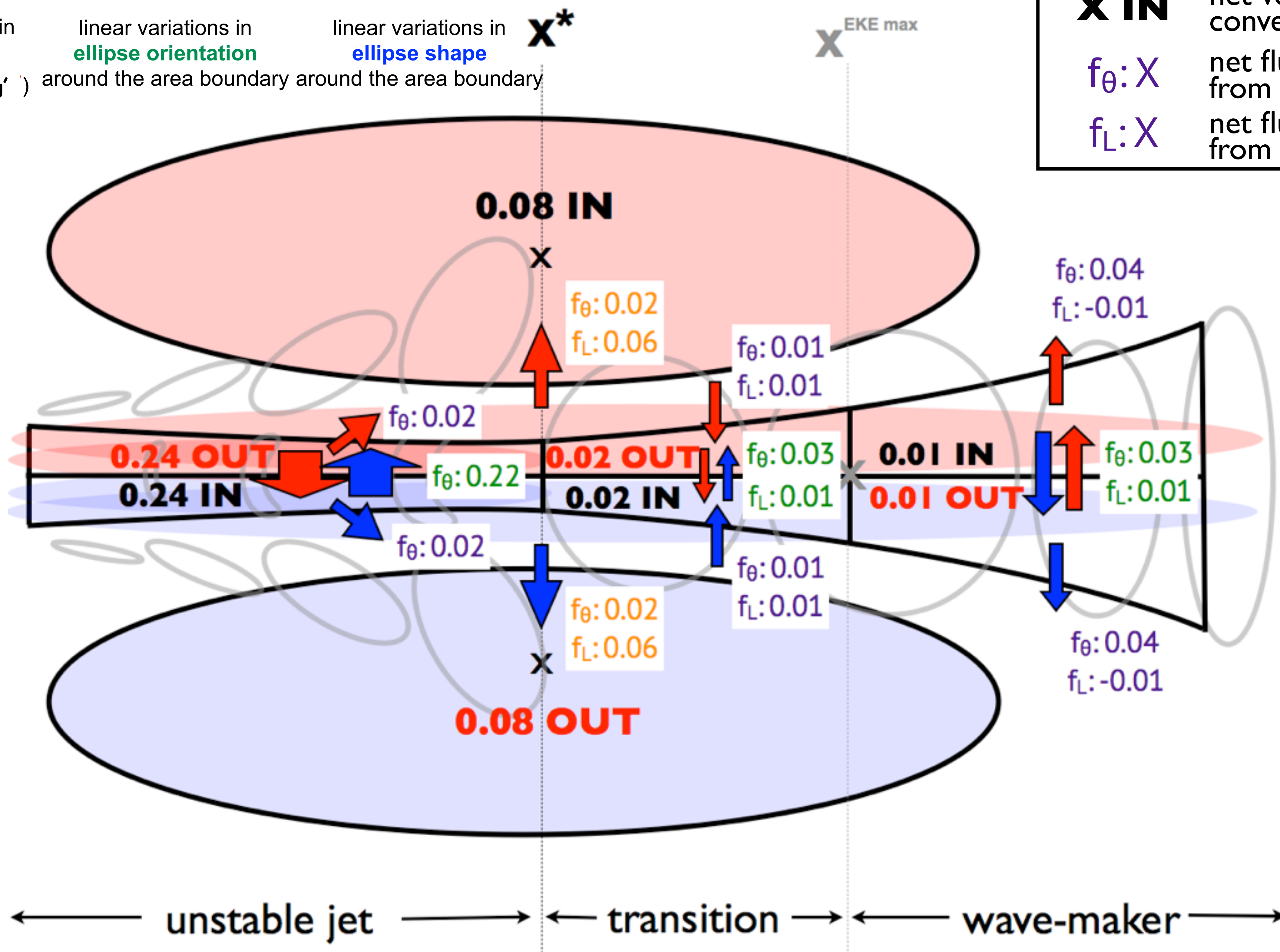
$$\int_A F\,dA = \oint_C \mathbf{f}_\theta \cdot \hat{\mathbf{n}}\,ds + \oint_C \mathbf{f}_L \cdot \hat{\mathbf{n}}\,ds$$

net eddy forcing in the area A ('the eddy forcing')

linear variations in **ellipse orientation** around the area boundary

linear variations in **ellipse shape** around the area boundary

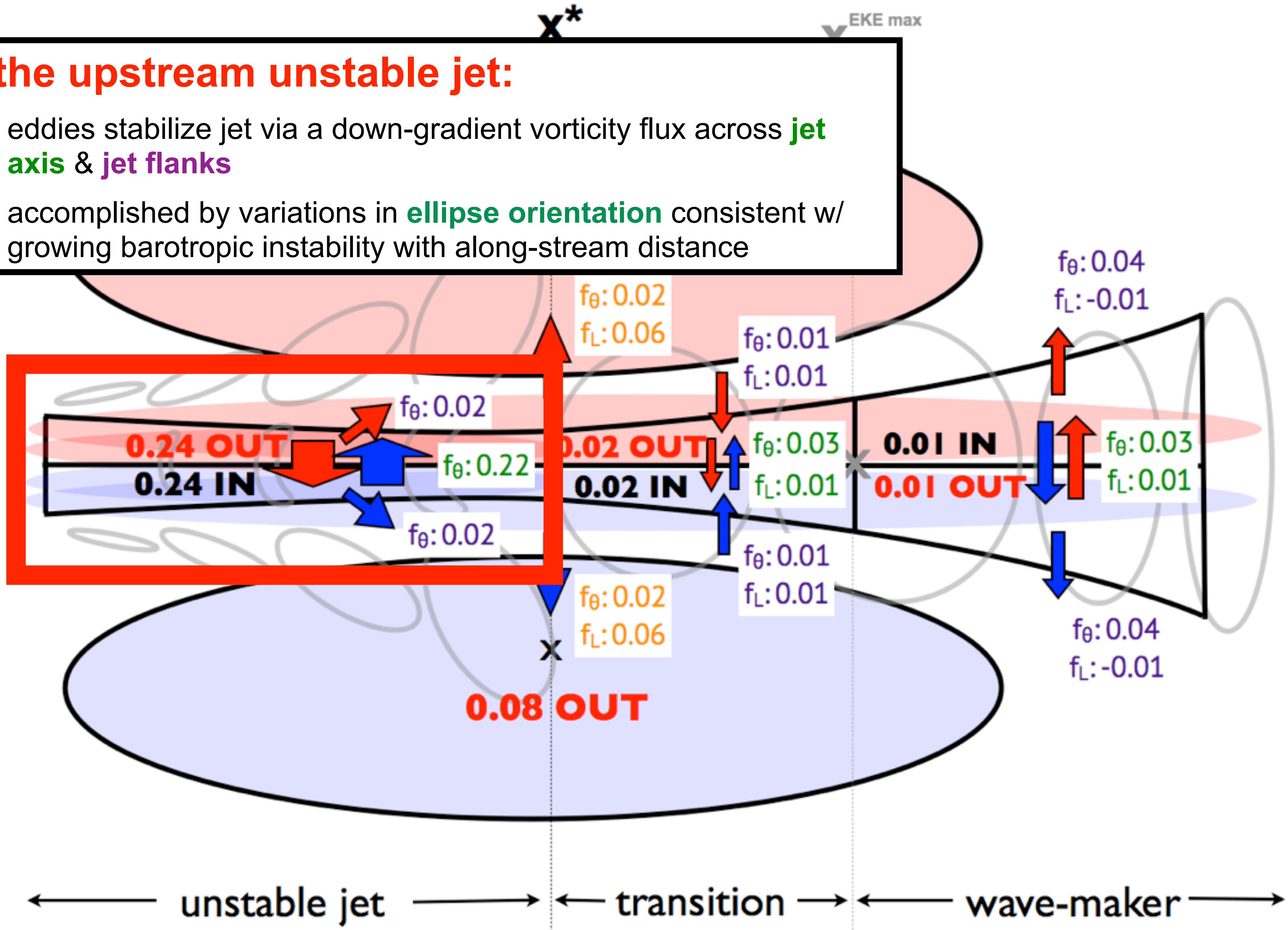
X OUT	net vorticity divergence
X IN	net vorticity convergence
f_θ: X	net flux arising from θ variations
f_L: X	net flux arising from L variations



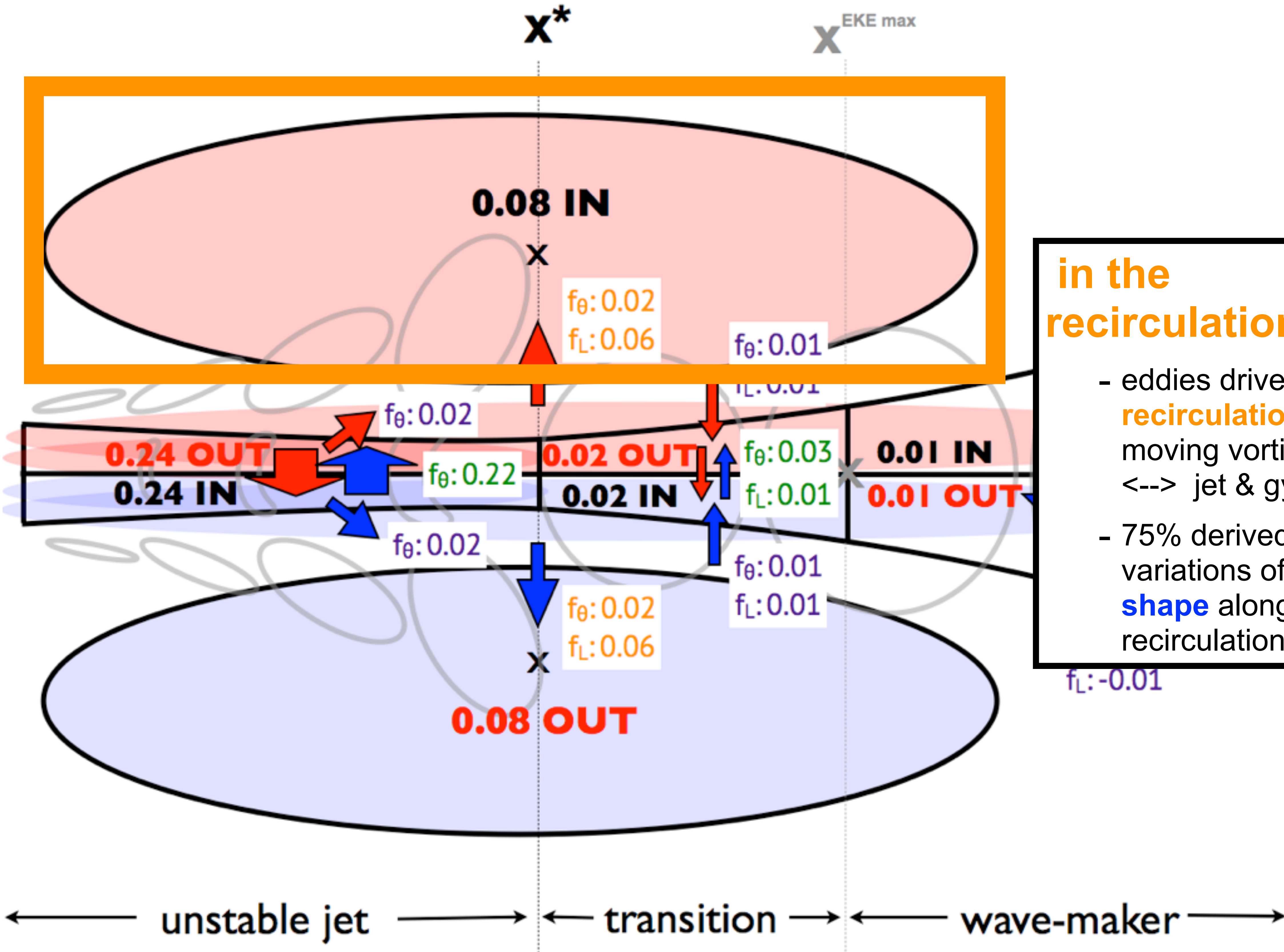
3. via the integral form

in the upstream unstable jet:

- eddies stabilize jet via a down-gradient vorticity flux across **jet axis** & **jet flanks**
- accomplished by variations in **ellipse orientation** consistent w/ growing barotropic instability with along-stream distance



3. via the integral form

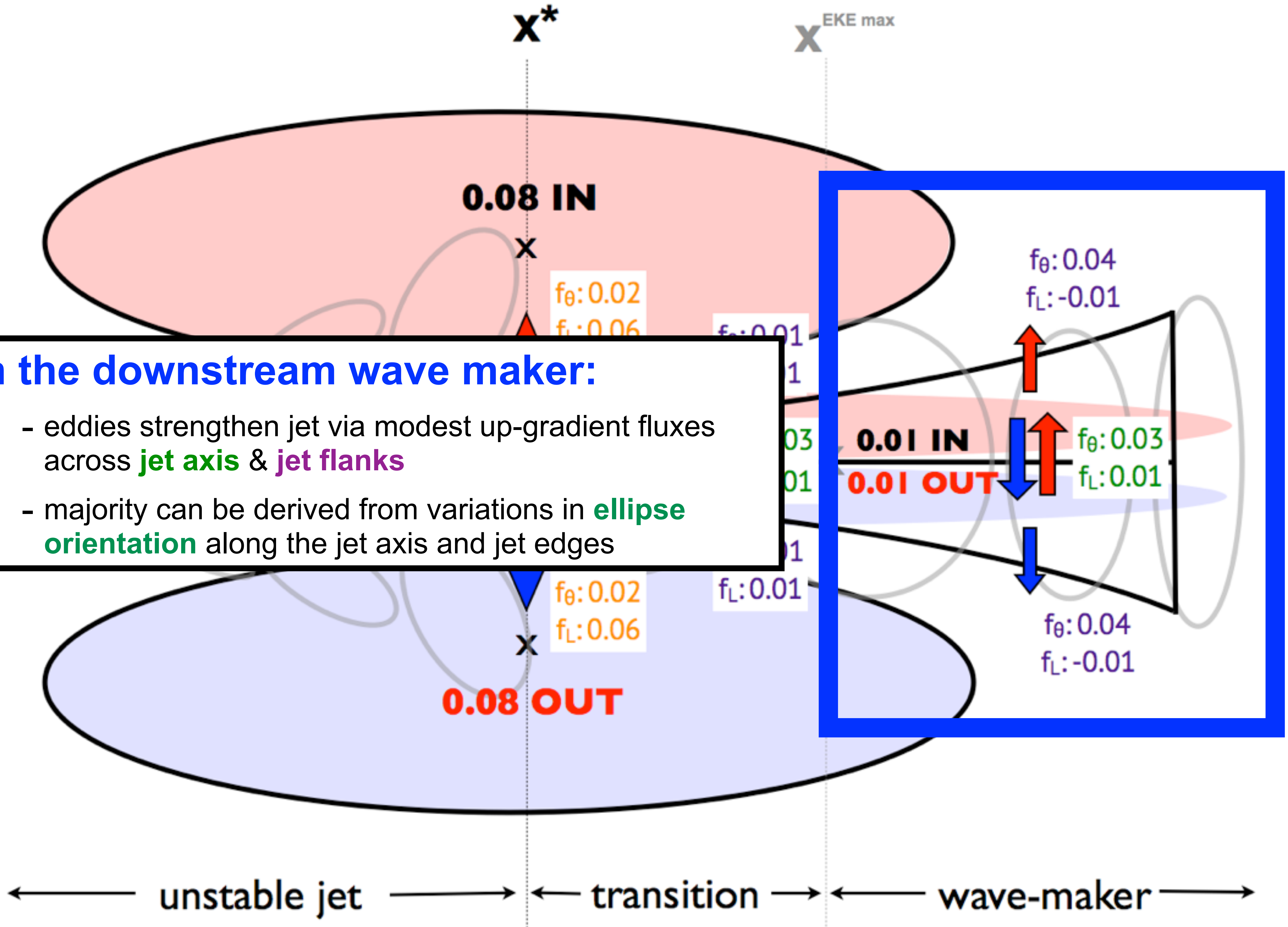


in the recirculations:

- eddies drive **recirculations** by moving vorticity <--> jet & gyre
- 75% derived from variations of **ellipse shape** along the recirculation edge

THE GEOMETRIC DECOMPOSITION: APPLICATION

3. via the integral form

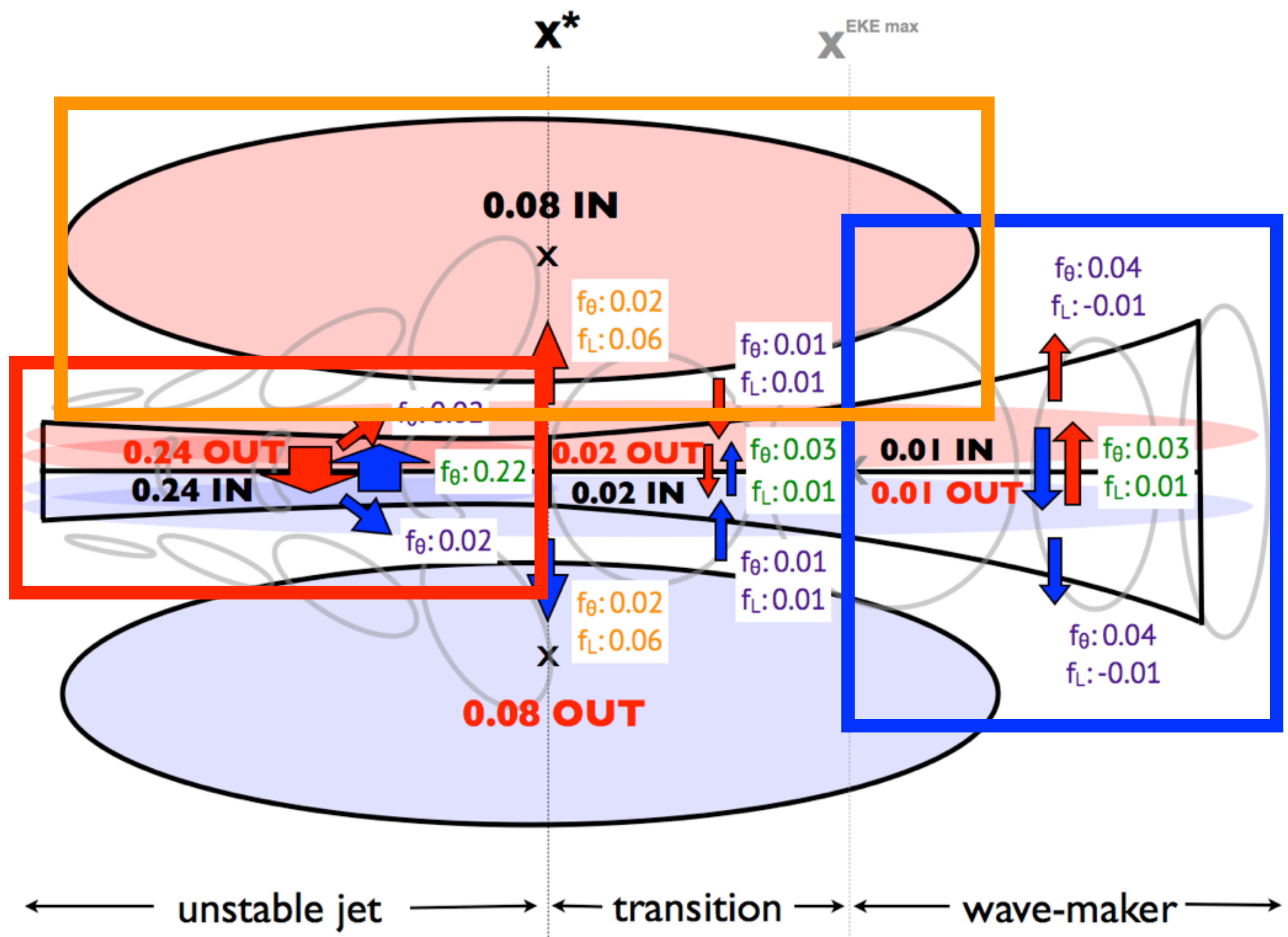


in the downstream wave maker:

- eddies strengthen jet via modest up-gradient fluxes across **jet axis** & **jet flanks**
- majority can be derived from variations in **ellipse orientation** along the jet axis and jet edges

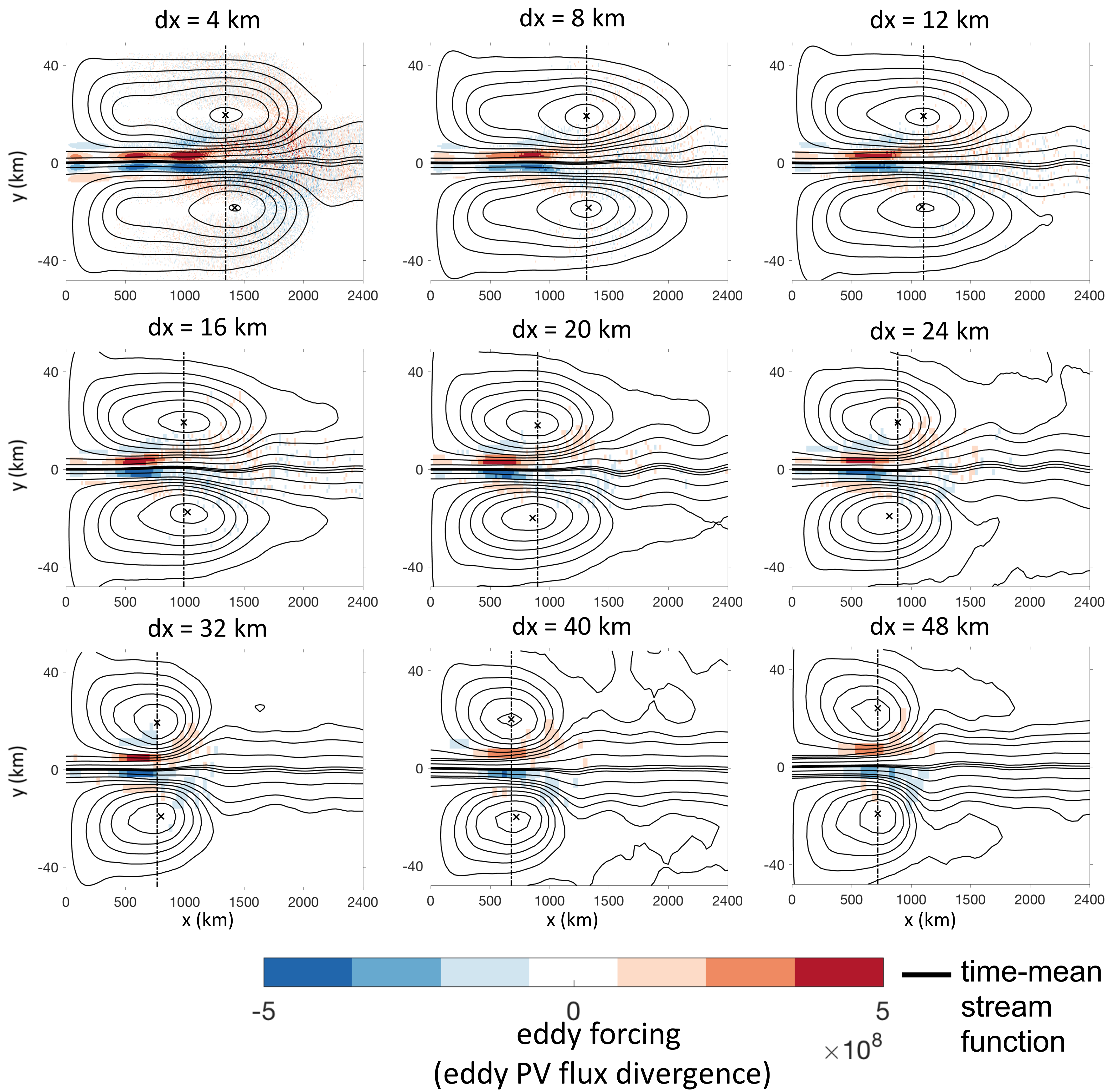
Summary

- 3 dominant eddy effects well approximated by large-scale linear variations of a single geometric property along well-defined mean flow boundaries
- these variations often consistent with expectations from stability theory & models of wave radiation
- demonstrates promise of framework to:
  1. infer eddy forcing from relatively coarsely-resolved fields
  2. link eddy forcing to physical processes
  3. suggest ingredients of an eddy parameterization



