The interaction between atmospheric gravity waves and large-scale flows:
an efficient description beyond the nonacceleration paradigm

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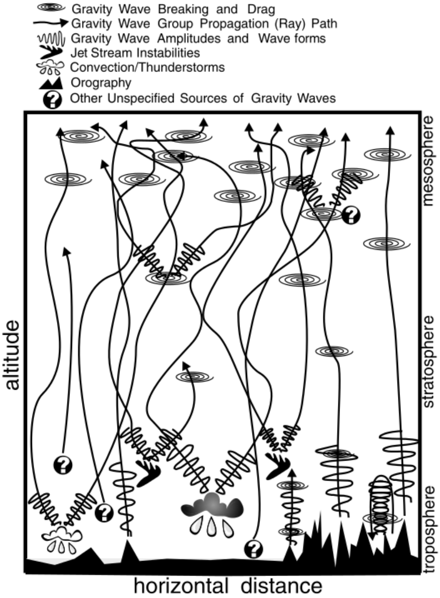
Gravity Waves in the Atmosphere

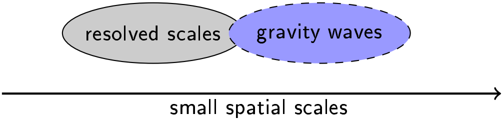


- main sources: orography, convection, jets/fronts
- mainly vertical energy (momentum) transport with $\vec{c}_g \Rightarrow$ interaction with the large scale flow ("drag")
- wave breaking $\Rightarrow$ turbulence, dissipation, energy transfer to large scale flow ("drag")

(*Kim et al.*, 2003)

Motivation

Parametrization of atmospheric GWs



- GWs are not fully resolved by GCMs and NWP models ⇒ parametrization ⇒ (Wentzel–Kramers–Brillouin) WKB theory
- Currently used parametrizations: steady state approximation ⇒ instantenous propagation through constant resolved flow ⇒ instantenous drag via wave breaking only!
- Proposal for improvement: direct weakly-nonlinear coupling between the GW and the resolved flow ⟺ transient propagation ⟺ continuous drag on the resolved flow during propagation + drag through wave breaking

WKB theory

Wave resolving system (2-D Euler equations, no rotation):

$$\begin{aligned}
\frac{Du}{Dt} + c_p\theta\frac{\partial\pi}{\partial x} &= 0\\
\frac{Dw}{Dt} + c_p\theta\frac{\partial\pi}{\partial z} + g &= 0\\
\frac{D\theta}{Dt} &= 0\\
\frac{D\pi}{Dt} + \frac{\kappa}{1-\kappa}\pi\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) &= 0
\end{aligned}$$

with $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + w\frac{\partial}{\partial z}$

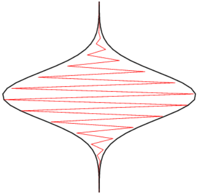
Exner pressure $\pi = (p/p_0)^\kappa$

Pot. temperature $\theta = T(p_0/p)^\kappa = T/\pi$

$\kappa = R/c_p$

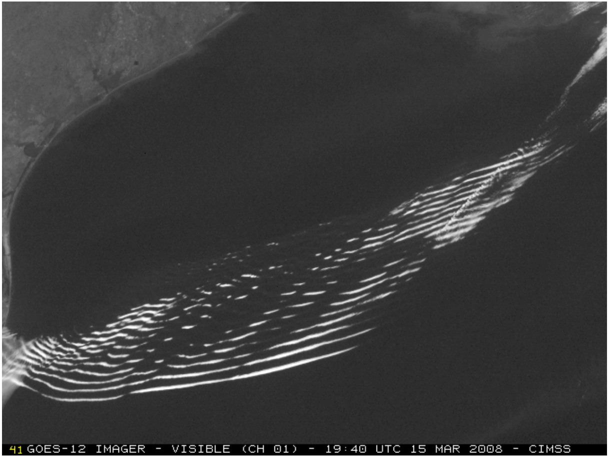
Simplification ingredients:

- Decomposition of the fields: $f = f_b + f_w$
- WKB assumption: $f_w(x,z,t) = \mathrm{Re}F_w(Z,T)e^{i\left[kx + \frac{\phi(Z,T)}{\epsilon}\right]}$ with $Z = \epsilon z, T = \epsilon t$, $m = \partial\phi/\partial Z$ and $\omega = -\partial\phi/\partial T$
- Scaling for the gravity waves: $\epsilon = \mathrm{L}_w/H_\theta << 1$: weak stratification



