Multi-scale interaction equations of the inverse energy transfer in atmospheric flows

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1. Introduction

Atmospheric motions in mid-high latitudes has been widely recognized to show variability on multiple time scales such as the low-frequency variability (7-30 days) of mid-latitude westerly jet and blocking or North Atlantic Oscillation (NAO) and the synoptic-scale (2-7days) variability of baroclinic eddies (Benedict et al. 2004; Franzke et al. 2004). Essentially speaking, the blocking and NAO are eddy-stirred turbulent flows in the atmosphere. The blocking and NAO events have been known to arise from the inverse energy transfer from small to large scales (Berggern et al.1949; Shutts 1983). At present, the multi-scale interaction of blocking and NAO events has been an important research topic.

In past years, the role of synoptic-scale eddies in the blocking and NAO flows were often considered as a time-mean effect of synoptic-scale eddies on the blocking (Hoskins et al. 1983; Shutts 1983) and NAO (Hurrell 1995; Vallis et al. 2004; Jin et al. 2006) using the multi-time frequency decomposition method. However, such a method ignored the instantaneous modulation of synoptic-scale eddies by the varying blocking and NAO flows, and thus cannot reflect the multi-scale interaction of the mean flow, NAO or blocking and synoptic-scale eddies. Luo and his collaborators (2000, 2005; 2007, 2014, 2015) proposed and developed the multi-space scale interaction model to describe how the multi-scale interaction leads to blocking and NAO flows. In this poster, we mainly describe the basic idea of the multi-space scale interaction and present a crude comparison with the multi-time scale interaction equations.

Observational results

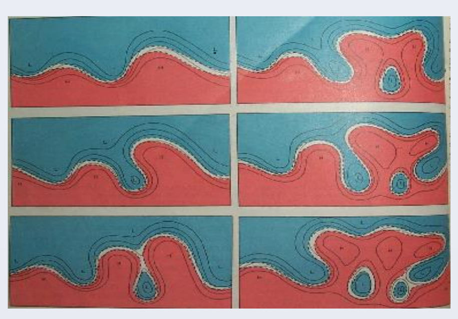
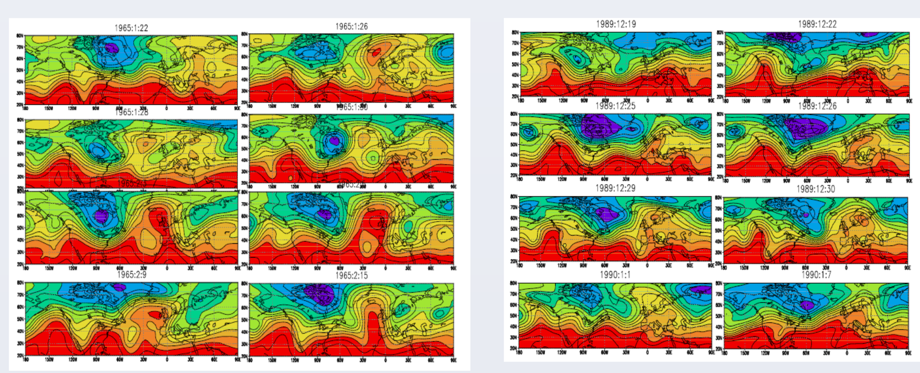


Figure 1. An observed blocking flow as a typical example of the inverse energy transfer from a small to a large scale (Berggren et al. 1949).



(a) (b)

Figure 2. Observed unfiltered daily fields of NAO events for negative (a) and (b) positive phases.

2. Multi-time and -space scale interaction models under a quasi-geostrophic barotropic approximation

For an atmospheric flow, the non-dimensional quasi-geostrophic barotropic potential vorticity (PV) equation can be written in the form of

$$\frac{\partial q_T}{\partial t} + J(\psi_T, q_T) + \beta \frac{\partial \psi_T}{\partial x} = F_f(\psi_T) + D(\psi_T) \ , \qquad (1)$$

where $q_T = \nabla^2 \psi_T - F\psi_T$ is the relative vorticity of the streamfunction ψ_T , $F = (L/R_d)^2$, L is the characteristic horizontal length, R_d is the radius of Rossby deformation, β is the meridional gradient of the Coriolis parameter, F_f is the forcing term that may depend on the streamfunction field and $D(\psi_T)$ is the dissipation term.

a). Multi-time scale interaction equations and their self-contradiction

To describe the contribution of high-frequency components to the low-frequency component, the often-used method is to consider the mean flow and low-frequency component as a time-mean flow with a period of $t > \tau$ (Shutts 1983; Hoskins et al. 1983). Correspondingly, the high-frequent components are assumed to have a period of $t \le \tau$ as defined by a derivation from the time-mean flow.

