The interaction between atmospheric gravity waves and large-scale flows: an efficient description beyond the nonacceleration paradigm

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Gravity Waves in the Atmosphere



- main sources: orography, convection, jets/fronts
- mainly vertical energy (momentum) transport with $\vec{c}_g \Rightarrow$ interaction with the large scale flow ("drag")
- wave breaking ⇒ turbulence, dissipation, energy transfer to large scale flow ("drag")

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(Kim et al., 2003)

Motivation

Parametrization of atmospheric GWs



- GWs are not fully resolved by GCMs and NWP models ⇒ parametrization
 ⇒ (Wentzel-Kramers-Brillouin) WKB theory
- Currently used parametrizations: steady state approximation
 instantenous propagation through constant resolved flow
 instantenous data via wave breaking only
 - \Rightarrow instantenous drag via wave breaking only!
- Proposal for improvement: direct weakly-nonlinear coupling between the GW and the resolved flow ⇐⇒ transient propagation ⇐⇒ continuous drag on the resolved flow during propagation + drag through wave breaking

WKB theory

Wave resolving system (2-D Euler equations, no rotation):

$$\frac{Du}{Dt} + c_p \theta \frac{\partial \pi}{\partial x} = 0$$
$$\frac{Dw}{Dt} + c_p \theta \frac{\partial \pi}{\partial z} + g = 0$$
$$\frac{D\theta}{Dt} = 0$$
$$\frac{D\theta}{Dt} = 0$$
$$\frac{D\pi}{Dt} + \frac{\kappa}{1 - \kappa} \pi \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) = 0$$

with Exner pressure Pot. temperature

$$\begin{split} \frac{D}{Dt} &= \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + w \frac{\partial}{\partial z} \\ \pi &= (p/p_0)^{\kappa} \\ \theta &= T(p_0/p)^{\kappa} = T/\pi \\ \kappa &= R/c_p \end{split}$$

Simplification ingredients:

- Decomposition of the fields: $f = f_b + f_w$
- WKB assumption: $f_w(x, z, t) = \operatorname{Re} F_w(Z, T) e^{i \left[kx + \frac{\phi(Z, T)}{\epsilon}\right]}$ with $Z = \epsilon z, T = \epsilon t, \ m = \partial \phi / \partial Z$ and $\omega = -\partial \phi / \partial T$
- Scaling for the gravity waves: $\epsilon = \mathcal{L}_w/H_\theta << 1$: weak stratification



WKB theory

WKB assumption in nature:



WKB theory

- At leading order $\mathcal{O}(\epsilon^2)$: dispersion-, and polarization relations \Rightarrow ray equations
- At next order $\mathcal{O}(\epsilon^3)$: wave action conservation and the mean-flow equations
- The coupled system (Achatz et al., 2010):



• Problem: if rays crossing \Rightarrow caustics: several m at same height $z \Rightarrow$ e.g. c_{gz} multivalued BUT! $\mathcal{A} = \mathcal{A}(z, t) \Rightarrow$ wave action conservation ill-defined \Rightarrow numerical problems

WKB theory in phase space



• Solution: extension of the model to a 2D phase space (z,m)

 "Slicing up" the wave action density to several m intervals ⇒ phase-space wave action density:

$$\mathcal{N}(z,m,t) = \int\limits_{R} \mathcal{A}_{\alpha}(z,t)\delta[m-m_{\alpha}(z,t)]d\alpha$$

Eulerian view

$$\frac{\partial \mathcal{N}}{\partial t} + \frac{\partial (c_{gz}\mathcal{N})}{\partial z} + \frac{\partial (\dot{m}\mathcal{N})}{\partial m} = 0$$

- ... and we have $\frac{\partial c_{gz}}{\partial z} + \frac{\partial \dot{m}}{\partial m} = 0$
- Lagrangian view

Hertzog et al., 2002, Muraschko et al., 2015

$$\frac{\partial \mathcal{N}(z,m,t)}{\partial t} + c_{gz} \frac{\partial \mathcal{N}(z,m,t)}{\partial z} + \dot{m} \frac{\partial \mathcal{N}(z,m,t)}{\partial m} = 0$$

WKB theory in phase space

• Coupled wave - meanflow equations in phase space:

- ullet phase space wave action density ${\cal N}$ conserved along ray trajectories
- multiple m values allowed at each location $z \rightarrow$ spectral (non-monochromatic) treatment \rightarrow no caustics problems
- extension to Muraschko et al., 2015: isothermal background with an atmosphere-like density profile, representation of turbulent wave breaking