WKB theory

WKB assumption in nature:



(cimss.ssec.wisc.edu)

WKB theory

- At leading order $\mathcal{O}(\epsilon^2)$: *dispersion-, and polarization relations* $\Rightarrow$ *ray equations*
- At next order $\mathcal{O}(\epsilon^3)$: *wave action conservation* and the *mean-flow equations*
- The coupled system (*Achatz et al.*, 2010):

Wave field

$$\frac{d_g z}{dt} = \mp \frac{Nkm}{(k^2+m^2)^{3/2}} \equiv c_{gz}$$

$$\frac{d_g m}{dt} = \mp \frac{k}{(k^2+m^2)^{1/2}} \frac{dN}{dz} - k \frac{d u_b}{dz} \equiv \dot{m}$$

$$\frac{d_g \mathcal{A}}{dt} = - \mathcal{A} \frac{\partial c_{gz}}{\partial z} \quad \left(\frac{d_g}{dt} = \frac{\partial}{\partial t} + c_{gz} \frac{\partial}{\partial z} \right)$$

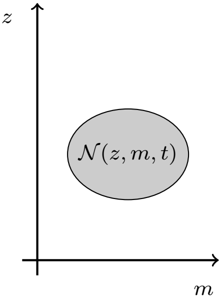
Mean flow

$$\frac{\partial u_b}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \left[\frac{\bar{\rho}}{2} \mathrm{Re}\left(U_w W_w^* \right) \right] = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (k c_{gz} \mathcal{A})$$

$$\bar{\rho} = \rho(z) = \rho_0 e^{-z/H} \quad H = \frac{RT_0}{g}$$

- *Problem*: if rays crossing $\Rightarrow$ *caustics*: several m at same height z $\Rightarrow$ e.g. c_{gz} multivalued BUT! $\mathcal{A} = \mathcal{A}(z, t)$ $\Rightarrow$ wave action conservation ill-defined $\Rightarrow$ numerical problems

WKB theory in phase space



Hertzog et al., 2002,
Muraschko et al., 2015

- Solution: extension of the model to a 2D phase space (z, m)
- "Slicing up" the wave action density to several m intervals $\Rightarrow$ phase-space wave action density:

$$\mathcal{N}(z,m,t) = \int_R \mathcal{A}_\alpha(z,t)\delta[m - m_\alpha(z,t)]d\alpha$$

- Eulerian view

$$\frac{\partial \mathcal{N}}{\partial t} + \frac{\partial (c_{gz}\mathcal{N})}{\partial z} + \frac{\partial (\dot{m}\mathcal{N})}{\partial m} = 0$$

- ... and we have $\frac{\partial c_{gz}}{\partial z} + \frac{\partial \dot{m}}{\partial m} = 0$
- *Lagrangian view*

$$\frac{\partial \mathcal{N}(z,m,t)}{\partial t} + c_{gz}\frac{\partial \mathcal{N}(z,m,t)}{\partial z} + \dot{m}\frac{\partial \mathcal{N}(z,m,t)}{\partial m} = 0$$

WKB theory in phase space

- Coupled wave - meanflow equations in phase space:

<u>Wave field</u>

$$\frac{\mathrm{d}_r z}{\mathrm{d}t} = \mp \frac{Nkm}{(k^2+m^2)^{3/2}} \equiv c_{gz}$$

$$\frac{\mathrm{d}_r m}{\mathrm{d}t} = \mp \frac{k}{(k^2+m^2)^{1/2}} \frac{\mathrm{d}N}{\mathrm{d}z} - k\frac{\mathrm{d}\,u_b}{\mathrm{d}z} \equiv \dot{m}$$

$$\frac{\mathrm{d}_r \mathcal{N}}{\mathrm{d}t} = 0 \quad \left(\frac{\mathrm{d}_r}{\mathrm{d}t} = \frac{\partial}{\partial t} + c_{gz}\frac{\partial}{\partial z} + \dot{m}\frac{\partial}{\partial m}\right)$$