If one divides the atmospheric flow into a time-mean flow ($t > \tau$) and high-frequency components ($t \le \tau$) in the form of $\psi_T = \bar{\psi}(x,y,t)(t>\tau) + \psi'(x,y,t)(t \le \tau)$, then substituting $\psi_T = \bar{\psi} + \psi'$ into Eq. (4) and making a time averaging yield

$$\frac{\partial \bar{q}}{\partial t} + J(\bar{\psi}, \bar{q}) + \beta \frac{\partial \bar{\psi}}{\partial x} = -\nabla \cdot \overline{(\mathbf{v}' q')} + F_f(\bar{\psi}) + D(\bar{\psi}) \ , \qquad (2a)$$

$$\frac{\partial q'}{\partial t} + J(\psi', q') + \beta \frac{\partial \psi'}{\partial x} = -J(\bar{\psi}, q') - J(\psi', \bar{q}) + \nabla \cdot \overline{(\mathbf{v}' q')} + F_f(\psi') + D(\psi') \ , \qquad (2b)$$

Where $\bar{q} = \nabla^2 \bar{\psi} - F\bar{\psi}$, $q' = \nabla^2 \psi' - F\psi'$, and $\mathbf{v}' = (-\partial\psi'/\partial y, \partial\psi'/\partial x)$ is the horizontal wind vector. Note that $F_f(\bar{\psi}+\psi') = F_f(\bar{\psi}) + F_f(\psi')$ and $D(\bar{\psi}+\psi') = D(\bar{\psi}) + D(\psi')$ have been used in the derivation of Eqs. (5a-b). The overbar denotes a time average during the range of a period from 0 to τ as well.

It is obvious that ψ' has a low-frequency timescale $t > \tau$ because its equation (2b) contains the variable $\bar{\psi}(x,y,t)(t>\tau)$. This contradicts the time frequency decomposition method in the form of $\psi_T = \bar{\psi}(x,y,t)(t>\tau) + \psi'(x,y,t)(t \le \tau)$.

b). Multi-space scale interaction equations and the same low-frequency variability

Split the streamfunction of atmospheric motions ψ_T in Eq. (1) into three parts: mean flow $\psi_o = -u_0 y$, large-scale ψ (low zonal wavenumber) and small-scale ψ' (high zonal wavenumber) components, then we can obtain the following three equations from Eq. (1):

$$\left(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}\right)(\nabla^2 \psi - F\psi) + J(\psi, \nabla^2 \psi) + (\beta + Fu_0)\frac{\partial \psi}{\partial x} = -\nabla \cdot (\mathbf{v}' q')_P + F_f(\psi) + D(\psi) \ , \qquad (3a)$$

$$\left(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}\right)(\nabla^2 \psi' - F\psi') + (\beta + Fu_0)\frac{\partial \psi'}{\partial x} = -J(\psi, \nabla^2 \psi') - J(\psi', \nabla^2 \psi) + F_f(\psi') + D(\psi') \ , \qquad (3b)$$

where $J(\psi', \nabla^2 \psi')_P = -\nabla \cdot (\mathbf{v}' q')_P$ has been used.

Eqs. (3a-b) are first derived by Luo (2000, 2005) and Luo and Li (2000) for quasi-geostrophic barotropic and two-layer fluid motions when the dissipation is neglected.