New Insights

Eddy effects as a function of model resolution



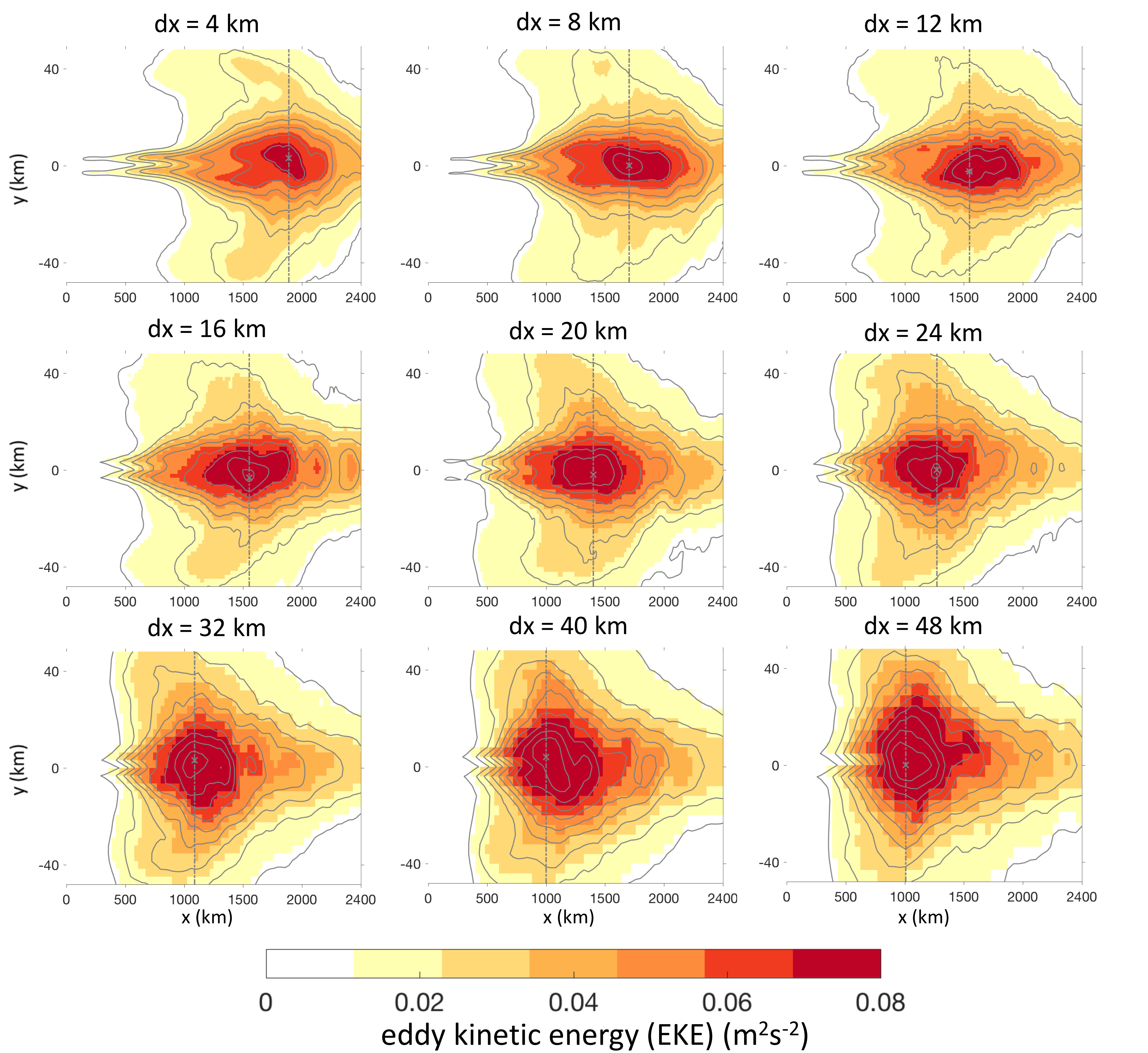
We see a rapid break-down in the eddy forcing & eddy effect as the spatial resolution is degraded:

- significant degradation in magnitude & extent of the eddy forcing along the jet axis & in the wave maker region
- eddy enhancement to jet strength is weakened
- eddy-driven recirculations become weaker
- eddy-driven recirculations have reduced zonal extent

Eddy effects as a function of model resolution

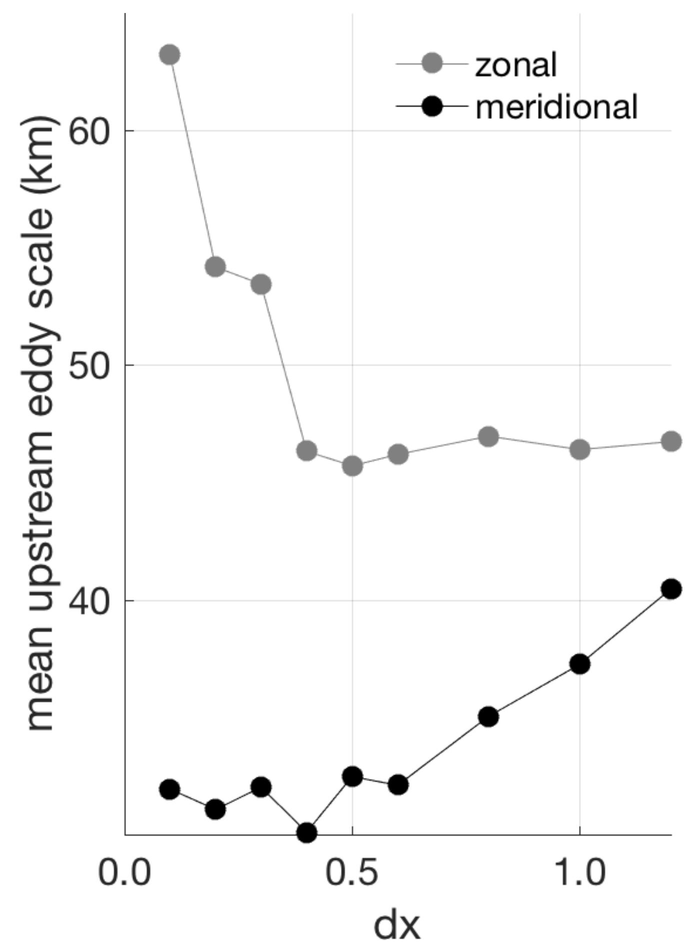
We see a rapid break-down in the eddy forcing & eddy effect as the spatial resolution is degraded

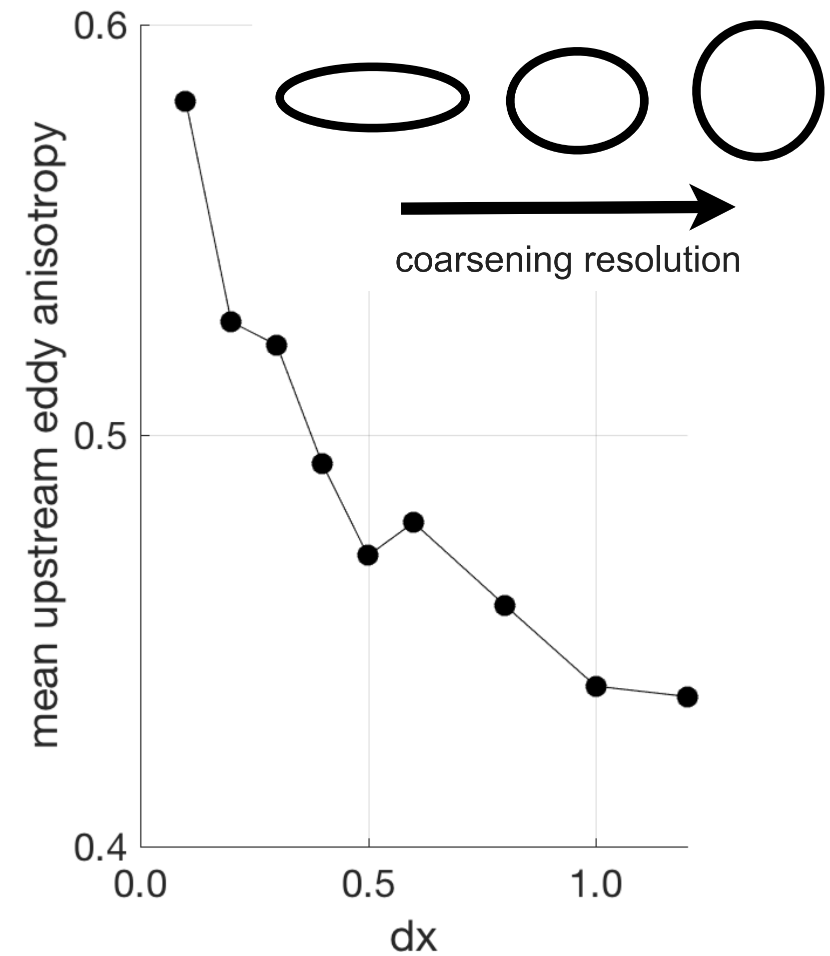
DESPITE the fact that EKE remains well-resolved.



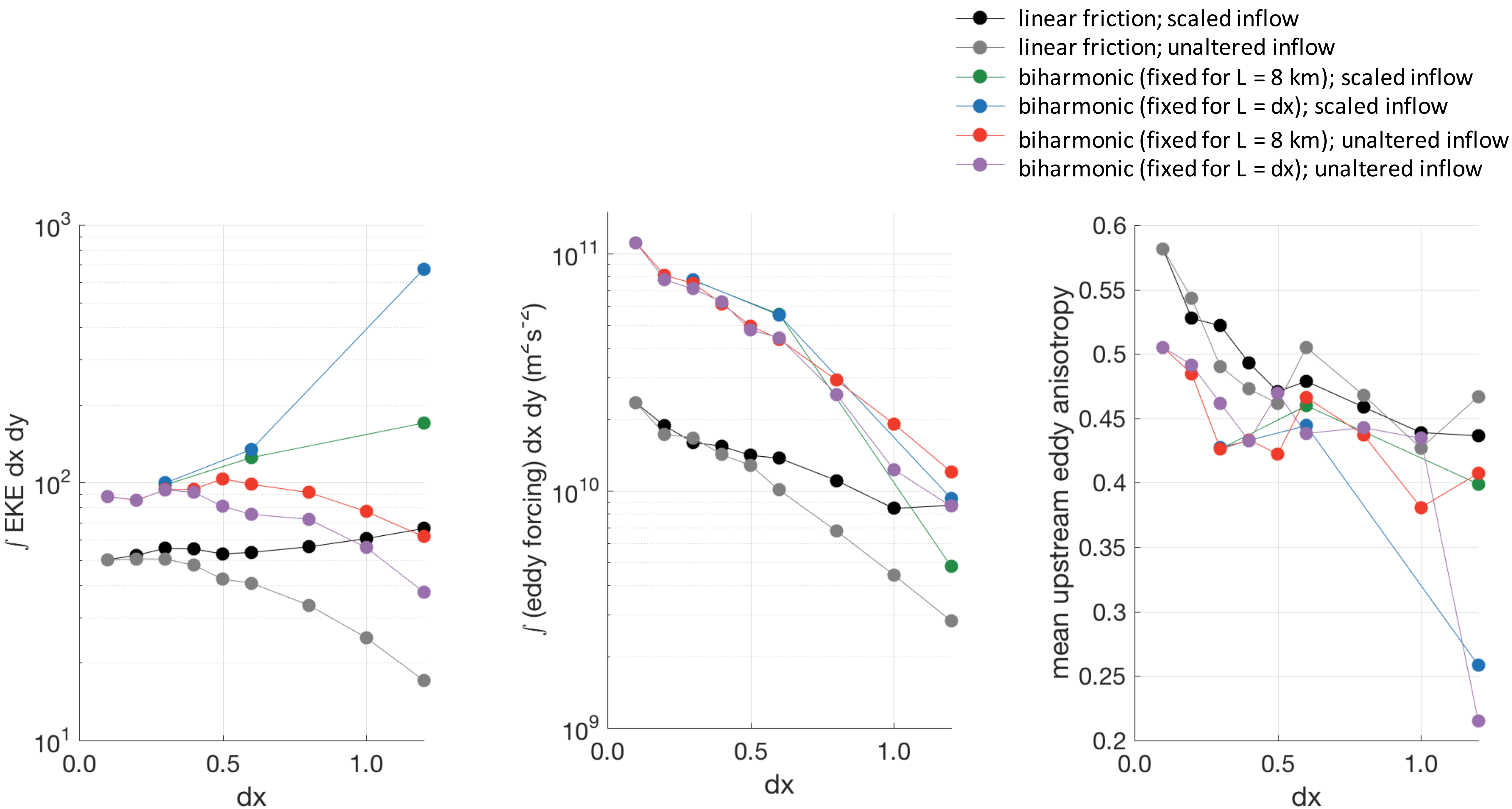
Eddy effects as a function of model resolution