Mean flow

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WKB theory in phase space + wave breaking

• saturation occurs if static instability sets in (*Lindzen 1981*), i.e. $\partial \theta_w / \partial z + d\overline{\theta} / dz < 0$ or, after an additional multiplication by $g/\overline{\theta}$

$$\frac{\partial b_w}{\partial z} + N^2 < 0$$

• with the ansatz $b_w(x,z,t) = \operatorname{Re} B_w(Z,T) e^{i \left\lfloor kx + \frac{\phi(Z,T)}{\epsilon} \right\rfloor}$ with $m = \partial \phi / \partial Z$ this amounts in

$$|m||B_w| > N^2$$

• monochromatic \Rightarrow spectral

$$\int_{-\infty}^{\infty} m^2 \frac{d|B_w|^2}{dm} dm = \frac{2N^2}{\bar{\rho}} \int_{-\infty}^{\infty} m^2 \hat{\omega} \mathcal{N} dm > \alpha^2 N^4$$

where α is a parameter accounting for the uncertainty of the criterion

 If saturation ⇒ wave action density N is reset to a value that sets back stability (height,- and scale-dependent eddy diffusivity coefficient).

Methodology

- LES: fully non-linear wave resolving reference (PincFloit, Rieper et al., 2013)
- WKB-eu: Eulerian WKB model
- WKB-la: Lagrangian WKB model
- WKB-st: steady-state WKB model

Idealized cases (Bölöni et al., 2016)

- hydorostatic and nonhydrostatic wavepackets: $1 < rac{\lambda_x}{\lambda_z} < 30$
- static instability: $|m|B_w > N^2$
- ullet modulational instability: $|m|pprox |k| \Rightarrow$ wave packet is shrinking, its amplitude growing
- ullet critical layer: $U_{jet}pprox -c_p \Rightarrow m$ grows to infinity, wavepacket collapses
- refraction by a jet: U_{jet} weak $\Rightarrow m$ only slightly modified

• reflection from a jet:
$$U_{jet} \geq rac{N}{k} \left(1 - rac{k}{\sqrt{k^2 + m^2}}
ight) \Rightarrow m$$
 and c_{gz} changes sign

 $Main \ question: \ relative \ importance \ of \ direct \ wave-meanflow \ interaction \ and \ wave \ breaking$

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Induced wind, refraction by a jet $(\lambda_x = 10km, \lambda_z = 1km)$



Wave energy, reflection from a jet ($\lambda_x = 10 km, \lambda_z = 1 km$) Bölöni et al., 2016









with the vertically integrated energy:

$$\bar{E}_w = \int_0^{L_z} dz E_w$$
$$\bar{E}_m = \int_0^{L_z} dz E_m$$
$$\bar{E}_{tot} = \bar{E}_w + \bar{E}_m$$

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Bölöni et al., 2016

Static instability ($\lambda_x = \lambda_z = 1km$)





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WKB vs. WKB-steady-state

Wave field

Mean flow

WKB theory: transient coupled system

$$\begin{aligned} \frac{\mathrm{d}_{g}z}{\mathrm{d}t} &= \mp \frac{Nkm}{(k^2 + m^2)^{3/2}} \equiv c_{gz} \\ \frac{\mathrm{d}_{g}m}{\mathrm{d}t} &= \mp \frac{k}{(k^2 + m^2)^{1/2}} \frac{\mathrm{d}N}{\mathrm{d}z} - k \frac{\mathrm{d} \mathbf{u}_{b}}{\mathrm{d}z} \equiv \dot{m} \\ \frac{\mathrm{d}g}{\mathrm{d}t} &= -\mathbf{A} \frac{\partial c_{gz}}{\partial z} \end{aligned}$$

$$\frac{\partial u_b}{\partial t} = -\frac{1}{\overline{\rho}} \frac{\partial}{\partial z} (kc_{gz} \mathcal{A})$$

The steady state approximation

$$\begin{array}{lll} \displaystyle \frac{\mathrm{d}_g z}{\mathrm{d}t} & = & \displaystyle \mp \frac{Nkm}{(k^2+m^2)^{3/2}} \equiv c_{gz} \\ \displaystyle \frac{\mathrm{d}_g m}{\mathrm{d}t} & = & 0 \\ \displaystyle \frac{\mathrm{d}_g \mathcal{A}}{\mathrm{d}t} & = & 0 \Leftarrow \Rightarrow c_{gz}(z) \mathcal{A}(z) = const. \end{array}$$

$$\frac{\partial \mathbf{u}_{b}}{\partial t} = -\frac{1}{\overline{\rho}} \frac{\partial}{\partial z} (kc_{gz} \mathbf{A})$$

