

Spontaneous gravity wave emission in the differentially heated rotating annulus experiment

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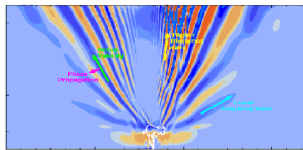
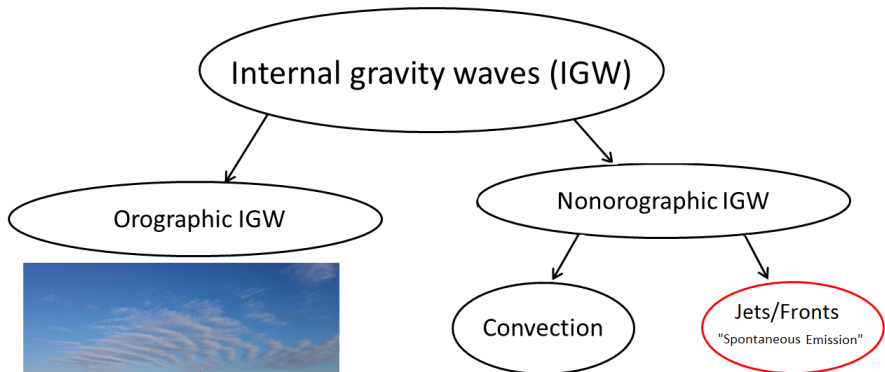
Lena Schoon & Christoph Zülicke

Leibniz-Institut für Atmosphärenphysik, University of Rostock

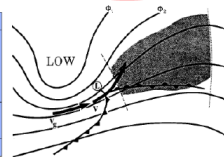
TRR 181 Workshop "Energy transfers in Atmosphere and Ocean",
Hamburg 2017



IGW in the atmosphere



M. Joen Alexander - CEDAR Tutorial Lectures 09/5/192
(See Norton and Alexander (1999) for model description.)



Koch and OHandley (1997)

IGW emission from jets and fronts

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(Plougonven and Zhang, 2014)

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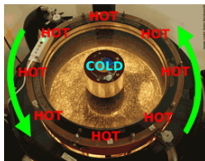
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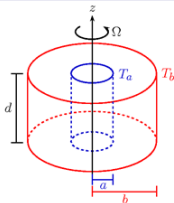
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- Possibility of comparison with corresponding laboratory studies

Differentially heated rotating annulus experiment

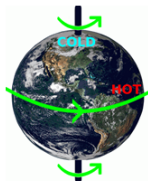
Simple laboratory experiment to reproduce dynamics of mid-latitudes



Laboratory experiment



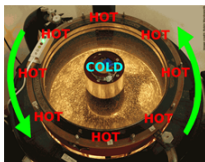
schematic view



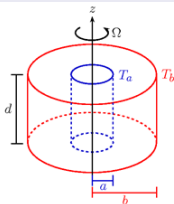
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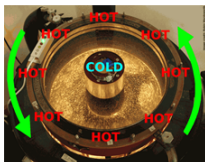
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Parameter setting

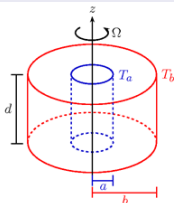
- $a = 20$ cm, $b = 70$ cm, $d = 4$ cm

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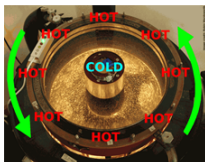
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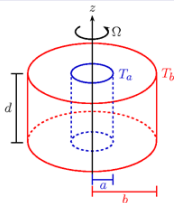
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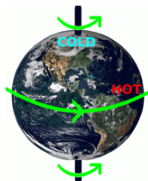
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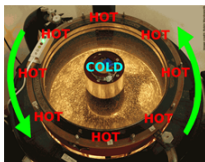
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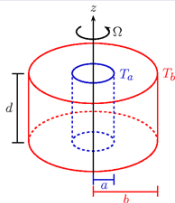
- $a = 20$ cm, $b = 70$ cm, $d = 4$ cm
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- $\Omega = 0.08$ rad/s