- we find average eddy **size** (proportional to EKE) is well resolved but the average eddy **shape** is not
- as dx ⬆, zonal eddy scale ⬇, meridional eddy scale ⬆
- average eddy anisotropy declines as characteristic eddy shape becomes increasingly circular
- appreciation of the importance of eddy anisotropy to eddy feedbacks suggests failure to resolve small scales of highly anisotropic eddies underpins the quick loss of eddy forcing





Eddy effects as a function of model resolution

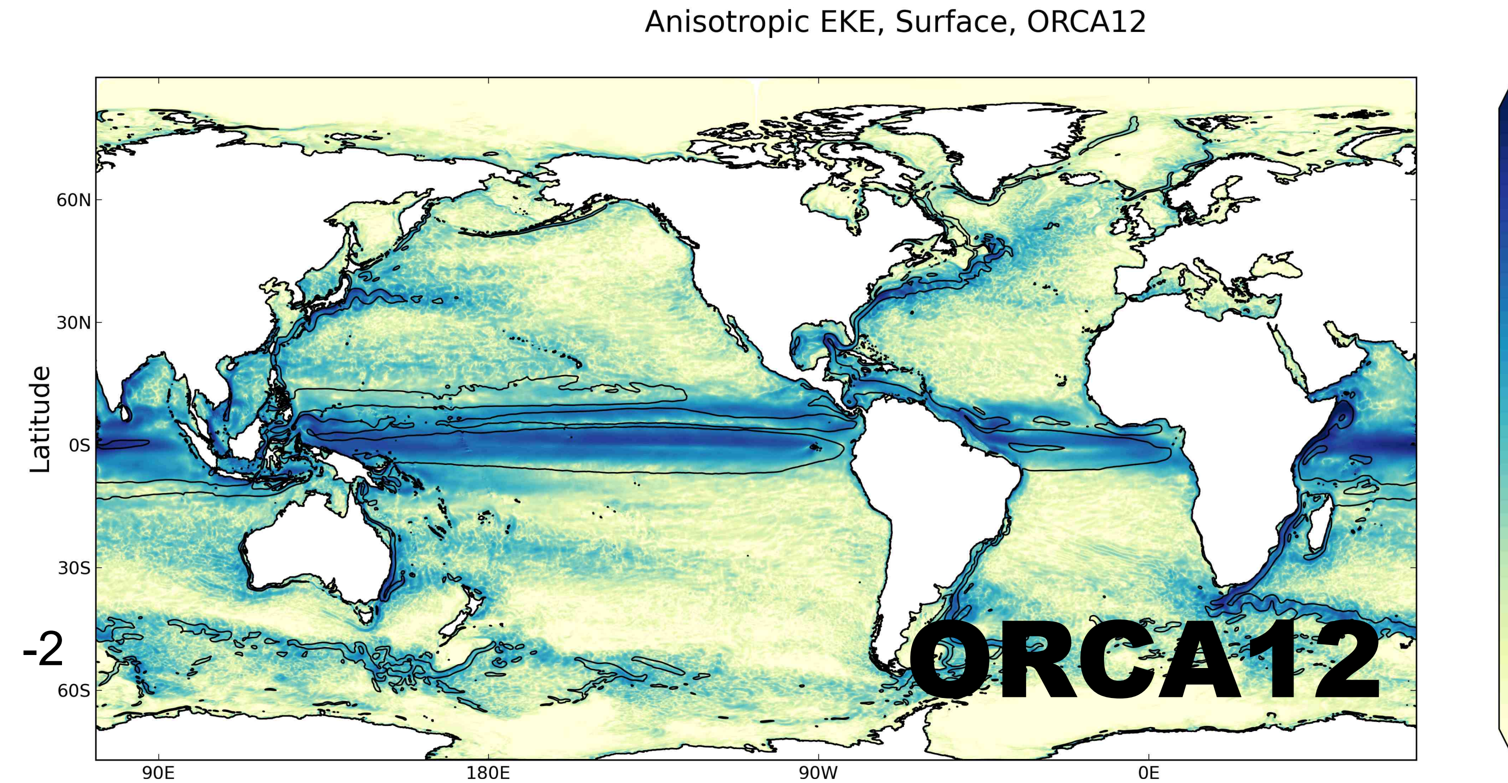


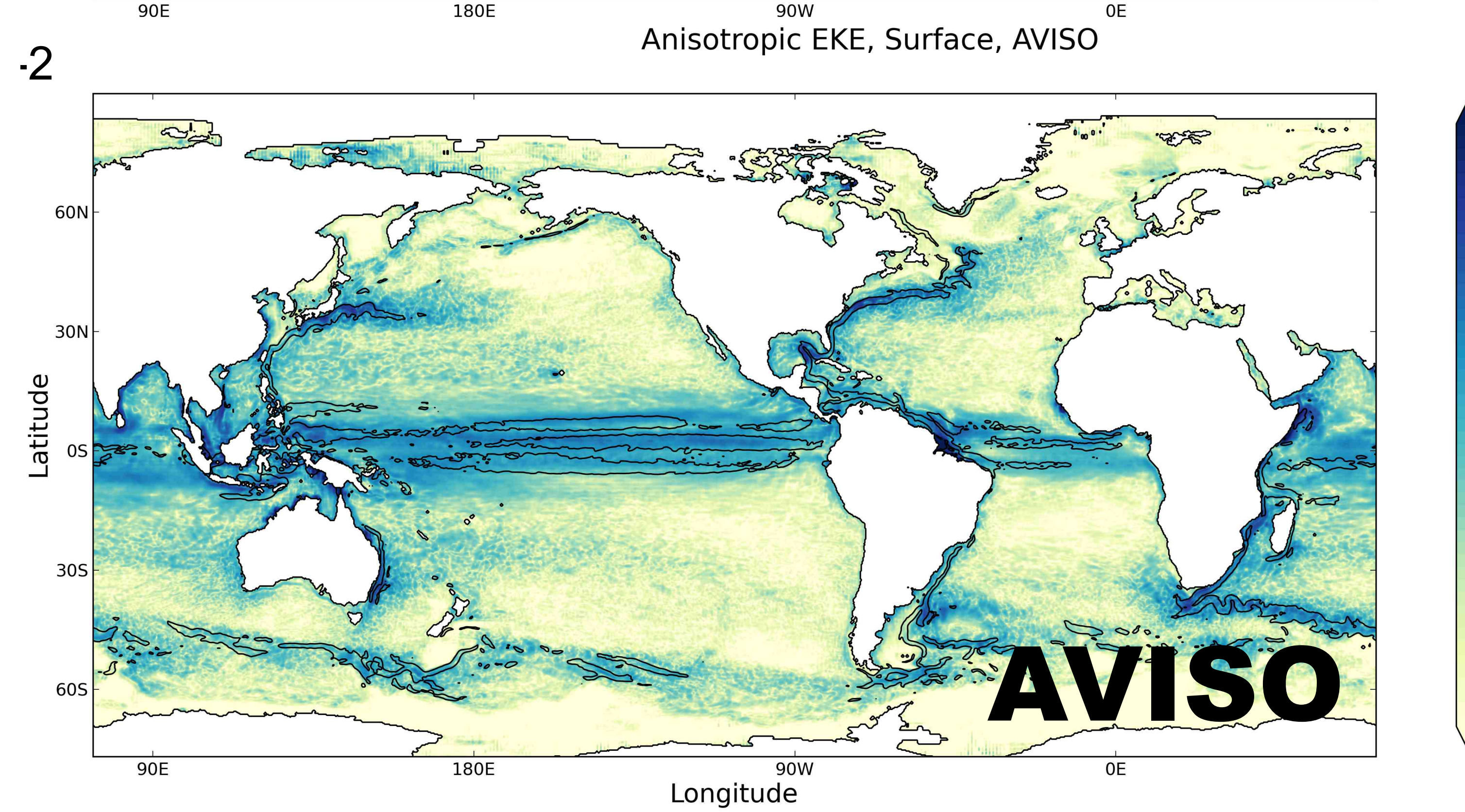
- we find these trends are ~ consistent across multiple simulation series suggesting the association of a decline in eddy forcing with coarsening resolution with a decline in resolved eddy anisotropy is robust

Application to global fields

- NEMO Ocean Model at 1/4^{o} and 1/12^{o} resolution ("ORCA025" and "ORCA12")
  - horizontal velocities form global ocean-ice model hindcasts 2003-2012
  - 5 day means to compute u' and v'

- AVISO satellite altimetry data at 1/4^{o} nominal resolution
  - compute geostrophic velocities from ΔSSH for same period (2003-2012)
  - "daily" fields to compute u' and v'



