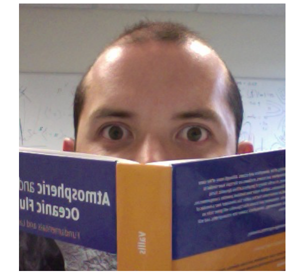
Geometry and Energetics of Ocean Mesoscale Eddies and Their Representation in Climate models (GEOMETRIC)



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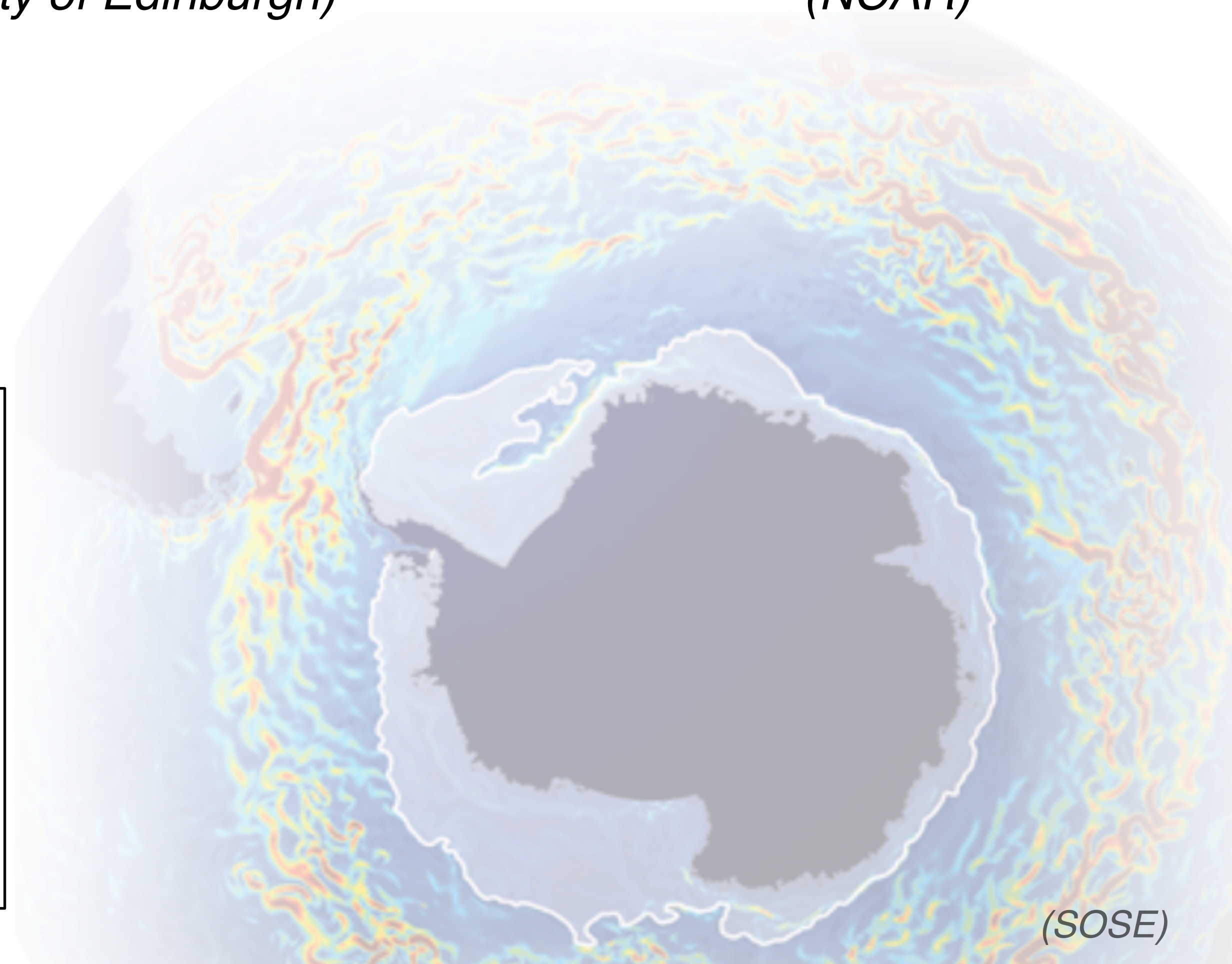
Structure:

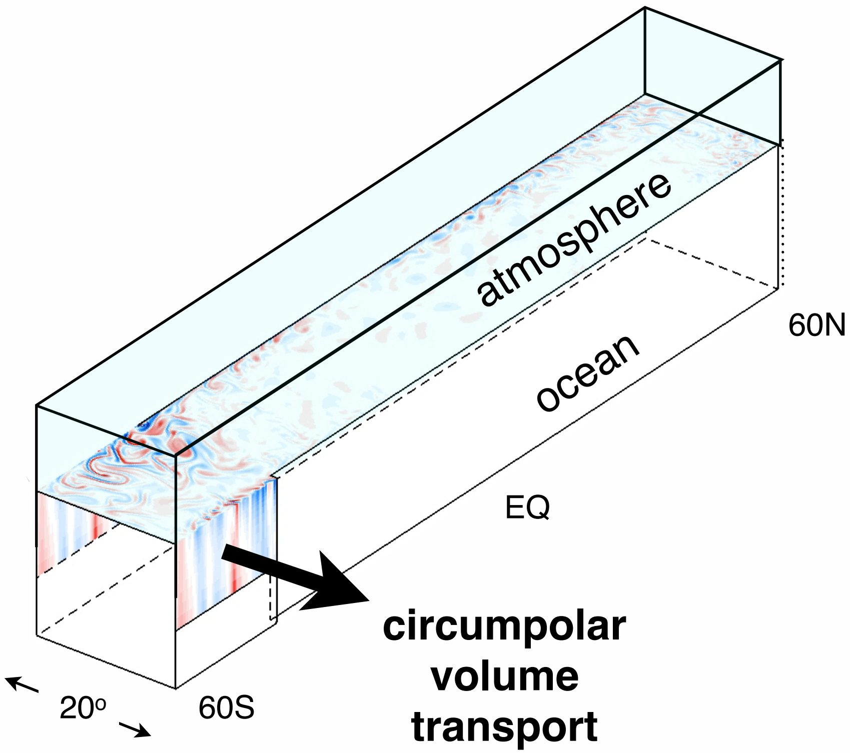
- Motivation and background
- GEOMETRIC
- Proofs of concept
- Recipe for implementation