 \Rightarrow no wave-mean-flow interaction! \Rightarrow wave breaking (constraining $\mathcal{A}(z)$) is necessary to get an induced wind! < ロ > (同 > (回 > (回 >))

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WKB vs. WKB-steady-state

Static instability ($\lambda_x = \lambda_z = 1km$)

Bölöni et al., 2016



TRR181 workshop, Hamburg, 4 May 2017 Interactions of GWs

Interactions of GWs and large-scale flows

Based on the idealized numerical simulations presented here...

- The direct weakly-nonlinear coupling between the GW and the meanflow is an important mechanism of wave-meanflow interactions.
- Wave breaking is also important but has a second order role in describing the interaction betwen the GW and the meanflow.
- The weakness of building GW parametrizations only on wave breaking has been demonstrated.
- The Lagrangian WKB model is very efficient: factor of 10-100 compared to a corresponding Eulerian model and factor of 1000-10000 compared to LES
- There is a good reason to try out coupled transient WKB approaches in gravity wave parametrizations

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Experiment	Wavepacket	Background	Domain size	Resolution
REFR	Cosine shape	non-Boussinesq	WKB Euler:	WKB Euler:
Refraction	$\lambda_x = 10 km, \lambda_z = 1 km$	T = 300K	$L_z = 40 km$	nz = 400, nm = 70
by a jet	$k=2\pi/\lambda_{\rm x}, m=2\pi/\lambda_{\rm z}$	$N \approx 0.018$	$m \in [0.001, 0.008]$	$dz \approx 100m, dm = 10^{-4}s^{-1}$
	$z_0=10 km, \Delta_{wp}=10 km$	$u_0 = 5m/s$	WKB Lagrange:	WKB Lagrange:
	$branch = -1, a_0 = 0.1$	$z_{\alpha} = 25km$	$L_z = 40 km$	$nz = 400, dz_{smooth} \approx 600m$
		$\Delta_u = 10 km$		$dz \approx 100m, n_{ray} = 4000$
			LES:	LES:
			$L_{\rm c}=40 km, L_{\rm x}=10 km$	nz = 1280, nx = 32
				$dz \approx 31m, dx = 310m$
REFL	Cosine shape	non-Boussinesq	WKB Euler:	WKB Euler:
Reflection	$\lambda_x = 10 km, \lambda_z = 1 km$	T = 300K	$L_z = 40 km$	nz = 400, nm = 180
from a jet	$k=2\pi/\lambda_{\rm x}, m=2\pi/\lambda_{\rm c}$	$N \approx 0.018$	$m \in [-0.01, 0.008]$	$dz \approx 100m, dm = 10^{-4}s^{-1}$
	$z_0=10 km, \Delta_{wp}=10 km$	$u_0=40m/s$	WKB Lagrange:	WKB Lagrange:
	$branch=-1, a_0=0.1$	$z_n = 25km$	$L_z = 40 km$	$nz = 400, dz_{smooth} \approx 600m$
		$\Delta_u = 10 km$		$dz\approx 100m, n_{ray}=4000$
			LES:	LES:
			$L_{\rm z}=40km, L_{\rm x}=10km$	nz = 2500, nx = 64
				$dz \approx 16m, dx = 156m$
PREFL	Cosine shape	non-Boussinesq	WKB Lagrange:	WKB Lagrange:
Partial	$\lambda_x = 6km, \lambda_z = 3km$	T = 300K	$L_z = 50 km$	$nz = 166, dz_{smooth} \approx 1800m$
Reflection	$k=2\pi/\lambda_{\rm x}, m=2\pi/\lambda_{\rm z}$	$N \approx 0.018$		$dz \approx 300m, n_{ray} = 4320$
from a jet	$z_0=10 km, \Delta_{wp}=10 km$	$u_0=9,75m/s$	LES:	LES:
	$branch=-1, a_0=0.1$	$z_{\alpha} = 25km$	$L_{\rm Z}=50 km, L_{\rm X}=6 km$	nz = 538, nx = 32
		$\Delta_u = 10 km$		$dz \approx 93m, dx = 187m$

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Interactions of GWs and large-scale flows