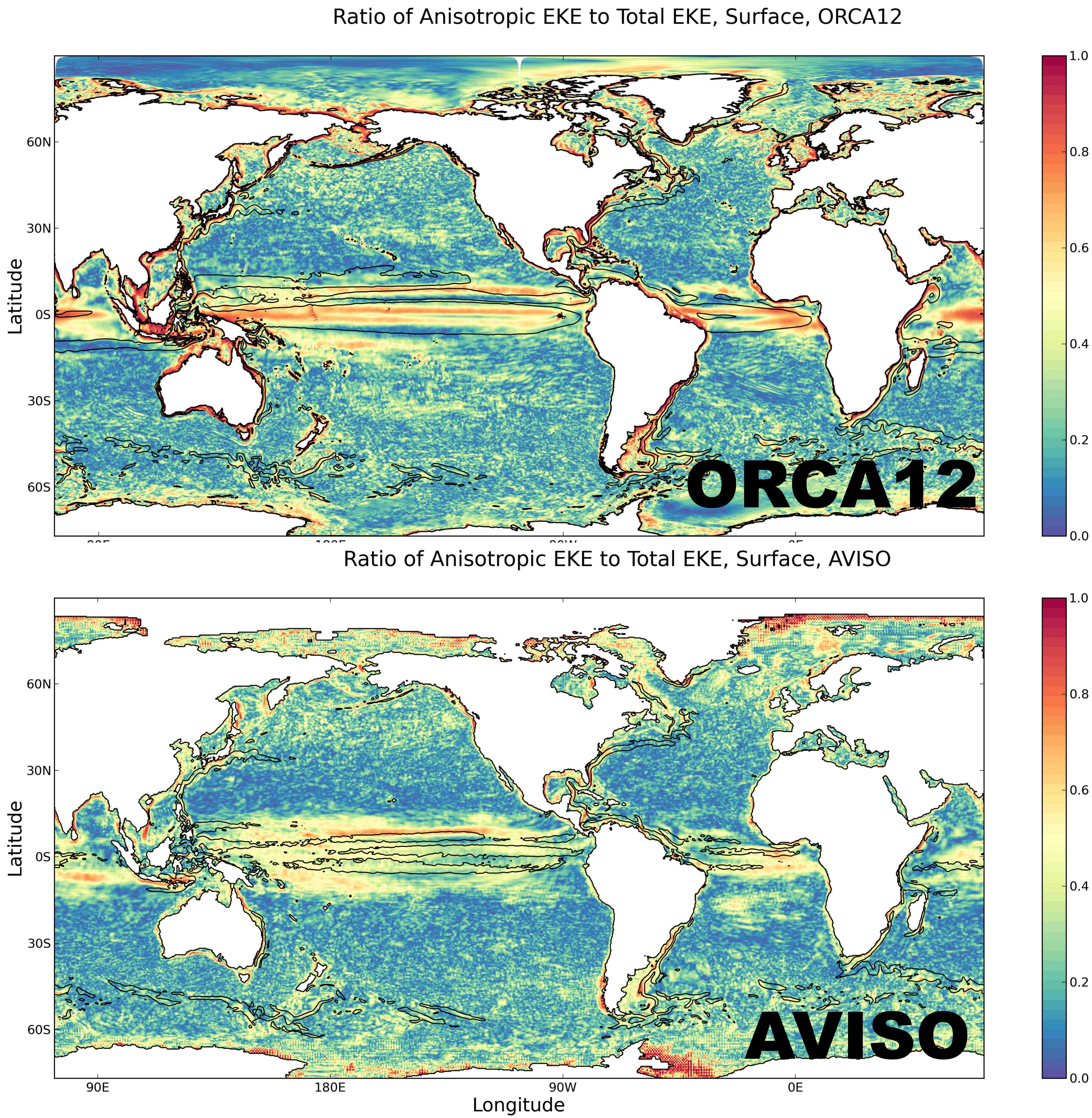


Work led by Kial Stewart, ANU

THE GEOMETRIC DECOMPOSITION: APPLICATION TO GLOBAL FIELDS

L/K at the surface

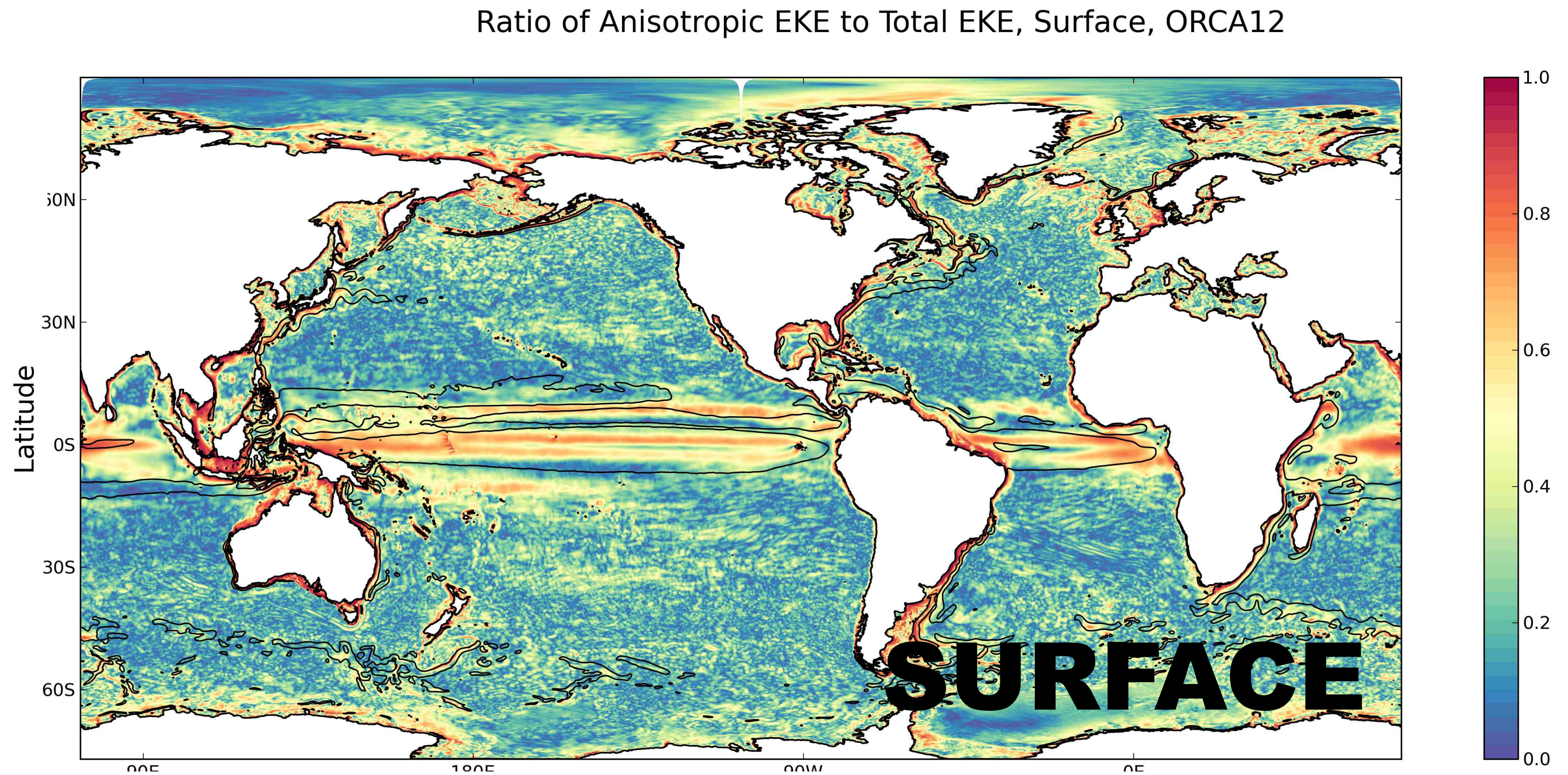
- large in and surrounding regions of large flow speeds, at the equator, & along coastlines & shelf breaks
- small inside basin gyres
- many characteristics familiar to the distribution of EKE but more localized to large flow speed regions and richer in small-scale structure

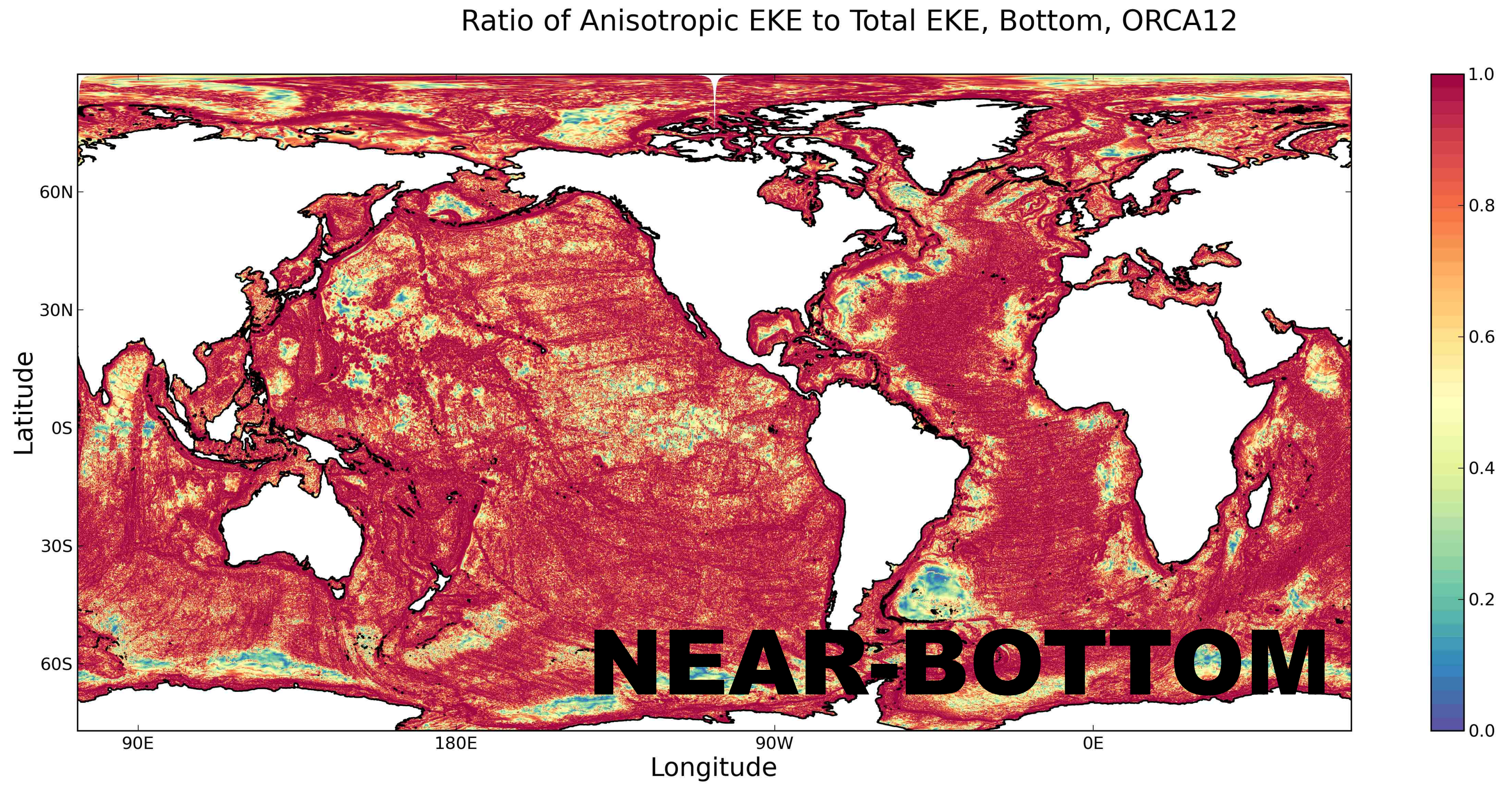


THE GEOMETRIC DECOMPOSITION: APPLICATION TO GLOBAL FIELDS

L/K at the bottom

- near-bottom eddy field is highly anisotropic almost everywhere: average L/K=0.65