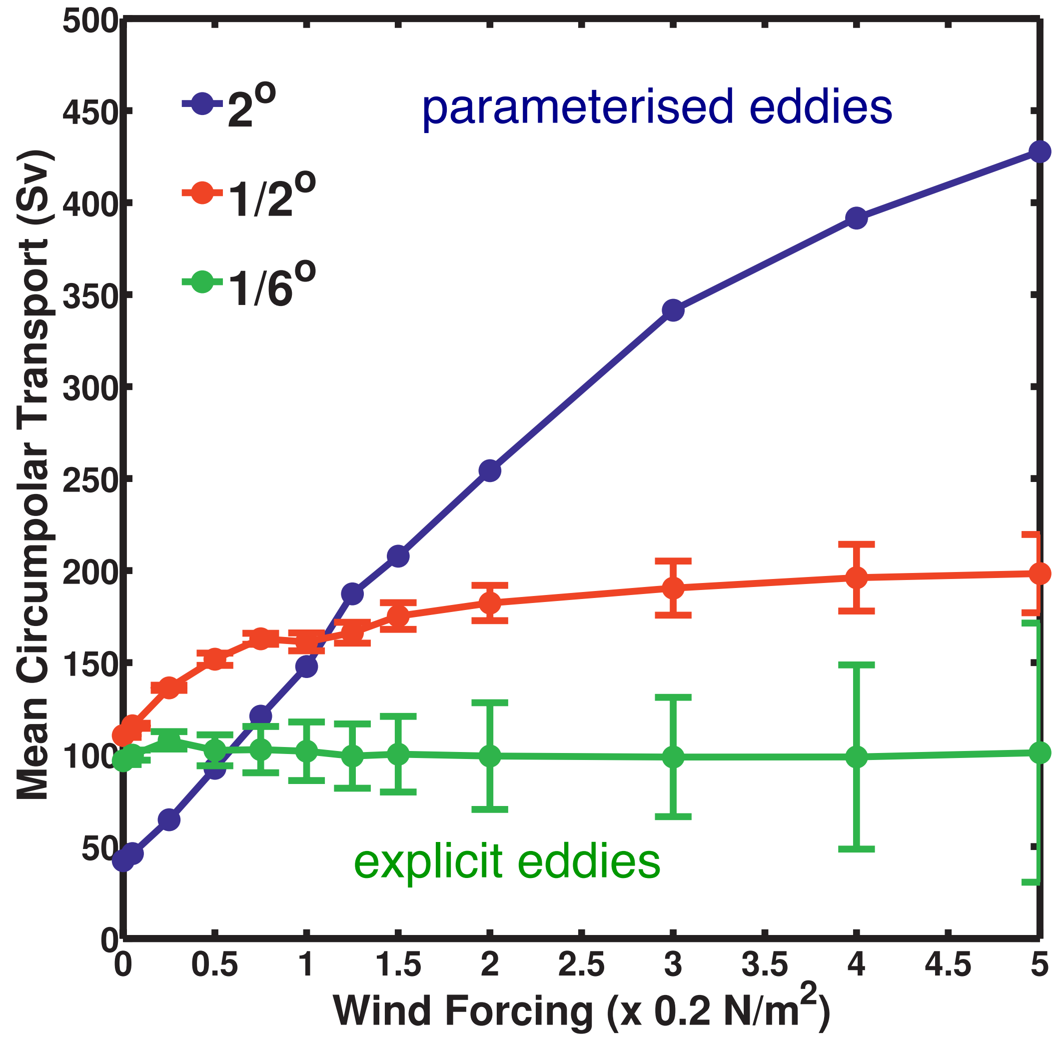


(SOSE)

Motivation and background

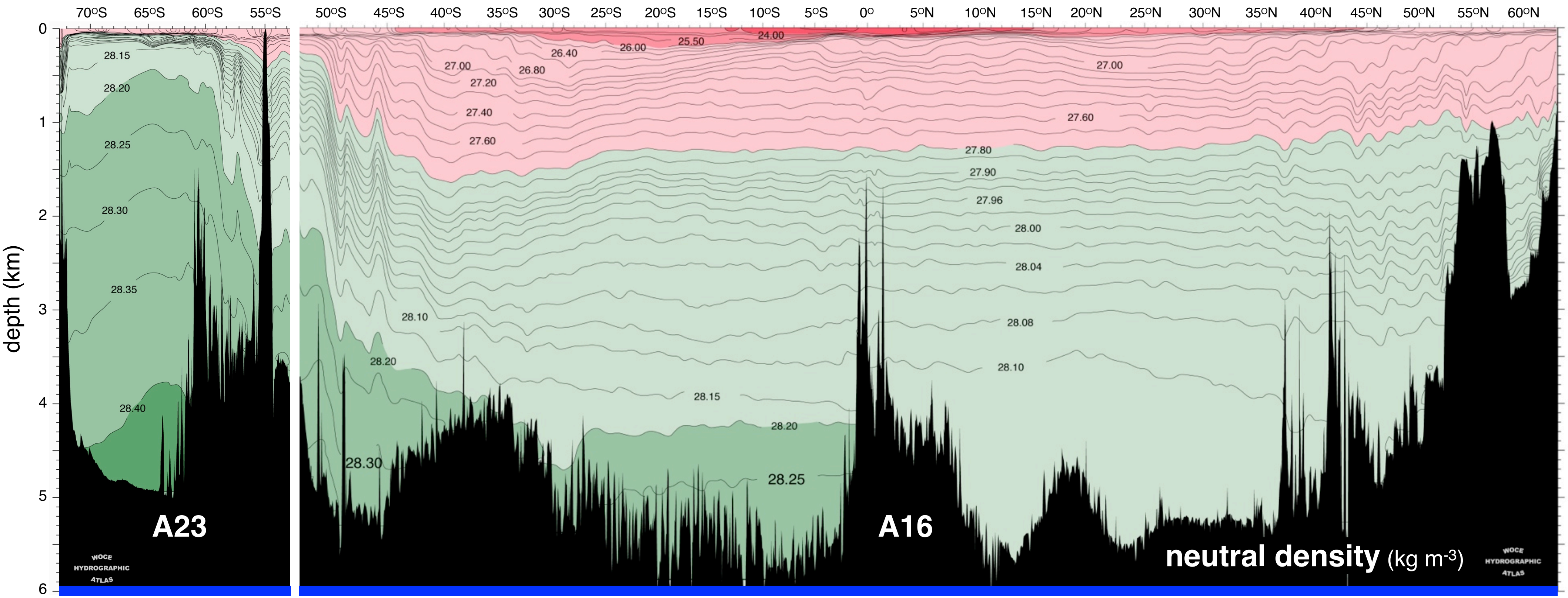
Circumpolar volume transport - “eddy saturation” *(Munday et al., 2013)*