Differentially heated rotating annulus experiment

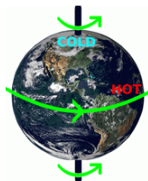
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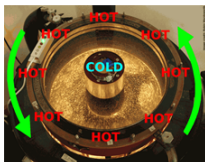
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- $\Omega = 0.08$ rad/s
- \Rightarrow Atmosphere-like conditions

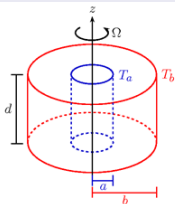
$$\frac{\langle N \rangle}{f} > 1$$

Differentially heated rotating annulus experiment

Simple laboratory experiment to reproduce dynamics of mid-latitudes



Laboratory experiment



schematic view



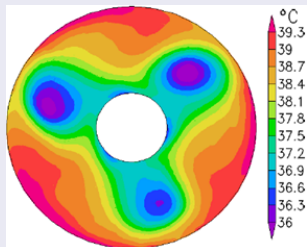
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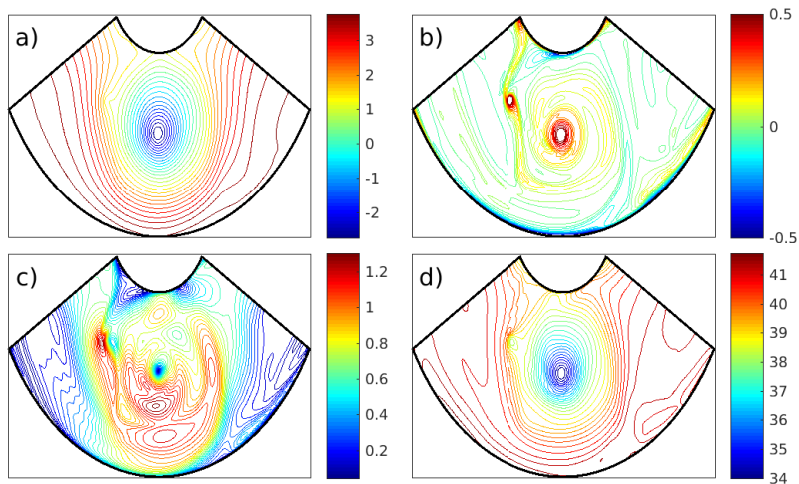
$$\frac{\langle N \rangle}{f} > 1$$

Baroclinic waves (Borchert et al., 2015)



Direct numerical simulations: large-scale flow

- Clear baroclinic wave structure including a jet-front system



a) pressure, b) vertical vorticity, c) $||\mathbf{u}||$ and d) temperature

Direct numerical simulations: IGW activity

- Reduced horizontal velocity divergence δ used as IGW indicator

$$\delta = \underbrace{\delta_{total}}_{\nabla_{\mathbf{h}} \cdot \mathbf{u}_{\mathbf{h}}} - \delta_{\text{bal}},$$

where

$$\delta_{\text{bal}} = -\frac{\partial w_{\text{bal}}}{\partial z}.$$

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where

$$\delta_{bal} = -\frac{\partial w_{bal}}{\partial z}.$$

- w_{bal} is diagnosed from the quasi-geostrophic omega equation (Hoskins et al. (1978), Danioux et al. (2012))

$$\nabla_{qg}^2 w_{bal} = -\frac{2}{N^2} \nabla_h \cdot \mathbf{Q},$$

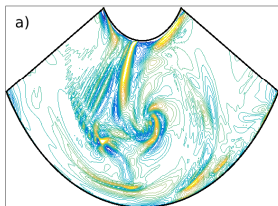
where

$$\mathbf{Q} = \nabla_h \mathbf{u}_g \cdot \nabla_h B_g.$$

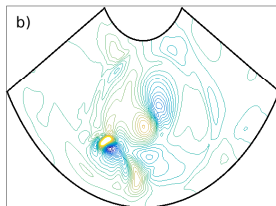
only depends on quasi-geostrophic fields.