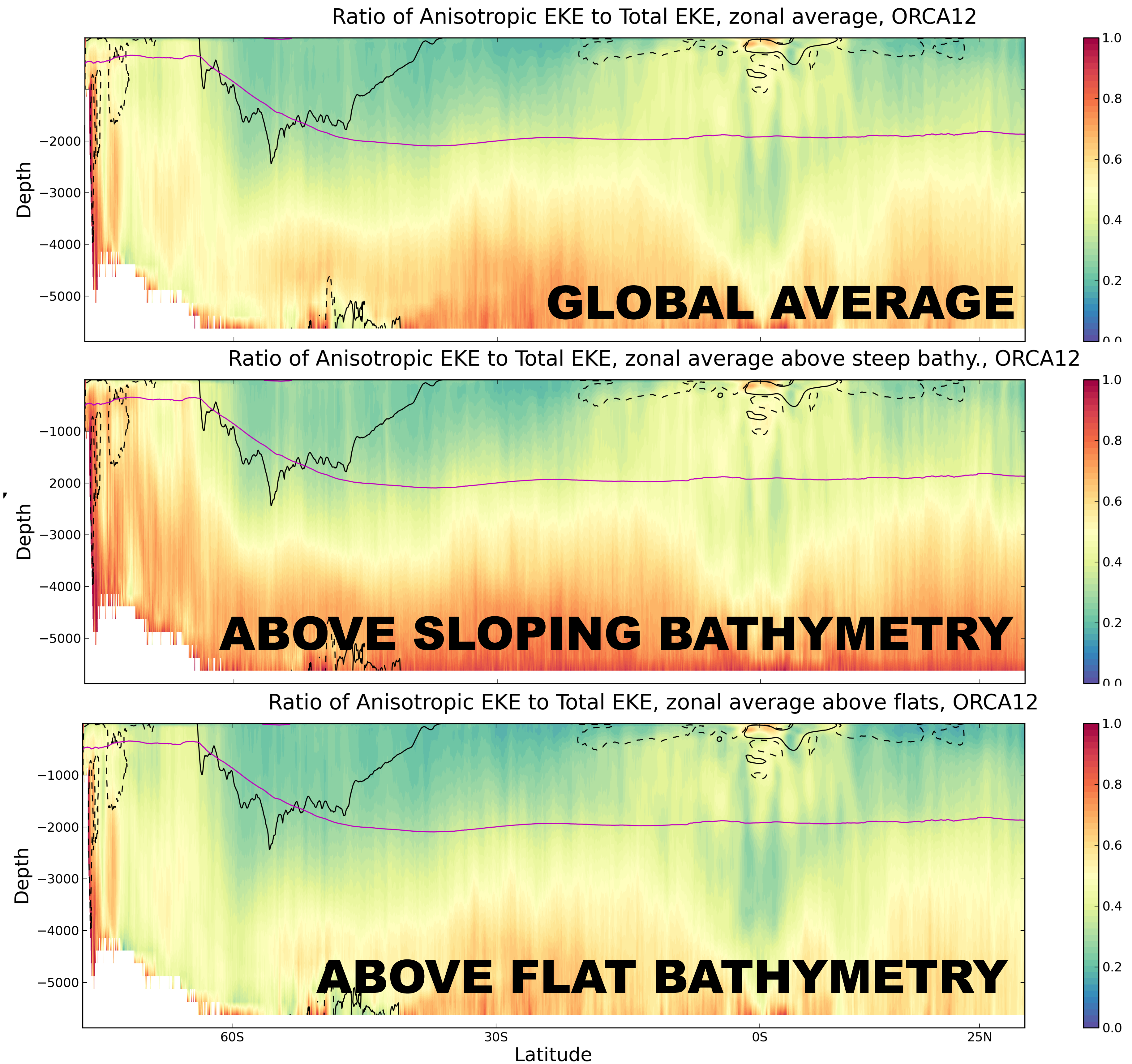




THE GEOMETRIC DECOMPOSITION: APPLICATION TO GLOBAL FIELDS

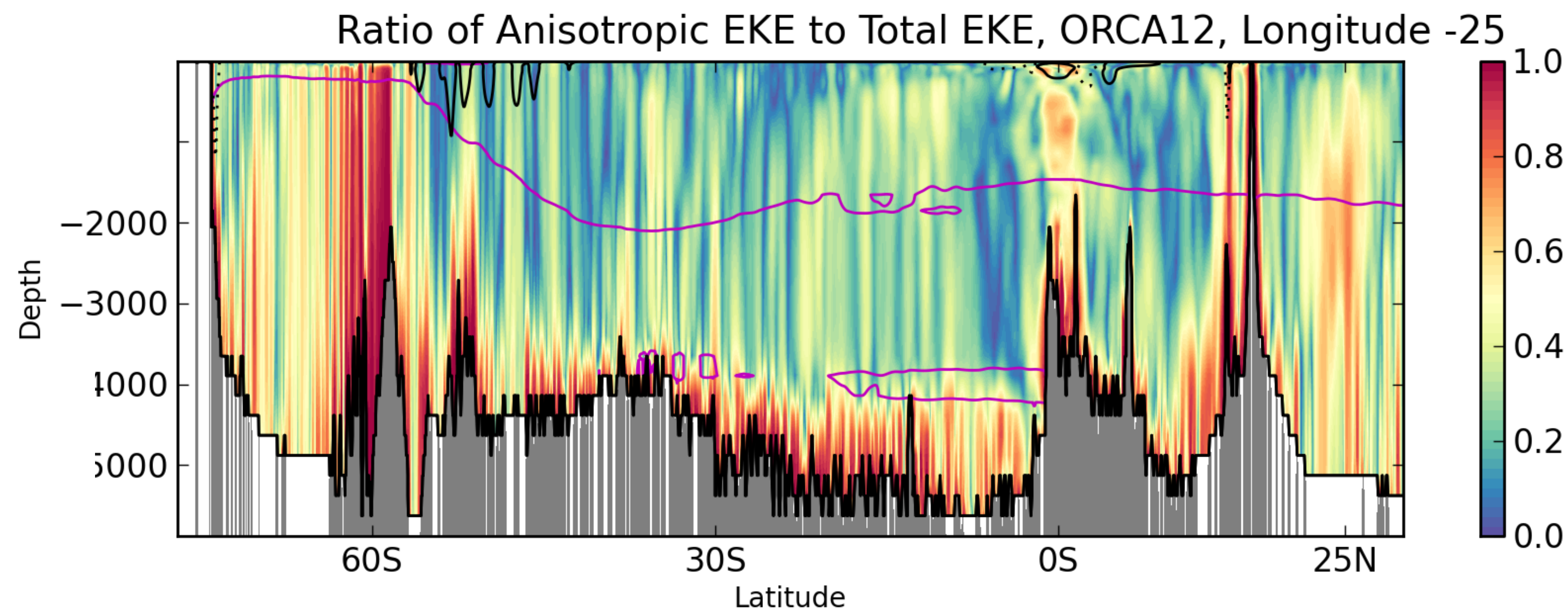
L/K at the bottom

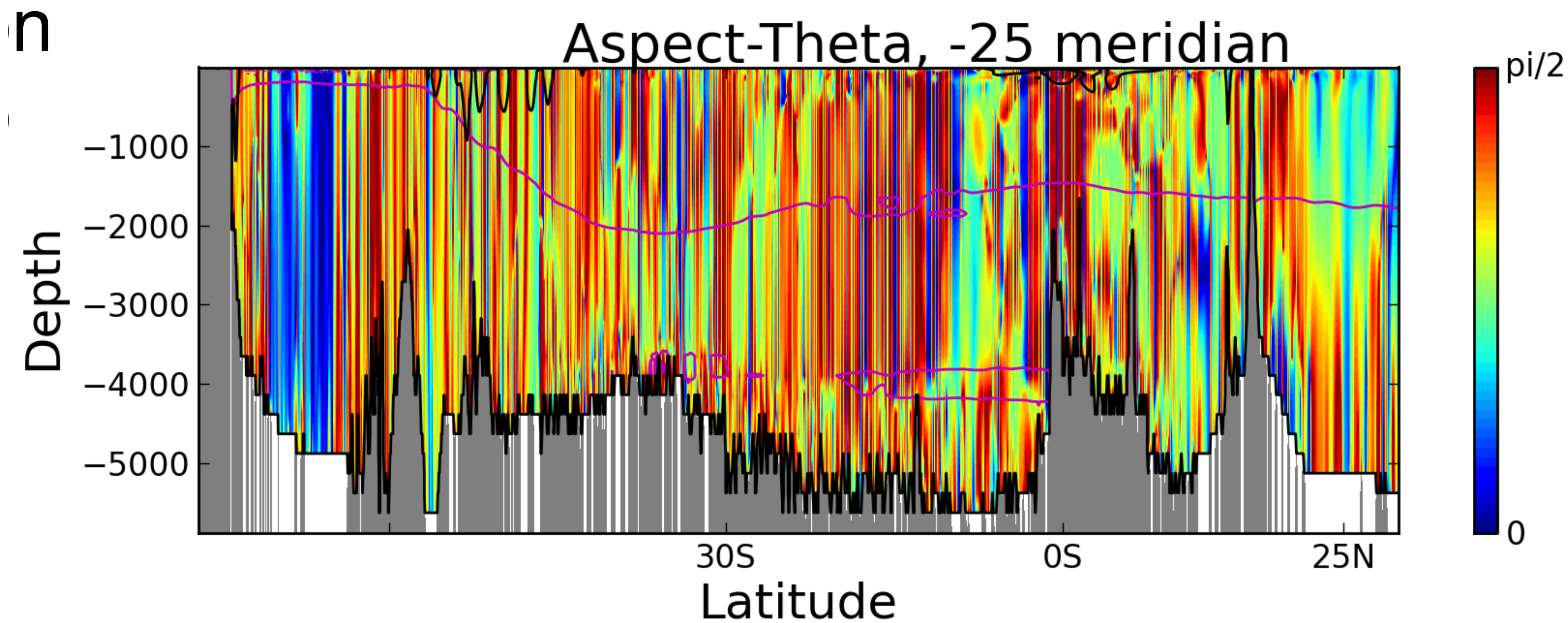
- near-bottom eddy field is highly anisotropic almost everywhere: average L/K=0.65
- near-bottom anisotropy intensifies above sloping bathymetry and reduces for regions of locally flat bathymetry; shows a strong vertical coherence



L/K at the bottom

- near-bottom eddy field is highly anisotropic almost everywhere: average L/K=0.65
- near-bottom anisotropy intensifies above sloping bathymetry and reduces for regions of locally flat bathymetry shows a strong vertical coherence
- eddy orientation tends to align with the underlying isobath and also remain vertically coherent with depth