It is seen that the large- and small-scale components tend to possess the same low-frequency variability during their interaction because $\nabla \cdot (\mathbf{v}' q')_P$ is a low-frequency timescale in Eq. (3a). This avoids the time-averaging assumption used in the derivation of Eq. (2a).

c). An analytical solution of multi-space scale interactions

Under the conditions $F_f(\psi) = D(\psi) = 0$ and $D(\psi') = 0$, we can solve Eq. (3). Its analytical solution can be derived by using a multi-scale expansion method similar to those in Luo (2000), Luo and Li (2000) when one assumes synoptic-scale eddies being of the form of $\psi' = \psi'_1 + \psi'_2$ where ψ'_1 is the preexisting synoptic-scale eddies prior to the block or NAO onset and ψ'_2 is the deformed eddies that result from the feedback of intensified large-scale flow on preexisting eddies.
Similar to Luo (2000, 2005), Luo and Li (2000) and Luo et al. (2007, 2015), the analytical solution to Eq. (3) can be obtained as

$$\psi_T = -u_0 y + \psi + \psi' = \psi_P + \psi' \ , \qquad (4a)$$

$$\psi_P = -u_0 y + \psi \approx -u_0 y + \psi_{NAO} + \psi_m \ , \qquad (4b)$$

$$\psi_{NAO} = B\sqrt{\frac{2}{L_y}} \exp[i(kx - \omega t)]\sin(my) + cc, \qquad (4c)$$

$$\psi_m = -|B|^2 \sum_{n=1}^{\infty} q_n g_n \cos(n + 1/2)my, \qquad (4d)$$

$$\psi' \approx \varepsilon^{3/2}(\tilde{\psi}'_0 + \varepsilon \tilde{\psi}'_1) = \psi'_1 + \psi'_2 \ , \qquad (4e)$$

$$\psi'_1 = \varepsilon^{3/2}\tilde{\psi}'_0 = f_0(x)\{\exp[i(\tilde{k}_1 x - \tilde{\omega}_1 t)] + \alpha \exp[i(\tilde{k}_2 x - \tilde{\omega}_2 t)]\}\sin(\frac{m}{2}y) + cc \ , \qquad (4f)$$

$$\psi'_2 = -\frac{m}{4}\sqrt{\frac{2}{L_y}} B f_0 \sum_{j=1}^{2} Q_j \alpha_j \exp\{i(\tilde{k}_j + k)x - (\tilde{\omega}_j + \omega)t]\}[p_j \sin(\frac{3m}{2}y) + r_j \sin(\frac{m}{2}y)]$$
$$+\frac{m}{4}\sqrt{\frac{2}{L_y}} B^* f_0 \sum_{j=1}^{2} Q_j \alpha_j \exp\{i(\tilde{k}_j - k)x - (\tilde{\omega}_j - \omega)t]\}[s_j \sin(\frac{3m}{2}y) + h_j \sin(\frac{m}{2}y)] + cc \ , \qquad (4g)$$

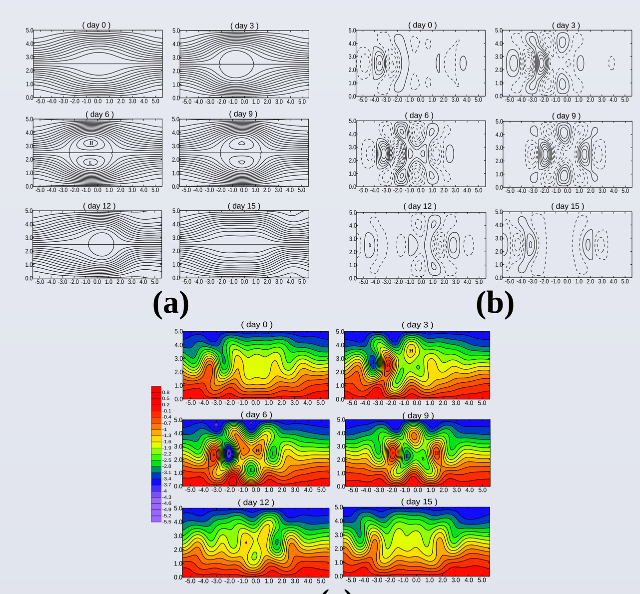
$$i\left(\frac{\partial B}{\partial t} + C_g \frac{\partial B}{\partial x}\right) + \lambda \frac{\partial^2 B}{\partial x^2} + \delta |B|^2 B + G f_0^{\,2} \exp[-i(\Delta k x + \Delta \omega t)] = 0 \ . \qquad (4h)$$