Balanced and unbalanced flow and their interaction

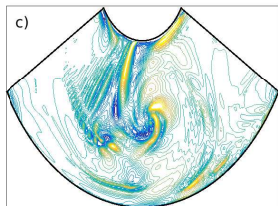
Diagnosing balanced part in horizontal divergence: omega equation



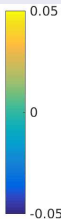
δ_{total}



δ_{bal}

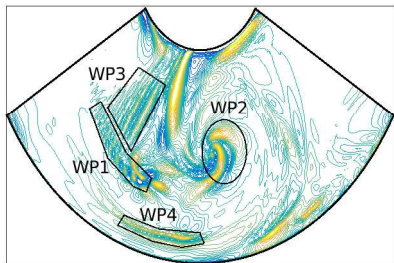


δ_{unbal}

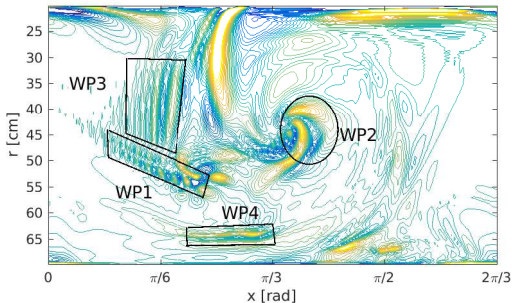


Direct numerical simulations: IGW activity

- Four distinct wave packets (**WP1 – WP4**) can be identified
(Hien et al., submitted)



Annulus geometry



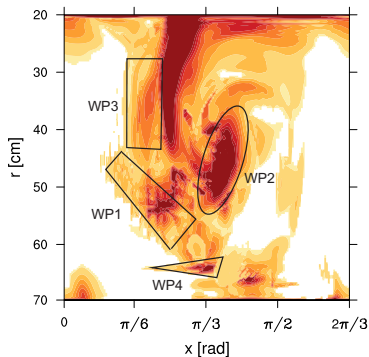
Cartesian projection

Direct numerical simulations: IGW activity

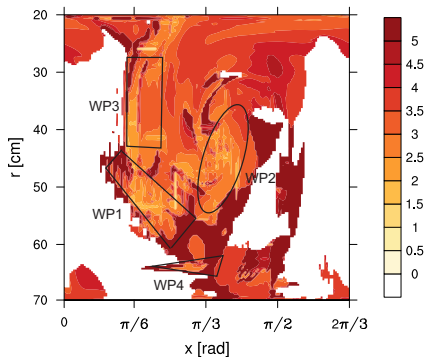
- Wave packet analysis (\mathbf{k} , \mathbf{A} ,...) using **UWaDi (Unified Wave Diagnostics)** based on Hilbert-transform algorithm
- Reference for related laboratory studies

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Amplitudes [1/s]



wave numbers [1/cm]

Balanced and unbalanced flow and their interaction

Decomposition of flow into **geostrophic** and **ageostrophic** part

$$\mathbf{v} = \mathbf{u}_g + \mathbf{v}_a$$

$$B = B_g + B_a$$

$$\rho = \rho_g + \rho_a$$

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$$\begin{aligned} \mathbf{v} &= \mathbf{u}_g + \mathbf{v}_a \\ B &= B_g + B_a \\ \rho &= \rho_g + \rho_a \end{aligned}$$

$$f \mathbf{e}_z \times \mathbf{u}_g + \nabla_h p_g = 0$$

$$B_g - \frac{\partial p_g}{\partial z} = 0$$

$$\Pi_g = \zeta + \frac{f}{N^2} \frac{\partial B}{\partial z} = \frac{1}{f} \left(\nabla_h^2 + \frac{f^2}{N^2} \frac{\partial^2}{\partial z^2} \right) p_g$$

$$\zeta_a + \frac{f}{N^2} \frac{\partial B_a}{\partial z} = 0$$

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$$\begin{aligned} f \mathbf{e}_z \times \mathbf{u}_g + \nabla_h p_g &= 0 \\ B_g - \frac{\partial p_g}{\partial z} &= 0 \\ \mathbf{v} &= \mathbf{u}_g + \mathbf{v}_a \\ B &= B_g + B_a \\ p &= p_g + p_a \\ \Pi_g &= \zeta + \frac{f}{N^2} \frac{\partial B}{\partial z} = \frac{1}{f} \left(\nabla_h^2 + \frac{f^2}{N^2} \frac{\partial^2}{\partial z^2} \right) p_g \\ \zeta_a + \frac{f}{N^2} \frac{\partial B_a}{\partial z} &= 0 \end{aligned}$$