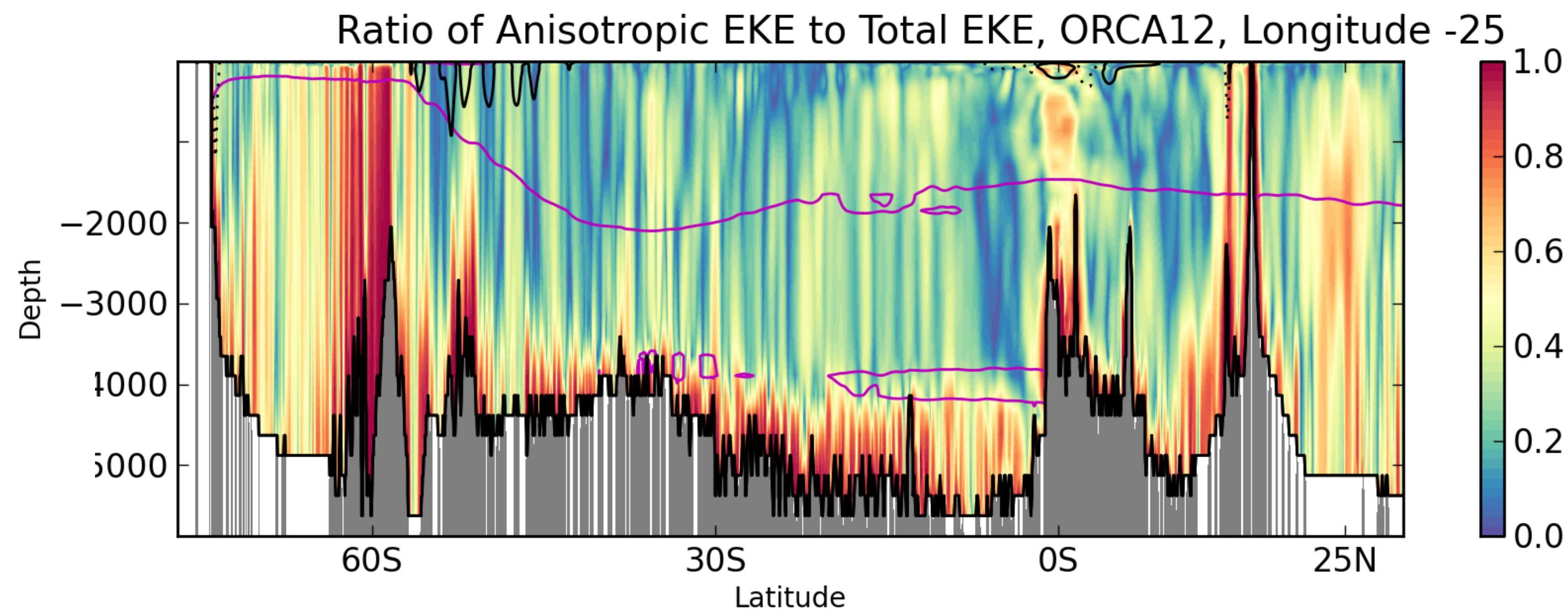


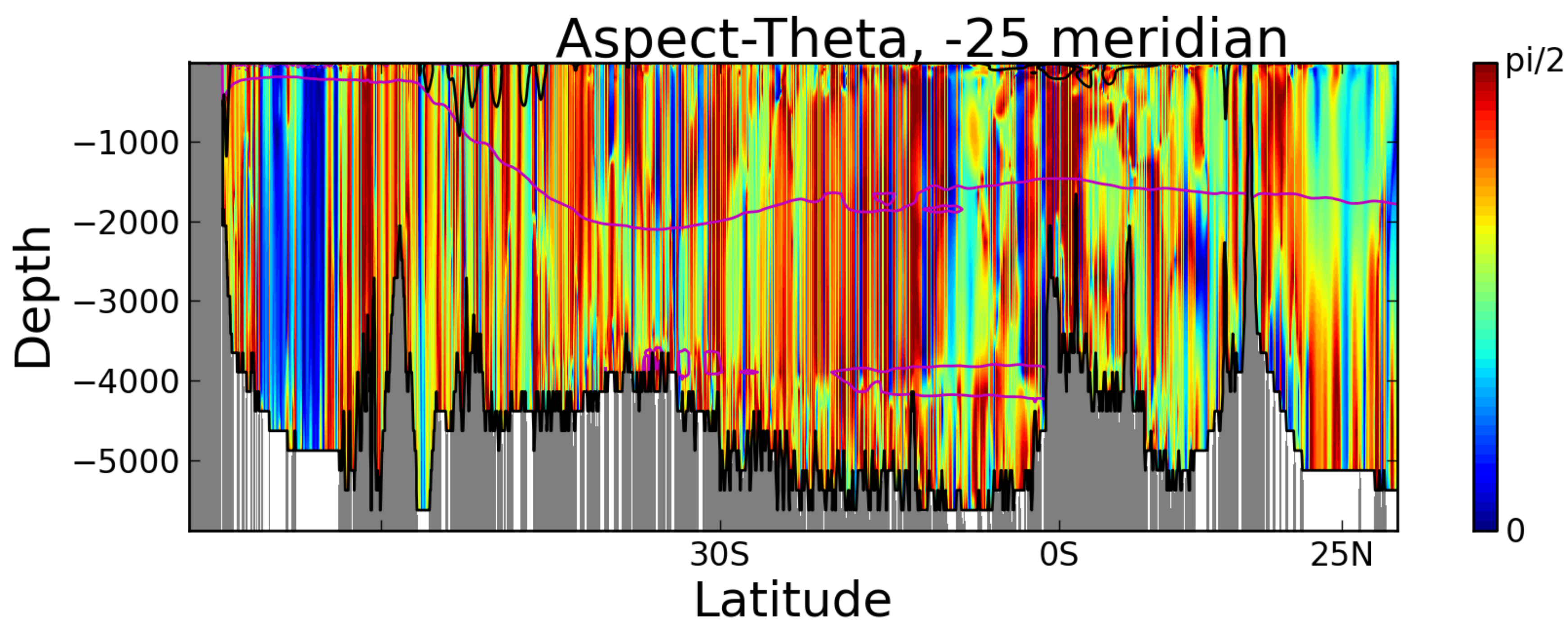


RED: along isobath
BLUE: across isobath

L/K at the bottom

- vertical coherence suggests a significant portion of the anisotropic signs is barotropic
- —> promise for a parameterization based on EKE & underlying bathymetry to operate on the barotropic flow





RED: along isobath
BLUE: across isobath

Extension to 3D

$$\Sigma = \begin{bmatrix} M+P & N & 0 \\ N & -M-P & 0 \\ -S & R & 0 \end{bmatrix}$$

EP flux tensor

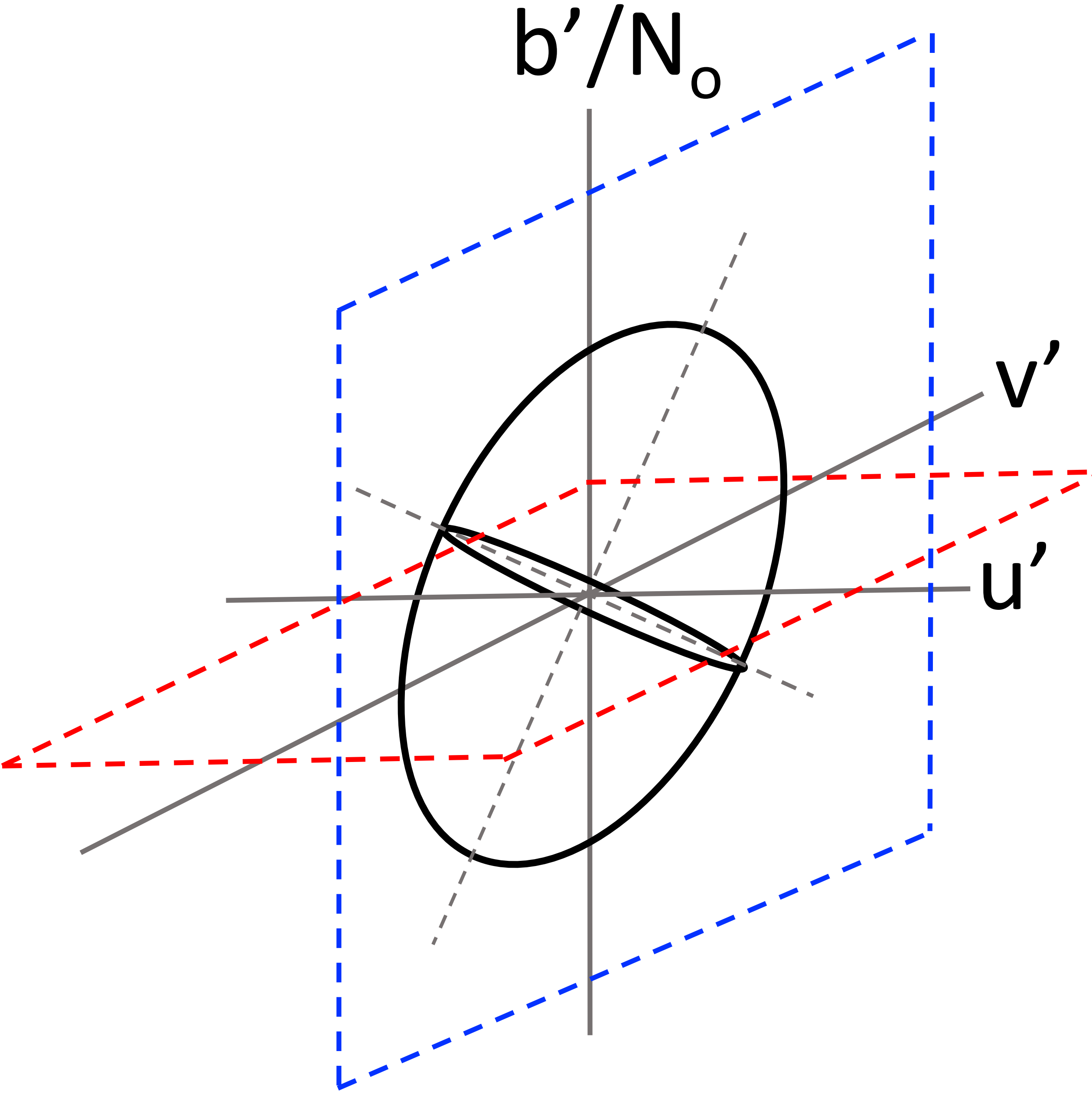
where

$$M = \frac{1}{2}\left(\overline{u'u'} - \overline{v'v'}\right), \quad N = \overline{u'v'}$$ **= eddy momentum fluxes**

$$P = \frac{1}{2N_0^2}\overline{b'b'}$$ **= eddy potential energy**

$$R = \frac{f_0}{N_0^2}\overline{b'u'}, \quad S = \frac{f_0}{N_0^2}\overline{b'v'}$$ **= eddy buoyancy fluxes**

1. Geometric Interpretation



eddy energy

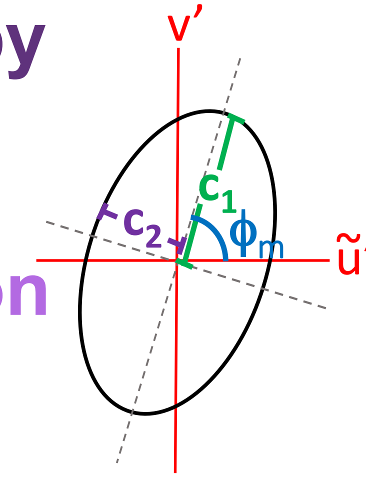
$$E = K + P$$

1. **momentum flux anisotropy**

$$\gamma_m = \frac{\sqrt{M^2 + N^2}}{K}$$

2. **momentum flux orientation**

$$\cos(2\phi_m) = \frac{-M}{\sqrt{M^2 + N^2}}$$

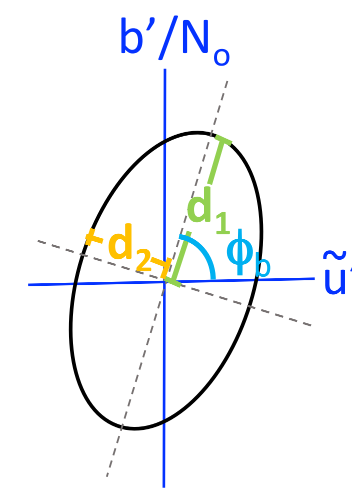


3. **buoyancy flux anisotropy**

$$\gamma_b = \frac{N_o}{2f_o}\frac{\sqrt{R^2 + S^2}}{KP}$$

4. **buoyancy flux orientation**

$$\cos(\phi_b) = \frac{R}{\sqrt{R^2 + S^2}}$$



5. **eddy energy partition angle**

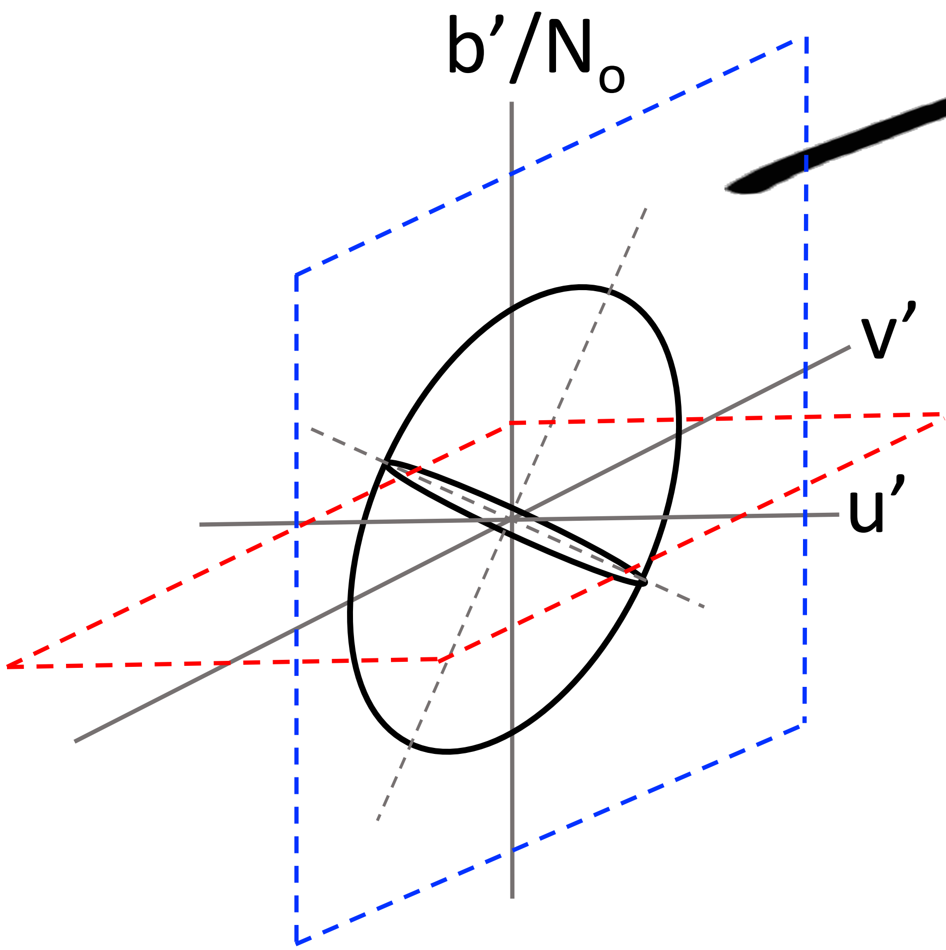
$$\frac{K}{E} = \cos^2\lambda, \frac{P}{E} = \sin^2\lambda$$

2. Dynamical Interpretation

$$\Sigma = \begin{bmatrix} M+P & N & 0 \\ N & -M-P & 0 \\ -S & R & 0 \end{bmatrix}$$

E-P fluxes	Σ
EP flux divergence (eddy momentum forcing)	$\nabla \cdot \Sigma$
eddy vorticity flux divergence (eddy vorticity forcing)	$\vec{k} \cdot \nabla \times [\nabla \cdot \Sigma]$