<u>Mean flow</u>

$$\frac{\partial\, u_b}{\partial t} = -\frac{1}{\bar\rho}\frac{\partial}{\partial z}\left[\frac{\bar\rho}{2}\mathrm{Re}\left(U_w W_w^*\right)\right] = -\frac{1}{\bar\rho}\frac{\partial}{\partial z}(kc_{gz}\mathcal{A}) = -\frac{1}{\bar\rho}\frac{\partial}{\partial z}\int_{-\infty}^{\infty} kc_{gz}\,\mathcal{N}(z,m,t)dm$$

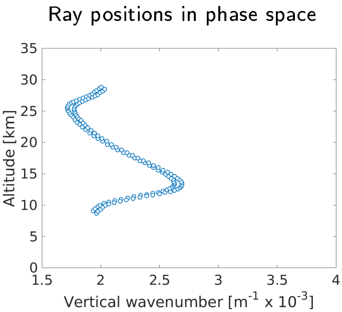
- phase space wave action density $\mathcal{N}$ conserved along ray trajectories

- multiple m values allowed at each location z $\rightarrow$ spectral (non-monochromatic) treatment $\rightarrow$ *no caustics problems*

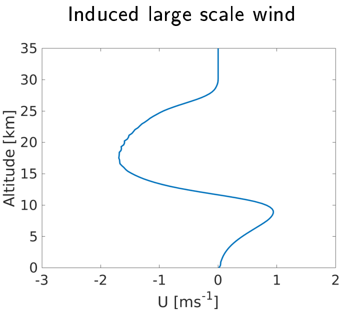
- extension to *Muraschko et al.*, 2015: *isothermal background with an atmosphere-like density profile, representation of turbulent wave breaking*

WKB theory in phase space

Wave field



Mean flow



WKB theory in phase space + wave breaking

- saturation occurs if static instability sets in (*Lindzen 1981*), i.e. $\partial\theta_w/\partial z + d\overline{\theta}/dz < 0$ or, after an additional multiplication by $g/\overline{\theta}$

$$\frac{\partial b_w}{\partial z} + N^2 < 0$$

- with the ansatz $b_w(x,z,t) = \mathrm{Re} B_w(Z,T) e^{i\left[kx + \frac{\phi(Z,T)}{\epsilon}\right]}$ with $m = \partial\phi/\partial Z$ this amounts in

$$|m||B_w| > N^2$$

- monochromatic $\Rightarrow$ spectral

$$\int_{-\infty}^{\infty} m^2 \frac{d|B_w|^2}{dm} dm = \frac{2N^2}{\bar{\rho}} \int_{-\infty}^{\infty} m^2 \hat{\omega} \mathcal{N} dm > \alpha^2 N^4$$

  where α is a parameter accounting for the uncertainty of the criterion

- If saturation $\Rightarrow$ wave action density $\mathcal{N}$ *is reset* to a value that sets back *stability* (height,- and scale-dependent eddy diffusivity coefficient).

Numerical experiments

Methodology

- LES: fully non-linear wave resolving reference (PincFloit, *Rieper et al.*, 2013)
- WKB-eu: Eulerian WKB model
- WKB-la: Lagrangian WKB model
- WKB-st: steady-state WKB model

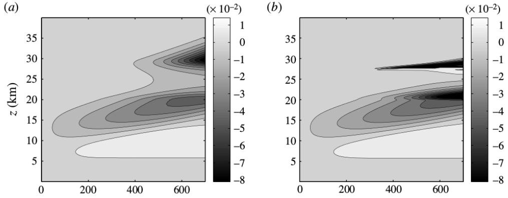
Idealized cases (*Bölöni et al.*, 2016)

- hydorostatic and nonhydrostatic wavepackets: $1 < \frac{\lambda_x}{\lambda_z} < 30$
- static instability: $|m| B_w > N^2$
- modulational instability: $|m| \approx |k| \Rightarrow$ wave packet is shrinking, its amplitude growing
- critical layer: $U_{jet} \approx -c_p \Rightarrow m$ grows to infinity, wavepacket collapses
- refraction by a jet: U_{jet} weak $\Rightarrow m$ only slightly modified
- reflection from a jet: $U_{jet} \geq \frac{N}{k}\left(1 - \frac{k}{\sqrt{k^2+m^2}}\right) \Rightarrow m$ and c_{gz} changes sign

Main question: relative importance of direct wave-meanflow interaction and wave breaking