... \Rightarrow Geostrophic forcing of ageostrophic flow

$$\frac{D\delta_a}{Dt} = -\frac{\partial B_a}{\partial z} + \frac{\partial^2 p_{aa}}{\partial z^2} + \frac{\partial \mathbf{v}}{\partial z} \cdot \nabla w_a - \boxed{\frac{\partial^2}{\partial z^2} \nabla^{-2} (\nabla \mathbf{u}_g \cdot \nabla \mathbf{u}_g)}$$

Tangent linear model for the ageostrophic flow: Principle

- Decomposition into balanced (large) and unbalanced (small) part

$$\mathbf{x} = \tilde{\mathbf{x}} + \mathbf{x}' \text{ with } |\mathbf{x}'| \ll |\tilde{\mathbf{x}}| \text{ (Unbalanced part } \approx \text{ gravity waves)}$$

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- Inserting this into the full equation system gives

$$\frac{\partial \mathbf{x}}{\partial t} = \frac{\partial \tilde{\mathbf{x}}}{\partial t} + \frac{\partial \mathbf{x}'}{\partial t} = \mathbf{N}(\mathbf{x}),$$

where $\mathbf{N}(\mathbf{x})$ is the nonlinear tendency of the full flow.

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- Taylor approximation leads to

$$\mathbf{N}(\tilde{\mathbf{x}} + \mathbf{x}') = \mathbf{N}(\tilde{\mathbf{x}}) + \mathbf{L}(\tilde{\mathbf{x}})\mathbf{x}' + \mathcal{O}(|\mathbf{x}'|^2)$$

with the linear, partial-differential operator $\mathbf{L}(\tilde{\mathbf{x}})\mathbf{x}'$

Tangent linear model for the ageostrophic flow: Principle

- Hence, the tangent linear evolution of \mathbf{x}' is given by

$$\frac{\partial \mathbf{x}'}{\partial t} = L(\tilde{\mathbf{x}})\mathbf{x}' + \underbrace{\mathbf{N}(\tilde{\mathbf{x}}) - \frac{\partial \tilde{\mathbf{x}}}{\partial t}}_{F(\tilde{\mathbf{x}})}$$

with a balanced forcing term $F(\tilde{\mathbf{x}})$.

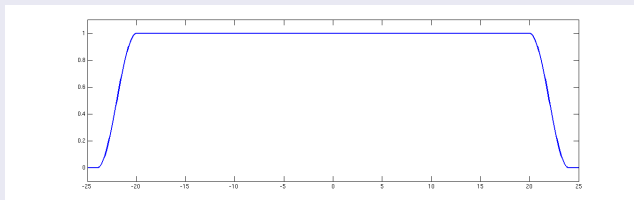
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- Implementation of window function to suppress instabilities and IGW generation at side walls



Tangent linear annulus equation

$$\frac{D\mathbf{u}_a}{Dt} = -f\mathbf{e}_z \times \mathbf{u}_a - \nabla_h \tilde{p}_a - \left(\frac{D\mathbf{u}_g}{Dt} \right)_a - \left\{ \left(\frac{D\mathbf{u}_g}{Dt} \right)_g \right\}$$

$$\frac{DB_a}{Dt} = -N^2 w_a - \left(\frac{DB_g}{Dt} \right)_a - \left\{ \left(\frac{DB_g}{Dt} \right)_g \right\}$$

$$\frac{Dw_a}{Dt} = B_a - \frac{\partial \tilde{p}_a}{\partial z}$$

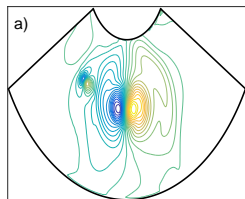
Source mechanism of IGW: Tangent linear analysis