Where $\Delta\omega = \tilde{\omega}_2 - \tilde{\omega}_2 - \omega$, $\tilde{\omega}_j = u_0 \tilde{k}_j - \frac{(\beta + Fu_0)\tilde{k}_j}{\tilde{k}_j^{\,2} + m^2/4 + F}$, $\omega = u_0 k - \frac{(\beta + Fu_0)k}{k^2 + m^2 + F}$ (j=1,2) , $\Delta k = k - (\tilde{k}_2 - \tilde{k}_1)$, $|B|^2 = BB^*$, B^* is the complex conjugate of B , $k = 2k_0$, $k_0 = \frac{1}{6.371\cos\phi_0}$, $m = \pm 2\pi/L_y$, ϕ_0 is the reference latitude, L_y is the width of the non-dimensional beta plane channel, $\alpha_1 = 1$ and $\alpha_2 = \alpha = \pm 1$, $f_0(x) = a_0 \exp[-\mu\varepsilon^2(x + x_0)^2]$ the spatial distribution of the eddy amplitude for $0 < \varepsilon \ll 1.0$ and $\mu \ge 0$, where a_0 is the maximum amplitude at $x = -x_0$ and other coefficients and notation can be found in Luo et al. (2007a-b; 2105).

In Eq. (4), ψ_{NAO} denotes the NAO anomaly and ψ_m represents the streamfunction of the mean flow change due to the presence of the NAO, whereas ψ' is the synoptic-scale eddy streamfunction being of the form of $\psi' = \psi'_1 + \psi'_2$, where ψ'_1 is the preexisting synoptic-scale eddies prior to the block or NAO onset and ψ'_2 is the deformed eddies that result from the feedback of intensified large-scale flow on preexisting eddies. When $\alpha = -1$ and $m = -2\pi/L_y$, it represents the solution of the blocking flow or NAO- event (Luo 2000, 2005; Luo et al. 2014). However, when $\alpha = 1$ and $m = 2\pi/L_y$, it represents the solution of a NAO+ event (Luo et al. 2007a-b; Luo et al. 2015). Thus, the model solution (4) may be referred to as a solution of the unified nonlinear multi-scale interaction (UNMI) model because it can not only describe the life cycle of blocking or NAO- events, but also the life cycle of NAO+ or intensified zonal jet events.

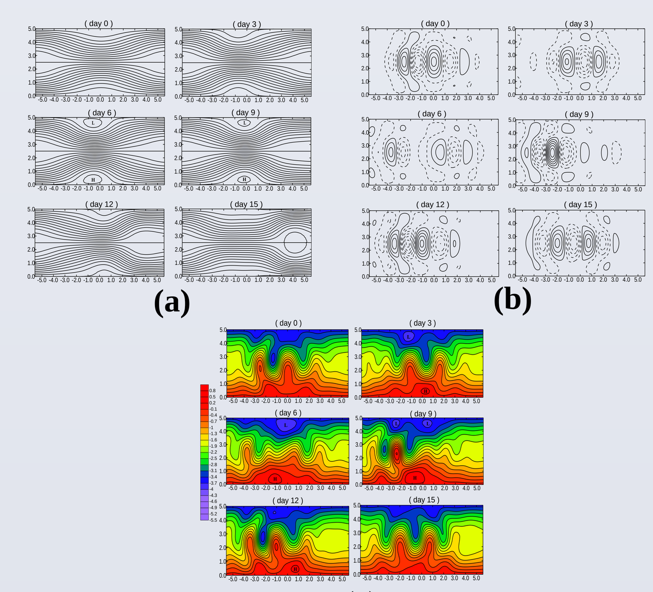
We can obtain $\frac{\partial q}{\partial t} \approx -\nabla \cdot (\mathbf{v}'_1 q'_1)_P$ from Eq. (3a) in the absence of dissipation and external forcing because the observations show that $J(\psi - u_0 y, \nabla^2 \psi + \beta y) \approx 0$ is approximately held during the NAO life cycle (Luo et al. 2007b). Prior to the NAO- onset, we have $\frac{\partial q}{\partial t} \approx -\nabla \cdot (\mathbf{v}'_1 q'_1)_P$ ($\mathbf{v}'_1 = (-\partial\psi'_1/\partial y, \partial\psi'_1/\partial x)$, $q'_1 = \nabla^2 \psi'_1$) because the NAO- is so weak that there is $\psi' = \psi'_1 + \psi'_2 \approx \psi'_1$ because of $\psi'_2 \approx 0$. This means that the NAO- is driven by preexisting synoptic-scale eddies ψ'_1 rather than by deformed eddies ψ'_2 . A similar mechanism works for the NAO+. Of course, the evolution of the NAO is also affected by deformed eddies ψ'_2 because of $\frac{\partial q}{\partial t} \approx -\nabla \cdot (\mathbf{v}'_1 q'_1)_P - \nabla \cdot (\mathbf{v}'_2 q'_2)_P$ ($\mathbf{v}'_2 = (-\partial\psi'_2/\partial y, \partial\psi'_2/\partial x)$, $q'_2 = \nabla^2 \psi'_2$), once it is stronger. The role of deformed eddies in the NAO evolution has been examined in Luo et al. (2015). Thus, here we do not focus on further examining this problem. Instead, we will use the model solution to confirm that the mean flow, NAO anomaly and synoptic-scale eddies will exhibit the same low-frequency variation once their interaction takes place.