Numerical experiments

Induced wind, refraction by a jet $(\lambda_x = 10km, \lambda_z = 1km)$

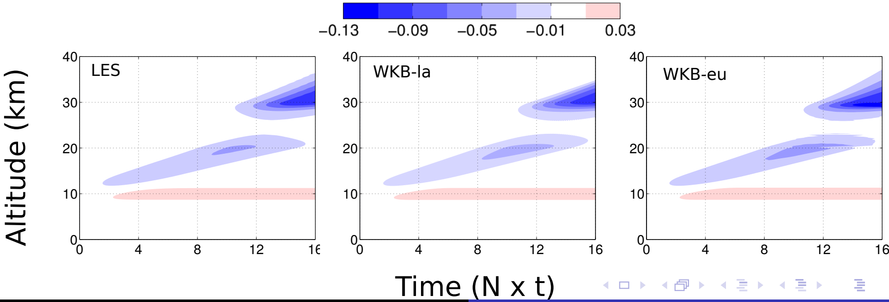


Rieper et al., 2013

a): LES

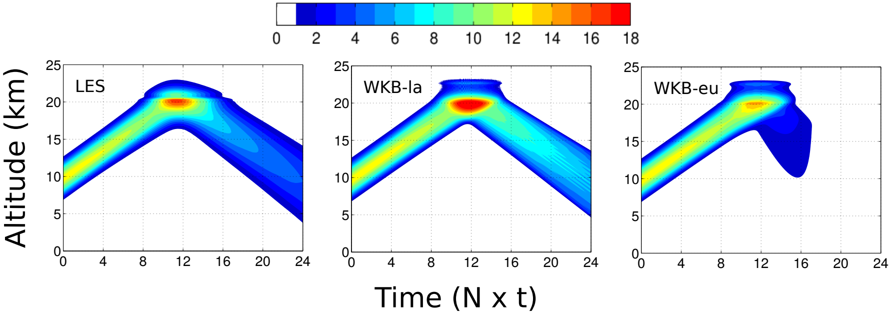
b): conventional WKB raytracer

Bölöni et al., 2016



Numerical experiments

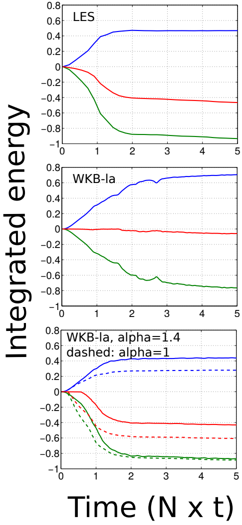
Wave energy, reflection from a jet $(\lambda_x = 10km, \lambda_z = 1km)$ ***Bölöni et al.***, **2016**



Numerical experiments

Static instability ($\lambda_x = \lambda_z = 1km$)

Bölöni *et al.*, 2016



Blue: meanflow kinetic energy $\bar{E}_m$

Green: wave energy $\bar{E}_w$

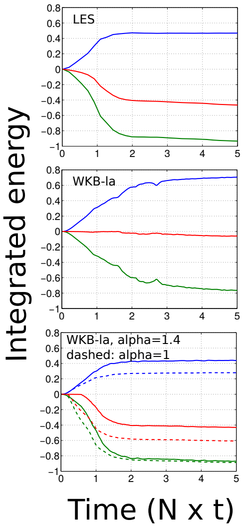
Red: total energy $\bar{E}_{tot}$

with the vertically integrated energy:

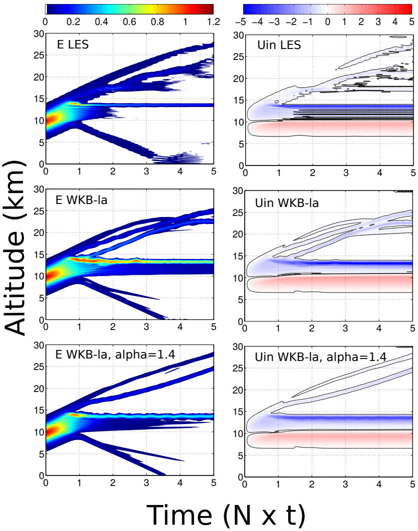
$$
\begin{aligned}
\bar{E}_w &= \int_0^{L_z} dz\, E_w \\
\bar{E}_m &= \int_0^{L_z} dz\, E_m \\
\bar{E}_{tot} &= \bar{E}_w + \bar{E}_m
\end{aligned}
$$