Tangent linear annulus equation

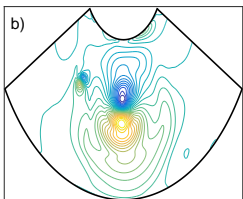
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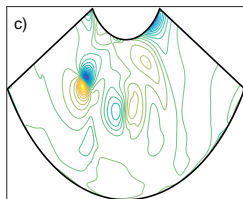
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forcing of u_a



forcing of v_a



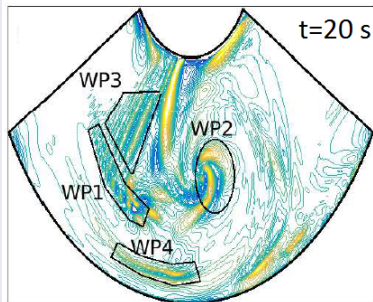
forcing of B_a

Sources of IGW in the rotating annulus experiment

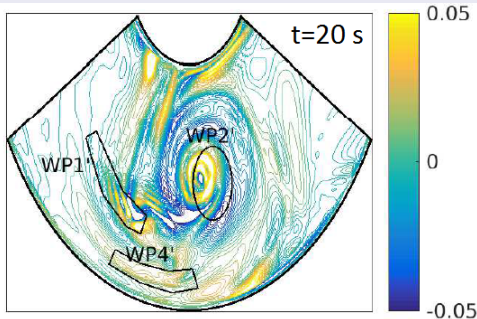
Initialisation of linear model with zero ageostrophic part at $t = 0$ s

Sources of IGW in the rotating annulus experiment

Initialisation of linear model with zero ageostrophic part at $t = 0$ s



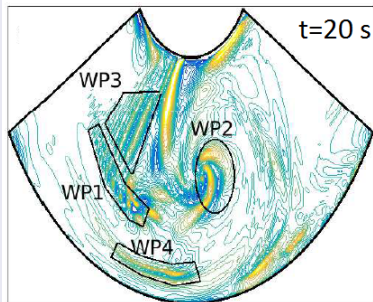
nonlinear model



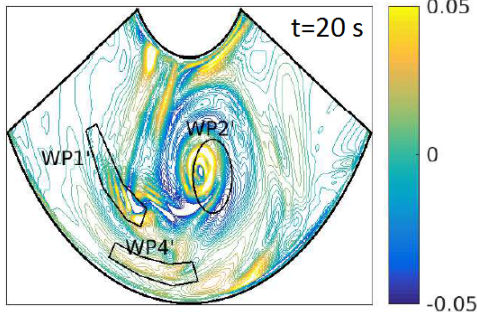
linear model

Sources of IGW in the rotating annulus experiment

Initialisation of linear model with zero ageostrophic part at $t = 0$ s



nonlinear model

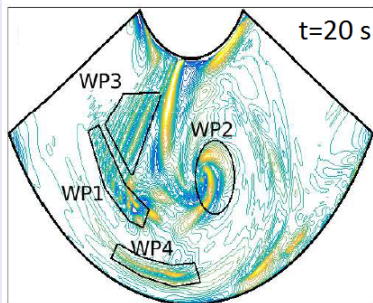


linear model

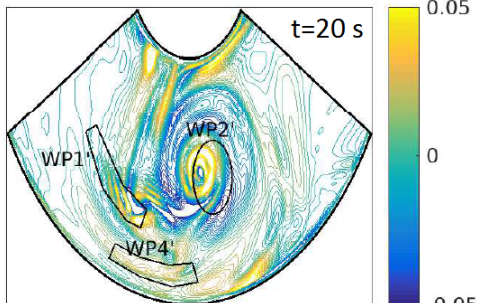
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Sources of IGW in the rotating annulus experiment

Initialisation of linear model with zero ageostrophic part at $t = 0$ s



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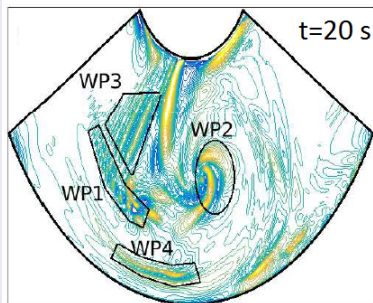