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State $\lambda_{e} = 30km, \lambda_{e} = 3km$ $T = 300k$ $L_{e} = 80km$ $n = 266, dz_{month} = 1800m$ Instability $k = 2\pi/\lambda_{a}, m = 2\pi/\lambda_{c}$ $N = 0.018$ LES: LES: Wavepacket $branch = -1, a_{0} = 0.5$ LES: LES: $dz = 300m, n_{eg} = 4320$ STNH Gaussian shape non-Beussines UKB Lagrange: $dz = 30km, dx = 940m$ Static $\lambda_{a} = 1km, \lambda_{a} = 1km$ $T = 300k$ $L_{c} = 30km, L_{a} = 30k$ $nz = 854, nx = 32$ Static $\lambda_{a} = 1km, \lambda_{c} = 1km$ $T = 300k$ $L_{c} = 30km$ $nz = 300, dz_{month} = 600m$ Instability $k = 2\pi/\lambda_{a}, m = 2\pi/\lambda_{c}$ $N = 0.018$ $L_{c} = 30km, L_{a} = 1km$ $nz = 960, nx = 32$ Non-hydrostatic $z_{0} = 10km, \Delta_{ep} = 10km$ LES: LES: $dz = 31m, dx \approx 310m$ Mit Osine shape non-Boussines KKB Lagrange: $nz = 600, nt_{month} = 600n$ Instability $k = 2\pi/\lambda_{a}, m = 2\pi/\lambda_{c}$ $N = 0.018$ $L_{c} = 00km, L_{a} = 10km, A_{c} = 400n$ Instability $k = 2\pi/\lambda_{a}, m = 2\pi/\lambda_{c}$ $N = 0.018$ Le = 00km, L_{a} = 10km, A_{c} = 30km $dz = 31$	STIH	Gaussian shape	non-Boussinesq	WKB Lagrange:	WKB Lagrange:	
Instability $k = 2\pi/\lambda_{x}, m = 2\pi/\lambda_{c}$ $N = 0.018$ LES: $LES:$ Hydrostatic $z_{0} = 10km, \Delta_{wp} = 25km$ LES: $LES:$ $LES:$ Wavepacket $branch = -1, a_{0} = 0.5$ LC $B0km, L_{u} = 30km$ $m = 854, nx = 32$ STINH Gaussian shape non-Boussinesq WKB Lagrange: $L_{z} = 30km, dx \approx 940m$ Static $\lambda_{u} = 1km, \lambda_{z} = 1km$ $T = 300K$ $L_{z} = 30km$ $m = 300, dz_{mouth} = 600m$ Instability $k = 2\pi/\lambda_{u}, m = 2\pi/\lambda_{z}$ $N = 0.018$ $L_{z} = 30km, L_{u} = 1km$ $m = 300, dz_{mouth} = 600m$ Non-hydrostatic $z_{0} = 10km, \Delta_{wp} = 10km$ IES: IES: $Lz = 31km, dx = 310m$ Marcypacket $branch = -1, a_{0} = 0.9$ $Lz = 30km, L_{u} = 1km$ $m = 960, nx = 32$ Modulational $\lambda_{v} = 1km$ $T = 300K$ $L_{z} = 60km, L_{u} = 1km$ $m = 600, dz_{mouth} = 600m$ Istability $k = 2\pi/\lambda_{u}, m = 2\pi/\lambda_{z}$ $N = 0.018$ $Lz = 60km, L_{u} = 1km$ $m = 1920, nx = 32$ $z_{0} = 10km, \Delta_{wp} = 2km$ $N = 0.018$ IES: IES: $Lz = 31km, dx = 310m$	Static	$\lambda_x = 30 km, \ \lambda_z = 3 km$	T = 300K	$L_z = 80 km$	$nz=266, dz_{amooth}\approx 1800m$	
Hydrostatic $z_0 = 10km, \Delta_{wp} = 25km$ LES: LES: LES: Wavepacket $branch = -1, a_0 = 0.5$ $L = 80km, L_u = 30km$ $n = 854, nx = 32$ Jacket $hranch = -1, a_0 = 0.5$ $u = 854, nx = 32$ $d z = 94m, d x = 940m$ STINH Gaussian shape non-Beussinesq WKB Lagrange: $u = 300, dz_{mouth} = 600m$ Instability $k = 2\pi/J_{n, m} - 2\pi/J_{k}$ $T = 300K$ $L_z = 30km$ $u = 300, dz_{mouth} = 600m$ Non-hydrostatic $z_0 = 10km, \Delta_{wp} = 10km$ $N = 0.018$ LES: LES: Wavepacket $branch = -1, a_0 = 0.9$ $L = 30km, L_u = 1km$ $n = 960, nx = 32$ Modulational $\lambda_v = 1km, \lambda_v = 1km$ $T = 300K$ $L_z = 60km, L_u = 1km$ $n = 600, dz_{mouth} = 600n$ Instability $k = 2\pi/J_{n, m}, A_z = 1km$ $T = 300K$ $L_z = 60km, L_u = 1km$ $n = 1920, nx = 32$ $z_0 = 10km, \Delta_{wp} = 20km$ $N = 0.018$ LES: LES: Les = 31m, dx $\approx 310m$ Instability $k = 2\pi/J_{n, m} = 2\pi/J_{n}$ $N = 0.018$ Leg = 60km, L_u = 14km $n = 1920, nx = 32$ $z_0 = 10km, \Delta_{wp} = 2km$	Instability	$k=2\pi/\lambda_x, m=2\pi/\lambda_z$	$N \approx 0.018$		$dz\approx 300m, n_{ray}=4320$	
Wavepacket branch = -1, a_0 = 0.5 L_z = 80km, L_z = 30km $nz = 884, nx = 32$ $dz = 940m, dx = 940m$ STINH Gaussian shape non-Bousinesq WKB Lagrange: WKB Lagrange: Static $\lambda_z = 1km, \lambda_z = 1km$ T = 300K $L_z = 30km$ $nz = 300, dz_{mouth} = 600m$ Instability $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ N = 0.018 Lz = 30km, $L_u = 10m, n_{erog} = 4000$ Non-hydrostatic $z_0 = 10km, \Delta_{wp} = 10km$ N = 0.018 LES: LES: Wavepacket branch = -1, a_0 = 0.9 N = 0.018 Lz = 30km, $L_u = 1km$ $nz = 960, nx = 32$ Modulational $\lambda_u = 1km, \lambda_z = 1km$ T = 300K Lz = 60km $nz = 660, dz_{mouth} = 600m$ Instability $k = 2\pi/\lambda_u, m = 2\pi/\lambda_z$ N = 0.018 Lz = 60km $nz = 1920, nz = 32$ $z_0 = 10km, \Delta_{wp} = 20km$ N = 0.018 Lz = 60km, Lu = 1km $nz = 1920, nz = 32$ $z_0 = 10km, \Delta_{wp} = 2m/\lambda_z$ N = 0.018 Lz = 60km, Lu = 1km $nz = 1920, nz = 32$ $z_0 = 10km, \Delta_{wp} = 2m/\lambda_z$ N = 0.018 Lz = 60km, Lu = 1km $nz = 1920, nz = 32$ $z_0 = 10km, \Delta_{wp} = 2m/\lambda_z$ N = 0.018 <t< td=""><td>Hydrostatic</td><td>$z_0=10 km, \Delta_{wp}=25 km$</td><td></td><td>LES:</td><td>LES:</td><td></td></t<>	Hydrostatic	$z_0=10 km, \Delta_{wp}=25 km$		LES:	LES:	
Image: Since the set of the set	Wavepacket	$branch = -1, a_0 = 0.5$		$L_z = 80 km, L_x = 30 km$	nz = 854, nx = 32	
STNH Gaussian shape non-Boussinesq WKB Lagrange: WKB Lagrange: State $\lambda_{a} = 1km, \lambda_{c} = 1km$ $T = 300k$ $L_{c} = 30km$ $n = 500, d_{cmouth} = 600m$ Instability $k = 2\pi/\lambda_{a}, m = 2\pi/\lambda_{c}$ $N = 0.018$ $L_{c} = 30km$ $dz = 100n, n_{rop} = 4000$ Non-hydrostatic $z_{0} = 10km, \Delta_{up} = 10km$ IES: IES: IES: Wavepacket $branch = -1, a_{0} = 0.9$ $L_{c} = 30km, L_{a} = 1km$ $n = 960, nx = 32$ MI Cosine shape non-Bousinesq WKB Lagrange: $dz = 31m, dx = 310m$ Modulational $\lambda_{c} = 1km$ $T = 300K$ $L_{c} = 60km$ $n = 600, dz_{mouth} = 600n$ Instability $k = 2\pi/\lambda_{a}, m = 2\pi/\lambda_{c}$ $N = 0.018$ IES: IES: $L_{c} = 00km, L_{m} = 1km$ $T = 300k$ $L_{c} = 60km, L_{m} = 1km$ $n = 1920, nx = 32$ $z_{0} = 10km, \Delta_{up} = 0.1$ $L = 60km, L_{m} = 1km$ $n = 1920, nx = 32$ $dz = 31m, dx \approx 310m$ Ct Cosine shape non-Boussinesq KKB Lagrange: $L = 30km, L_{m} = 1600, dz_{mouth} = 600n$ Layer $\lambda_{m} = 10km, \Delta_{n} = 1km$ <td></td> <td></td> <td></td> <td></td> <td>$dz \approx 94m, dx \approx 940m$</td> <td></td>					$dz \approx 94m, dx \approx 940m$	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	STINH	Gaussian shape	non-Boussinesq	WKB Lagrange:	WKB Lagrange:	