Theoretical results



(a) (b) (c)

Figure 3. Instantaneous (a) non-dimensional planetary-scale (CI=0.15), (b) synoptic-scale (CI=0.3) and (c) total (CI=0.3) fields of a NAO$^-$ event obtained from the UNMI model for $B(x,0) = 0.35$, $\mu = 1.2$ and $a_0 = 0.17$.



(a) (b) (c)

Figure 4. Same as Fig. 3 but for a NAO$^+$ event.

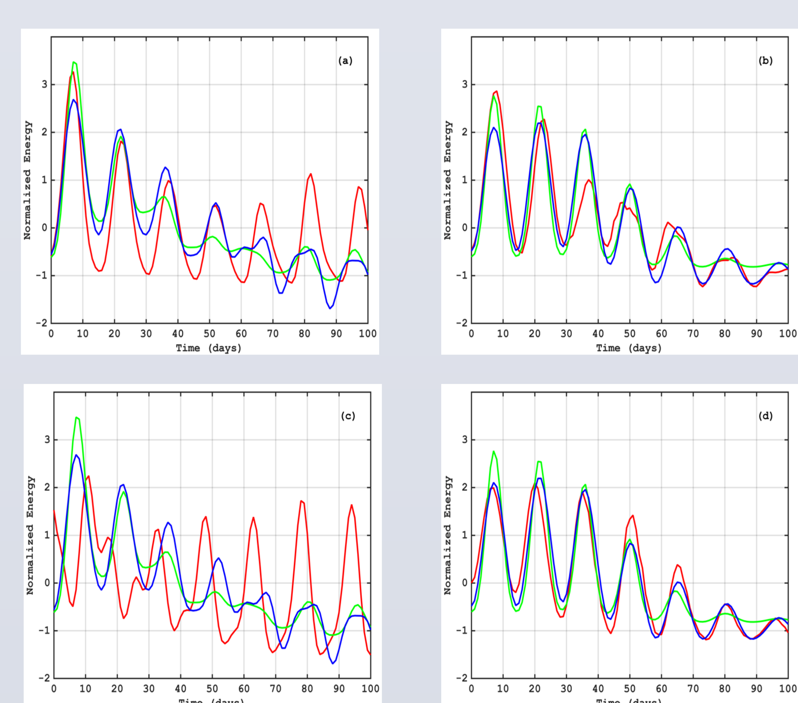


Figure 5. Time series of normalized non-dimensional kinetic energies of mean flow ψ_m(red), NAO anomaly ψ_{NAO} (blue) and synoptic-scale eddies ψ' (green) averaged in the NAO region $-2.5 \le x \le 2.5$ for (a, b) NAO- and (c, d) NAO+ events with (a, c) $\mu = 1.2$ and (b, d) $\mu = 0$. The other parameters are chosen to be the same as in Figs. 1-2.

3. Main conclusions

(1) The multi-time scale interaction equations cannot describe the multi-scale interaction of blocking and NAO flows because there is a self-contradiction between the time-frequency decomposition and their interaction equations.

(2) The multi-space scale interaction equations can represent the multi-scale interaction of blocking and NAO flows. The solution of the multi-space scale interaction equations can account for why the mean flow, large- and small-scale components possess the same low-frequency timescale variability when the large-scale components are driven by small-scale components.

4. Main References

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