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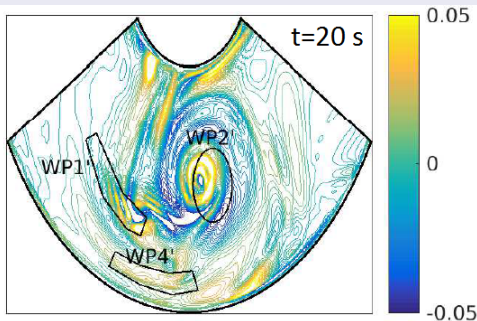
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nonlinear model

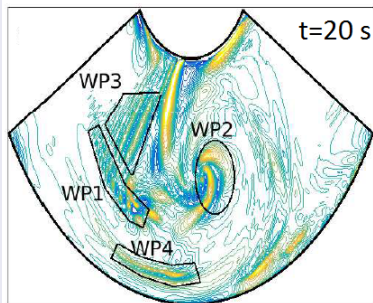


linear model

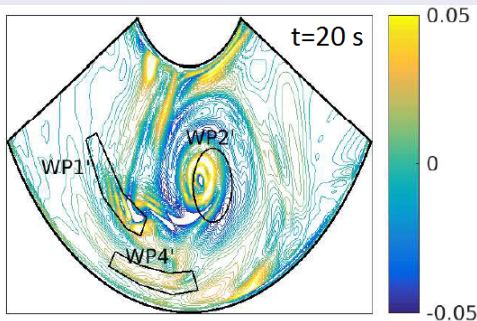
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Sources of IGW in the rotating annulus experiment

Initialisation of linear model with zero ageostrophic part at $t = 0$ s



nonlinear model



linear model

- WP1, WP2 and WP4 reproduced by linear model, WP3 is not present
- At $t=0$ s, (only) balanced forcing induces ageostrophic flow (and IGWs)
⇒ WP1, WP2 and WP4 emitted by balanced part of the flow
- WP3 is probably generated at inner side wall

(for details see Jacoby et al. (2011); Randriamampianina and Crespo del Arco (2015))

Initialisation of linear model with non-zero ageostrophic part

- Three model configurations: Fully nonlinear, forced linear and unforced linear ($F = 0$)

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- Initialisation with same (non-zero) ageostrophic field at $t \equiv 0$ s

Initialisation of linear model with non-zero ageostrophic part

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$$\mathbf{x}' = \mathbf{x} - \tilde{\mathbf{x}}$$

Initialisation of linear model with non-zero ageostrophic part

- Three model configurations: Fully nonlinear, forced linear and unforced linear ($F = 0$)
- Initialisation with same (non-zero) ageostrophic field at $t \equiv 0$ s

$$\mathbf{x}' = \mathbf{x} - \tilde{\mathbf{x}}$$

- Wave packets already present at initial time $t=0$ s

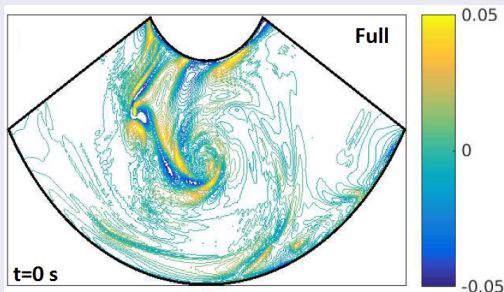
Sources of IGW and role of balanced forcing