3. The Geometric Decomposition



$$\Sigma$$

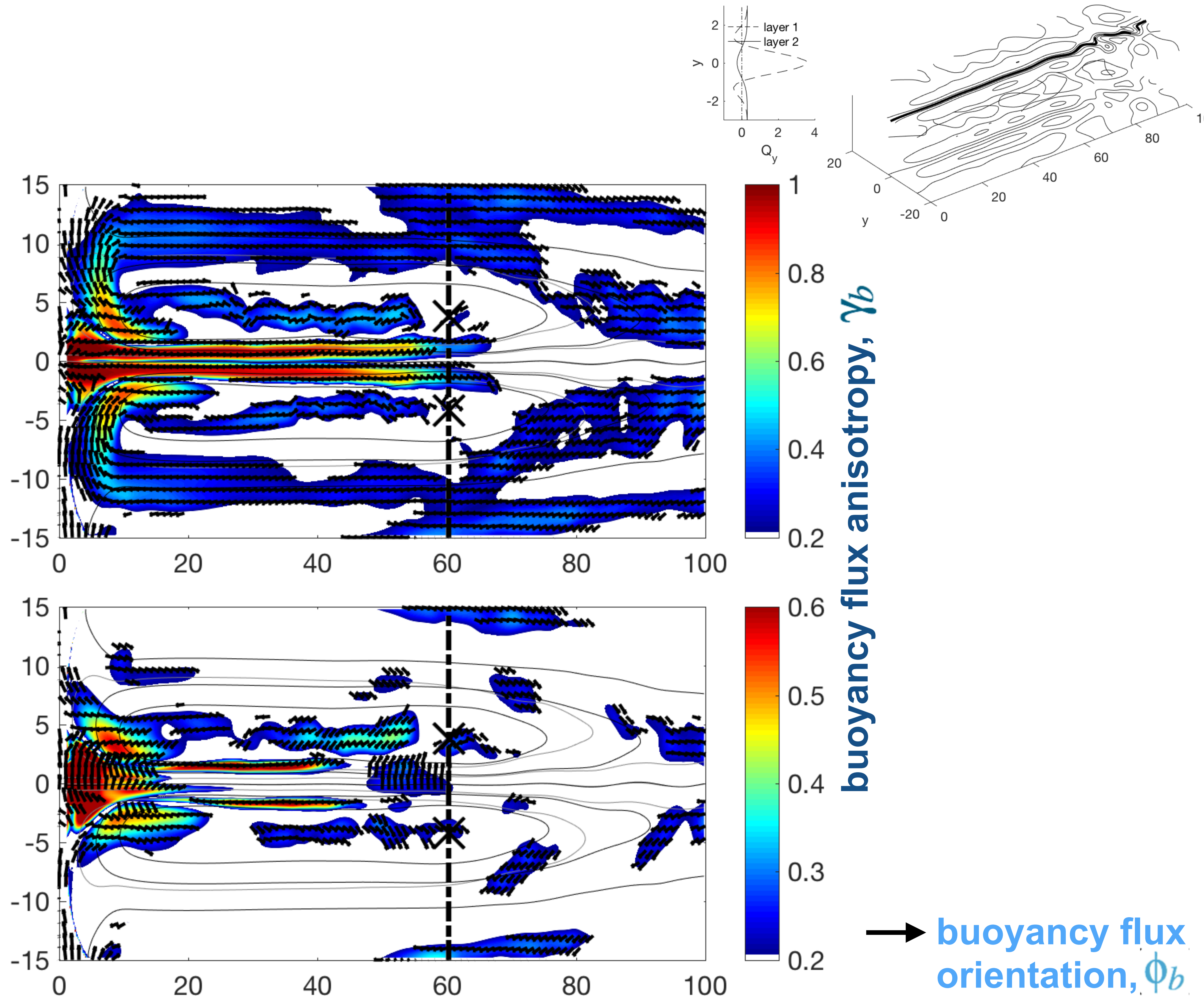
$$\nabla \cdot \Sigma$$

$$\vec{k} \cdot \nabla \times [\nabla \cdot \Sigma]$$

$$\Sigma = \begin{bmatrix} E\gamma_m\left(\cos(2\phi_m)\cos^2\lambda + \sin^2\lambda\right) & E\gamma_m \sin(2\phi_m)\cos^2\lambda & 0 \\ E\gamma_m \sin(2\phi_m)\cos^2\lambda & E\gamma_m\left(\sin^2\lambda - \cos(2\phi_m)\cos^2\lambda\right) & 0 \\ \frac{f_o}{N_o}E\gamma_b \cos\phi_b \sin(2\lambda) & \frac{f_o}{N_o}E\gamma_b \sin\phi_b \sin(2\lambda) & 0 \end{bmatrix}$$

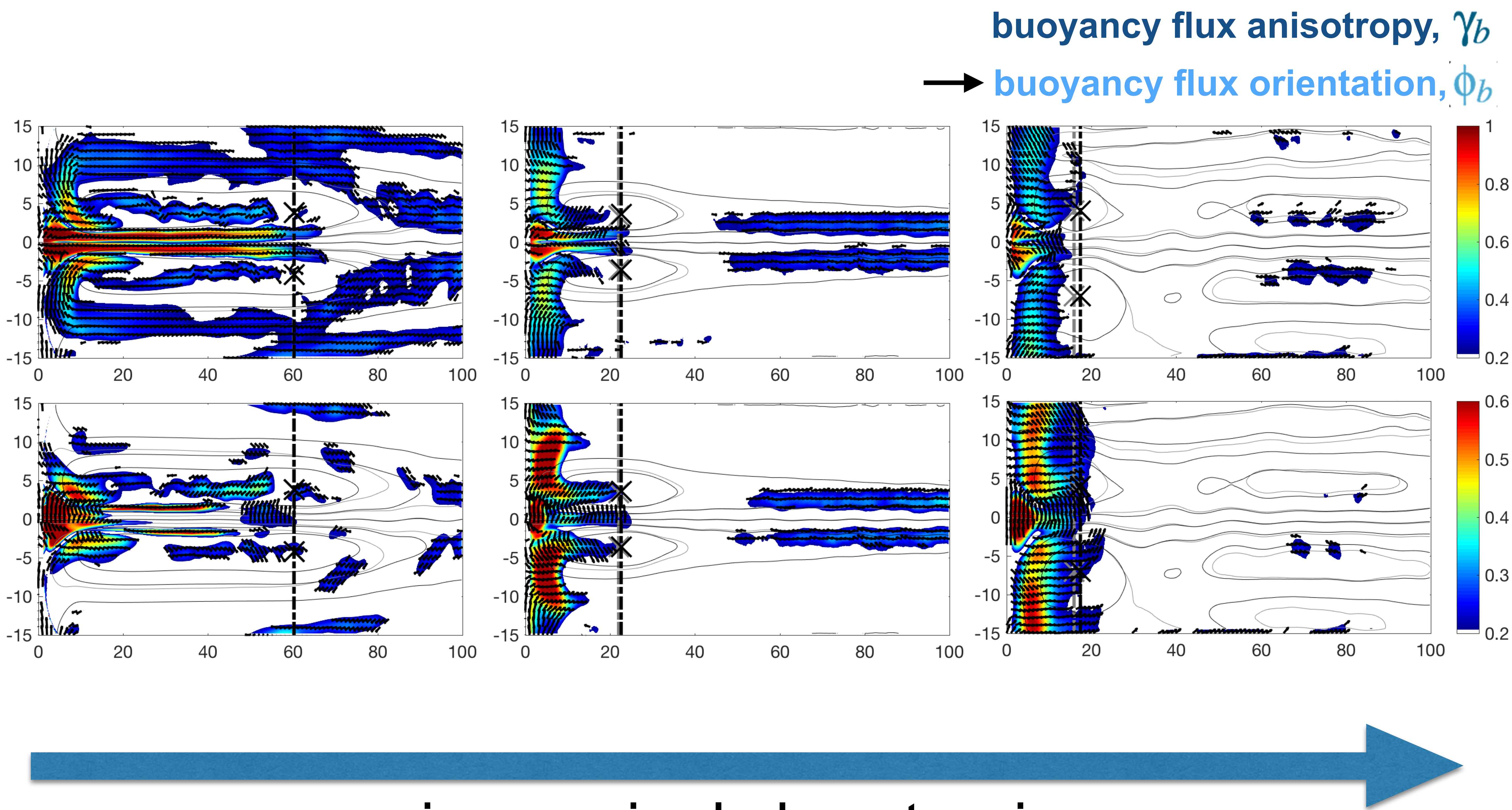
THE GEOMETRIC DECOMPOSITION: EXTENSION TO 3D

Application to a mixed instability jet



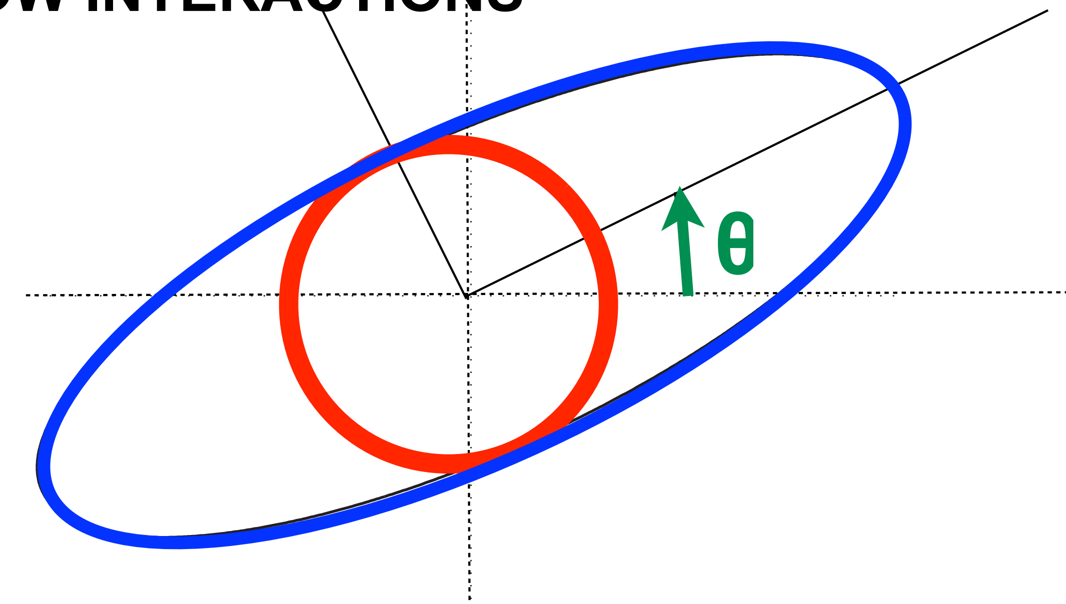
THE GEOMETRIC DECOMPOSITION: EXTENSION TO 3D

Application to a mixed instability jet



increasingly barotropic

In summary...



- a new framework to describe eddy-mean flow interactions in terms of spatial patterns of variance ellipse/ellipsoid geometry

- in 2D: describes the eddy vorticity flux divergence ('the eddy forcing') in terms of spatial patterns of ellipse **shape** & **orientation** - specifically linear variations of these properties around a region periphery in the integral form **= a significant simplification to the representation of the eddy forcing!**

- in 3D: ellipsoid geometry encodes info on dominant orientation of eddy **momentum** & **buoyancy** fluxes, **partitioning of eddy energy <—> kinetic & potential forms**, & efficiency of eddy forcing relative to eddy energy; as in 2D, spatial patterns of ellipsoid geometry are linked directly to the eddy forcing

- identifies the importance of resolving eddy shape (and not just eddy size/EKE) to resolve eddy effects

- application to global fields in a high-resolution model suggests a possible parameterization for eddy anisotropy & associated effects based on EKE and the underlying bathymetry to operate on the barotropic flow

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Waterman S. and J. M. Lilly (2015). Geometric Decomposition of Eddy Feedbacks in Barotropic Systems. J. Phys. Oceanogr., 45, 1009–1024.

Stewart, K. D., P. Spence, S. Waterman, J. Le Sommer, J.-M. Molines, J. M. Lilly and M. H. England (2015). Anisotropy of eddy variability in the global ocean. Ocean Modelling, 95, 53-65.