Numerical experiments

Static instability ($\lambda_x = \lambda_z = 1km$)



***Bölöni et al.*, 2016**



WKB vs. WKB-steady-state

Wave field

Mean flow

WKB theory: transient coupled system

$$\frac{d_g z}{dt} = \mp\frac{Nkm}{(k^2+m^2)^{3/2}} \equiv c_{gz}$$

$$\frac{d_g m}{dt} = \mp\frac{k}{(k^2+m^2)^{1/2}}\frac{dN}{dz} - k\frac{d u_b}{dz} \equiv \dot{m}$$

$$\frac{d_g \mathcal{A}}{dt} = -\mathcal{A}\frac{\partial c_{gz}}{\partial z}$$

$$\frac{\partial u_b}{\partial t} = -\frac{1}{\bar\rho}\frac{\partial}{\partial z}(k c_{gz}\mathcal{A})$$

The steady state approximation

$$\frac{d_g z}{dt} = \mp\frac{Nkm}{(k^2+m^2)^{3/2}} \equiv c_{gz}$$

$$\frac{d_g m}{dt} = 0$$

$$\frac{d_g \mathcal{A}}{dt} = 0 \Longleftrightarrow c_{gz}(z)\mathcal{A}(z) = const.$$

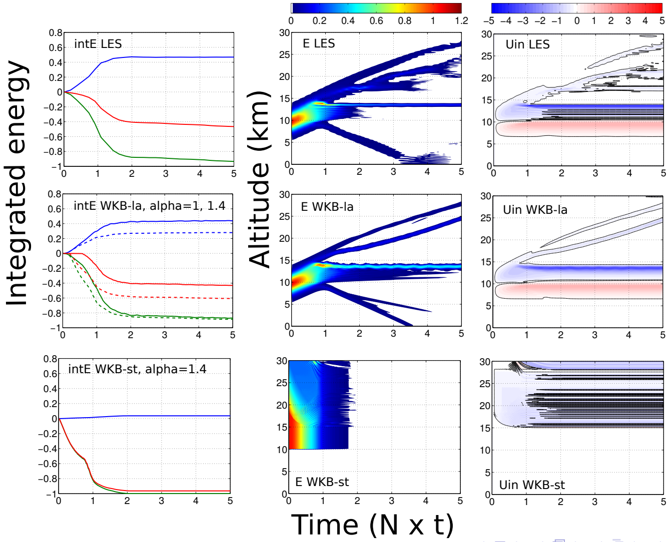
$$\frac{\partial u_b}{\partial t} = -\frac{1}{\bar\rho}\frac{\partial}{\partial z}(k c_{gz}\mathcal{A})$$

$\Rightarrow$ no *wave-mean-flow interaction*! $\Rightarrow$ *wave breaking* (constraining $\mathcal{A}(z)$) is necessary to get an induced wind!

WKB vs. WKB-steady-state

Static instability ($\lambda_x = \lambda_z = 1km$)

Bölöni et al., 2016



Conclusions

Based on the idealized numerical simulations presented here...

- The **direct weakly-nonlinear coupling** between the GW and the meanflow **is an important mechanism** of wave-meanflow interactions.
- **Wave breaking** is also important but **has a second order role** in describing the interaction betwen the GW and the meanflow.
- The weakness of building GW parametrizations only on wave breaking has been demonstrated.
- **The Lagrangian WKB model is very efficient**: factor of 10-100 compared to a corresponding Eulerian model and factor of 1000-10000 compared to LES
- There is a good reason to try out **coupled transient WKB approaches in gravity wave parametrizations**