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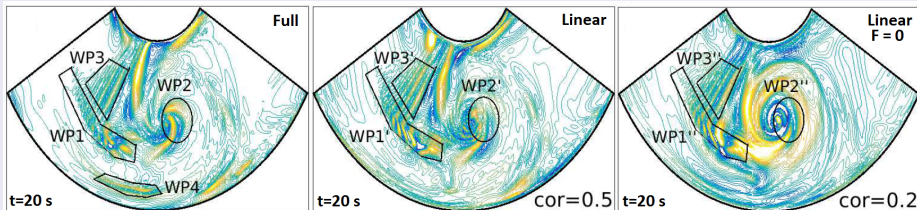
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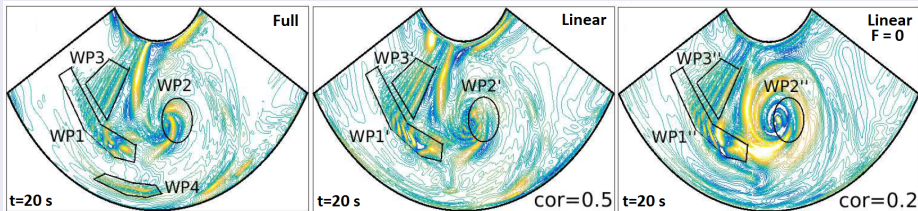
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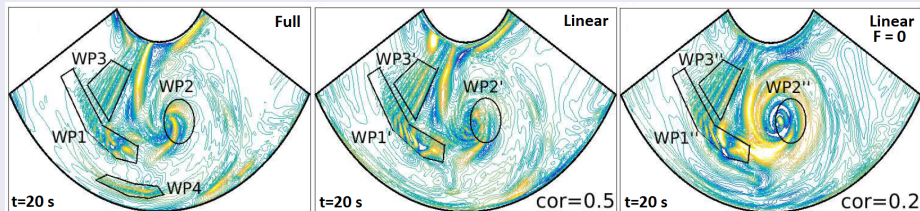
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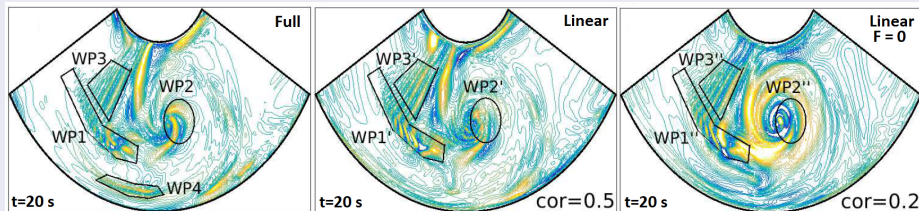
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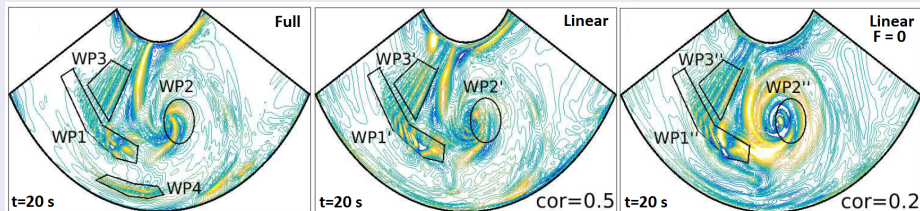
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- Propagation of WP1 and WP3 captured by linear models \Rightarrow Forcing controls over generation but only minor on propagation
- WP2 differs significantly in unforced linear model \Rightarrow WP2 is continuously affected by internal forcing
- WP4 only hardly identifiable \Rightarrow effect of window function?

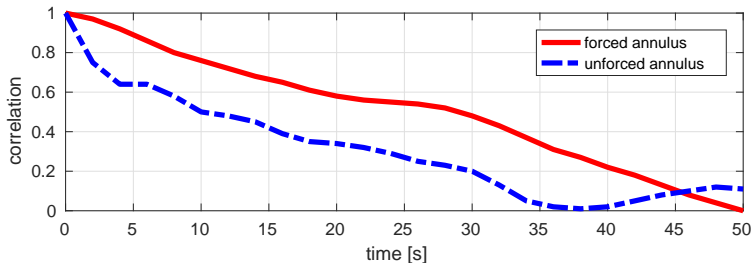
Time evolution of 3D correlation coefficient between (un)forced linear and nonlinear model

- Only grid points not affected by window function are taken into account

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Outlook

- Investigation of IGW radiation by jets and fronts in ever more realistic flow configurations
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- Physically based source parameterization of IGWs in large-scale background flow

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