$ \begin{array}{ c c c c } \mbox{Insubility} & k = 2\pi/\lambda_{x_1}m = 2\pi/\lambda_c & N = 0.018 & dz = 100m, n_{rop} = 4000 \\ \mbox{Non-hydrostatic} & z_0 = 10km, \Delta_{u_F} = 10km & LES: & LES: \\ \mbox{Wavepacket} & branch = -1, a_0 = 0.9 & L_c = 30km, L_u = 1km & nz = 960, nx = 32 \\ \mbox{d}zz = 31m, dx = 310m & dz = 31m, dx = 310m \\ \mbox{Modulational} & \lambda_u = 1km, \lambda_z = 1km & T = 300K & L_z = 60km & nz = 600, dz_{mooth} = 600m \\ \mbox{Insubility} & k = 2\pi/\lambda_u, m = 2\pi/\lambda_c & N = 0.018 & LES: & LES: \\ \mbox{d}z = 100m, n_{rop} = 4000 & LES: & LES: \\ \mbox{d}z = 00m, n_{rop} = 4000 & LES: & LES: \\ \mbox{d}z = 100m, n_{rop} = 4000 & LES: & LES: \\ \mbox{d}z = 00m, n_{rop} = 4000 & LES: & LES: \\ \mbox{d}z = 00m, n_{rop} = 4000 & LES: & LZ = 60km, L_u = 1km & nZ = 1920, nx = 32 \\ \mbox{d}z = 31m, dx \approx 310m & LZ = 30km, L_u = 10km & nZ = 30m, dz_{mooth} = 600m \\ \mbox{Layer} & k_1 = 10km, \lambda_c = 1km & T = 300K & L_c = 30km, L_u = 10km & nZ = 300, dz_{mooth} = 600m \\ \mbox{Layer} & k_1 = 2\pi/\lambda_u, m = 2\pi/\lambda_c & N = 0.018 & L_c = 30km & nZ = 300, dz_{mooth} = 600m \\ \mbox{Layer} & k_1 = 2\pi/\lambda_u, m = 2\pi/\lambda_c & N = 0.018 & L_c = 30km & nZ = 30m, dz_{mooth} = 600m \\ \mbox{Layer} & k_2 = 2\pi/\lambda_u, m = 2\pi/\lambda_c & N = 0.018 & L_c = 30km & nZ = 30m, dz_{mooth} = 600m \\ \mbox{Layer} & k_2 = 2\pi/\lambda_u, m = 2\pi/\lambda_c & N = 0.018 & L_c = 30km & L_c = 100, n_{rop} = 4000 \\ \mbox{Layer} & k_2 = 2\pi/\lambda_u, m = 2\pi/\lambda_c & N = 0.018 & L_c = 30km, L_u = 10km & nZ = 900, nX = 32 \\ \mbox{Layer} & k_2 = 10km, \Delta_{u_F} = 10km & n_0 = -11m/s & LES: & LES: \\ \mbox{Layer} & L_c = 30km, L_u = 10km & n_c = 900, nX = 32 \\ \mbox{Layer} & L_u = 30km, L_u = 10km & n_c = 900, nX = 32 \\ \mbox{Layer} & L_u = 30km, L_u = 10km & n_c = 900, nX = 32 \\ \mbox{Layer} & L_u = 30km, L_u = 10km & n_c = 900, nX = 32 \\ \mbox{Layer} & L_u = 30km, L_u = 10km & n_c = 900, nX = 32 \\ \mbox{Layer} & L_u = 30km, L_u = 10km & L_u = 30m, dX = 310m \\ \mbox{Layer} & L_u = 30km, L_u = 10km & L_u = 30m, dX = 310m \\ \mbox{Layer} & L_u = 30km, L_u = 10km & L_u = 30km, L_u = 30km, dX = 310m \\ \mbox{Layer} & L_u = 30km, L_u =$	Static	$\lambda_x = 1km, \lambda_z = 1km$	T = 300K	$L_z = 30km$	$nz=300, dz_{anooth}\approx 600m$	