References

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Experiment	Wavepacket	Background	Domain size	Resolution
REFR Refraction by a jet	Cosine shape $\lambda_x = 10km, \lambda_z = 1km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 10km$ $branch = -1, a_0 = 0.1$	non-Boussinesq $T = 300K$ $N \approx 0.018$ $u_0 = 5m/s$ $z_u = 25km$ $\Delta_u = 10km$	WKB Euler: $L_z = 40km$ $m \in [0.001, 0.008]$ WKB Lagrange: $L_z = 40km$ LES: $L_z = 40km, L_x = 10km$	WKB Euler: $nz = 400, nm = 70$ $dz \approx 100m, dm = 10^{-4}s^{-1}$ WKB Lagrange: $nz = 400, dz_{smooth} \approx 600m$ $dz \approx 100m, n_{ray} = 4000$ LES: $nz = 1280, nx = 32$ $dz \approx 31m, dx = 310m$
REFL Reflection from a jet	Cosine shape $\lambda_x = 10km, \lambda_z = 1km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 10km$ $branch = -1, a_0 = 0.1$	non-Boussinesq $T = 300K$ $N \approx 0.018$ $u_0 = 40m/s$ $z_u = 25km$ $\Delta_u = 10km$	WKB Euler: $L_z = 40km$ $m \in [-0.01, 0.008]$ WKB Lagrange: $L_z = 40km$ LES: $L_z = 40km, L_x = 10km$	WKB Euler: $nz = 400, nm = 180$ $dz \approx 100m, dm = 10^{-4}s^{-1}$ WKB Lagrange: $nz = 400, dz_{smooth} \approx 600m$ $dz \approx 100m, n_{ray} = 4000$ LES: $nz = 2500, nx = 64$ $dz \approx 16m, dx = 156m$
PREFL Partial Reflection from a jet	Cosine shape $\lambda_x = 6km, \lambda_z = 3km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 10km$ $branch = -1, a_0 = 0.1$	non-Boussinesq $T = 300K$ $N \approx 0.018$ $u_0 = 9,75m/s$ $z_u = 25km$ $\Delta_u = 10km$	WKB Lagrange: $L_z = 50km$ LES: $L_z = 50km, L_x = 6km$	WKB Lagrange: $nz = 166, dz_{smooth} \approx 1800m$ $dz \approx 300m, n_{ray} = 4320$ LES: $nz = 538, nx = 32$ $dz \approx 93m, dx = 187m$

STIH Static Instability Hydrostatic Wavepacket	Gaussian shape $\lambda_x = 30km,\ \lambda_z = 3km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 25km$ $branch = -1, a_0 = 0.5$	non-Boussinesq $T = 300K$ $N \approx 0.018$	WKB Lagrange: $L_z = 80km$ LES: $L_z = 80km, L_x = 30km$	WKB Lagrange: $nz = 266, dz_{smooth} \approx 1800m$ $dz \approx 300m, n_{ray} = 4320$ LES: $nz = 854, nx = 32$ $dz \approx 94m, dx \approx 940m$
STINH Static Instability Non-hydrostatic Wavepacket	Gaussian shape $\lambda_x = 1km,\ \lambda_z = 1km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 10km$ $branch = -1, a_0 = 0.9$	non-Boussinesq $T = 300K$ $N \approx 0.018$	WKB Lagrange: $L_z = 30km$ LES: $L_z = 30km,\ L_x = 1km$	WKB Lagrange: $nz = 300, dz_{smooth} \approx 600m$ $dz \approx 100m, n_{ray} = 4000$ LES: $nz = 960, nx = 32$ $dz \approx 31m, dx \approx 310m$
MI Modulational Instability	Cosine shape $\lambda_x = 1km,\ \lambda_z = 1km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 20km$ $branch = -1, a_0 = 0.1$	non-Boussinesq $T = 300K$ $N \approx 0.018$	WKB Lagrange: $L_z = 60km$ LES: $L_z = 60km,\ L_x = 1km$	WKB Lagrange: $nz = 600, dz_{smooth} \approx 600m$ $dz \approx 100m, n_{ray} = 4000$ LES: $nz = 1920, nx = 32$ $dz \approx 31m, dx \approx 310m$
CL Critical Layer	Cosine shape $\lambda_x = 10km,\ \lambda_z = 1km$ $k = 2\pi/\lambda_x, m = 2\pi/\lambda_z$ $z_0 = 10km, \Delta_{wp} = 10km$ $branch = -1, a_0 = 0.1$	non-Boussinesq $T = 300K$ $N \approx 0.018$ $u_0 = -11m/s$ $z_u = 25km$ $\Delta_u = 10km$	WKB Lagrange: $L_z = 30km$ LES: $L_z = 30km,\ L_x = 10km$	WKB Lagrange: $nz = 300, dz_{smooth} \approx 600m$ $dz \approx 100, n_{ray} = 4000$ LES: $nz = 960, nx = 32$ $dz \approx 31m, dx \approx 310m$