$ \begin{array}{c c c c c c c c } \mbox{Non-hydrostatic} & z_0 = 10km, \Delta_{u_F} = 10km \\ \mbox{Wavepacket} & branch = -1, a_0 = 0.9 \\ \mbox{Wavepacket} & branch = -1, a_0 = 0.9 \\ \mbox{Wavepacket} & branch = -1, a_0 = 0.9 \\ \mbox{MI} & Cosine shape & non-Boussinesq & WKB Lagrange: \\ \mbox{Modulational} & \lambda_{\pi} = 1km, \lambda_{\pi} = 1km & T = 300K & L_{\pi} = 60km & n\pi = 600, nt_{\pi = 00} = 600n \\ \mbox{Insubility} & k = 2\pi/\lambda_{\pi}, m = 2\pi/\lambda_{\pi} & N = 0.018 & LES: \\ \mbox{Issubility} & k = 2\pi/\lambda_{\pi}, m = 2\pi/\lambda_{\pi} & N = 0.018 & LES: \\ \mbox{Issubility} & L_{\pi} = 0.1 & LES: \\ \mbox{Issubility} & L_{\pi} = 10km, \Delta_{u_F} = 0.018 & LES: \\ \mbox{Issubility} & L_{\pi} = 10km, \Delta_{u_F} = 0.11 & LES: \\ \mbox{Issubility} & L_{\pi} = 10km, \Delta_{u_F} = 0.11 & LES: \\ \mbox{Issubility} & L_{\pi} = 10km, \Delta_{u_F} = 10km & T = 300K & L_{\pi} = 300, nt_{mog} = 4000 \\ \mbox{Layer} & k = 2\pi/\lambda_{\pi}, m = 2\pi/\lambda_{\pi} & N = 0.018 & L_{\pi} = 30km, L_{\pi} = 10km & n\pi = 300, nt_{mog} = 4000 \\ \mbox{Layer} & k = 2\pi/\lambda_{\pi}, m = 2\pi/\lambda_{\pi} & N = 0.018 & LES: \\ \mbox{Issubility} & L_{\pi} = 10km, \Delta_{u_F} = 10km & n_{0} = -11m/s & LES: \\ \mbox{Issubility} & L_{\pi} = 30km, L_{\pi} = 10km & n_{\pi} = 960, nx = 32 \\ \mbox{Issubility} & L_{\pi} = 30km, L_{\pi} = 10km & n_{\pi} = 960, nx = 32 \\ \mbox{Issubility} & L_{\pi} = 10km & n_{\pi} = 960, nx = 32 \\ \mbox{Issubility} & L_{\pi} = 10km & LES: \\ \mbox{Issubility} & L_{\pi} = 10km & n_{\pi} = 960, nx = 32 \\ \mbox{Issubility} & L_{\pi} = 30km, L_{\pi} = 10km & n_{\pi} = 960, nx = 32 \\ \mbox{Issubility} & L_{\pi} = 10km & n_{\pi} = 960, nx = 32 \\ \mbox{Issubility} & L_{\pi} = 10km & L_{\pi} = 10km & n_{\pi} = 960, nx = 32 \\ \mbox{Issubility} & L_{\pi} = 10km & L_{\pi} = 10km & L_{\pi} = 30m, dx = 310m \\ \mbox{Issubility} & L_{\pi} = 30km, L_{\pi} = 10km & n_{\pi} = 960, nx = 32 \\ \mbox{Issubility} & L_{\pi} = 10km & L_{\pi} = 30m, dx = 310m \\ \mbox{Issubility} & L_{\pi} = 10km & L_{\pi} = 30m, dx = 310m \\ \mbox{Issubility} & L_{\pi} = 10km & L_{\pi} = 10km & L_{\pi} = 30m, dx = 310m \\ \mbox{Issubility} & L_{\pi} = 10km & $	Instability	$k=2\pi/\lambda_{\rm x}, m=2\pi/\lambda_{\rm z}$	$N \approx 0.018$		$dz \approx 100m, n_{ray} = 4000$	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	Non-hydrostatic	$z_0=10 km, \Delta_{wp}=10 km$		LES:	LES:	
$ \begin{array}{ c c c c c } \hline & & & & & & & & & & & & & & & & & & $	Wavepacket	$branch = -1, a_0 = 0.9$		$L_z = 30km, L_x = 1km$	nz = 960, nx = 32	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$					$dz \approx 31m, dx \approx 310m$	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	МІ	Cosine shape	non-Boussinesq	WKB Lagrange:	WKB Lagrange:	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Modulational	$\lambda_x = 1km, \lambda_z = 1km$	T = 300K	$L_z = 60 km$	$nz=600, dz_{amooth}\approx 600m$	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Instability	$k=2\pi/\lambda_{\rm x}, m=2\pi/\lambda_{\rm z}$	$N \approx 0.018$		$dz \approx 100m, n_{ray} = 4000$	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		$z_0=10 km, \Delta_{wp}=20 km$		LES:	LES:	
CL Cosine shape non-Bousinesq WKB Lagrange: WKB Lagrange: Critical $\lambda_x = 10km, \lambda_x = 1km$ $T = 300k$ $L_z = 30km$ $nz = 300, dz_{mooth} = 600m$ Layer $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $N = 0.018$ $dz \approx 100, n_{my} = 4000$ $z_0 = 10km, \Delta_{wp} = 10km$ $u_0 = -11m/s$ LES: LES: branch = -1, a_0 = 0.1 $z_a = 25km$ $L_z = 30km, L_x = 10km$ $nz = 960, nx = 32$ $\Delta_x = 10km$ $L_z = 30km, L_x = 10km$ $nz = 960, nx = 32$		$branch = -1, a_0 = 0.1$		$L_{\rm z}=60km, L_{\rm x}=1km$	nz = 1920, nx = 32	
$ \begin{array}{cccc} {\bf CL} & {\bf Cosine shape} & {\bf non-Boussinesq} & {\bf WKB Lagrange:} & {\bf WKB Lagrange:} \\ {\bf Critical} & \lambda_{s} = 10km, \lambda_{c} = 1km & T = 300k & L_{c} = 30km & nc = 300, dz_{mooth} = 600m \\ {\bf Layer} & k = 2\pi/\lambda_{s}, m = 2\pi/\lambda_{c} & N = 0.018 & dz \approx 100, n_{my} = 4000 \\ z_{0} = 10km, \Delta_{wp} = 10km & u_{0} = -11m/s & {\bf IES:} & {\bf IES:} \\ branch = -1, a_{0} = 0.1 & z_{a} = 25km & L_{c} = 30km, L_{s} = 10km & nc = 960, nx = 32 \\ & \Delta_{s} = 10km & dz \approx 31m, dx \approx 310m \\ \end{array} $					$dz \approx 31m, dx \approx 310m$	
$ \begin{array}{c c} \mbox{Critical} & \lambda_{z} = 10km, \lambda_{z} = 1km & T = 300k & L_{z} = 30km & nz = 300, dz_{mooth} = 600m \\ \mbox{Layer} & k = 2\pi/\lambda_{z}, m = 2\pi/\lambda_{z} & N = 0.018 & dz \approx 100, n_{my} = 4000 \\ \mbox{z}_{0} = 10km, \Delta_{wp} = 10km & u_{0} = -11m/s & IES: & IES: \\ \mbox{branch} = -1, a_{0} = 0.1 & z_{a} = 25km & L_{z} = 30km, L_{z} = 10km & nz = 960, nx = 32 \\ & \Delta_{x} = 10km & dz \approx 31m, dx \approx 310m \end{array} $	CL	Cosine shape	non-Boussinesq	WKB Lagrange:	WKB Lagrange:	
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Critical	$\lambda_x = 10 km, \ \lambda_z = 1 km$	T = 300K	$L_z = 30 km$	$nz=300, dz_{smooth}\approx 600m$	
	Layer	$k=2\pi/\lambda_{\rm x}, m=2\pi/\lambda_{\rm z}$	$N \approx 0.018$		$dz\approx 100, n_{my}=4000$	
		$z_0=10 km, \Delta_{wp}=10 km$	$u_0=-11m/s$	LES:	LES:	
$\Delta_{a} = 10 km$ $dz \approx 3 1m, dx \approx 3 10m$		$branch = -1, a_0 = 0.1$	$z_u = 25 km$	$L_z = 30km, L_x = 10km$	nz = 960, nx = 32	
			$\Delta_{\alpha} = 10 km$		$dz \approx 31m, dx \approx 310m$	

TRR181 workshop, Hamburg, 4 May 2017

Interactions of GWs and large-scale flows

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