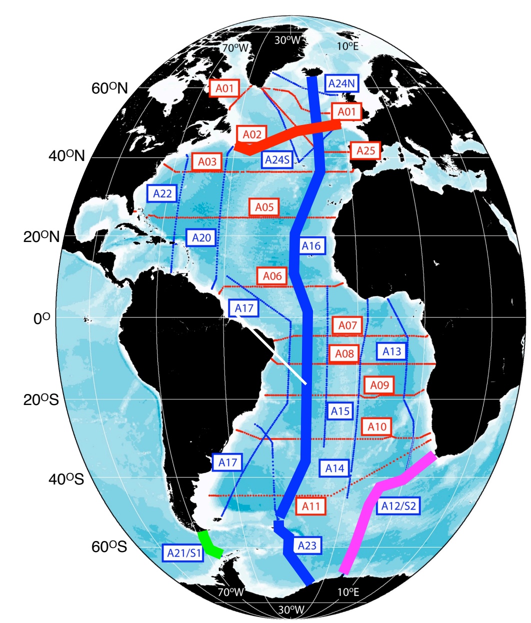
cf. Straub (1993), Hallberg and Gnanadesikan (2001), Tansley and Marshall (2001), Hallberg and Gnanadesikan (2006), Meredith and Hogg (2006), Hogg and Blundell (2010), Farneti et al. (2010), Farneti and Delworth (2010), Hogg and Munday (2014), ... , ...

meridional section of neutral density: *(Koltermann et al., 2011)*



circumpolar volume transport ⟷ global stratification

⇒ Southern Ocean eddies important for setting:
- global stratification;
- ocean heat content;
- equilibrium atmospheric CO_2;
- ocean heat and carbon uptake;
- global sea level change;
- ocean adjustment time scale.

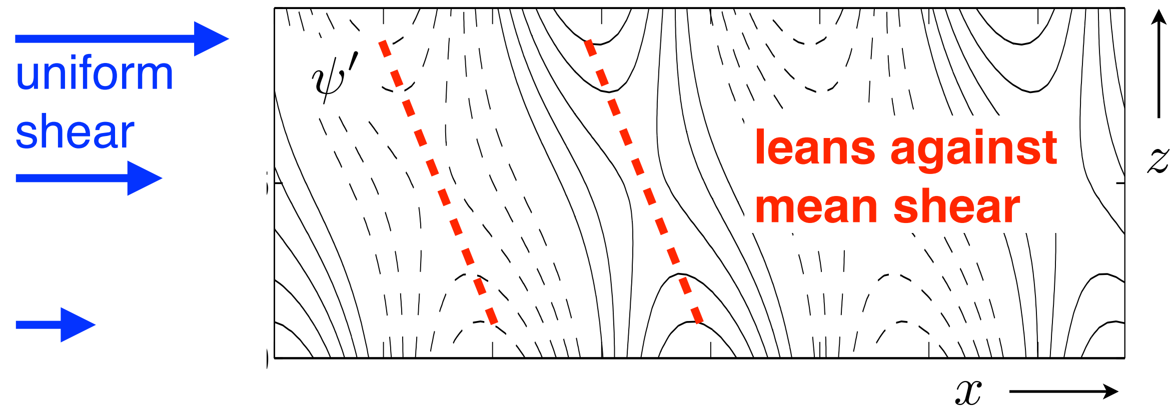


Classical paradigm for location/structure of ocean eddies:

Eady (1949) model of baroclinic instability

- uniform rotation
- uniform stratification
- uniform shear

most **unstable** mode:



(figure: adapted from Vallis 2006)

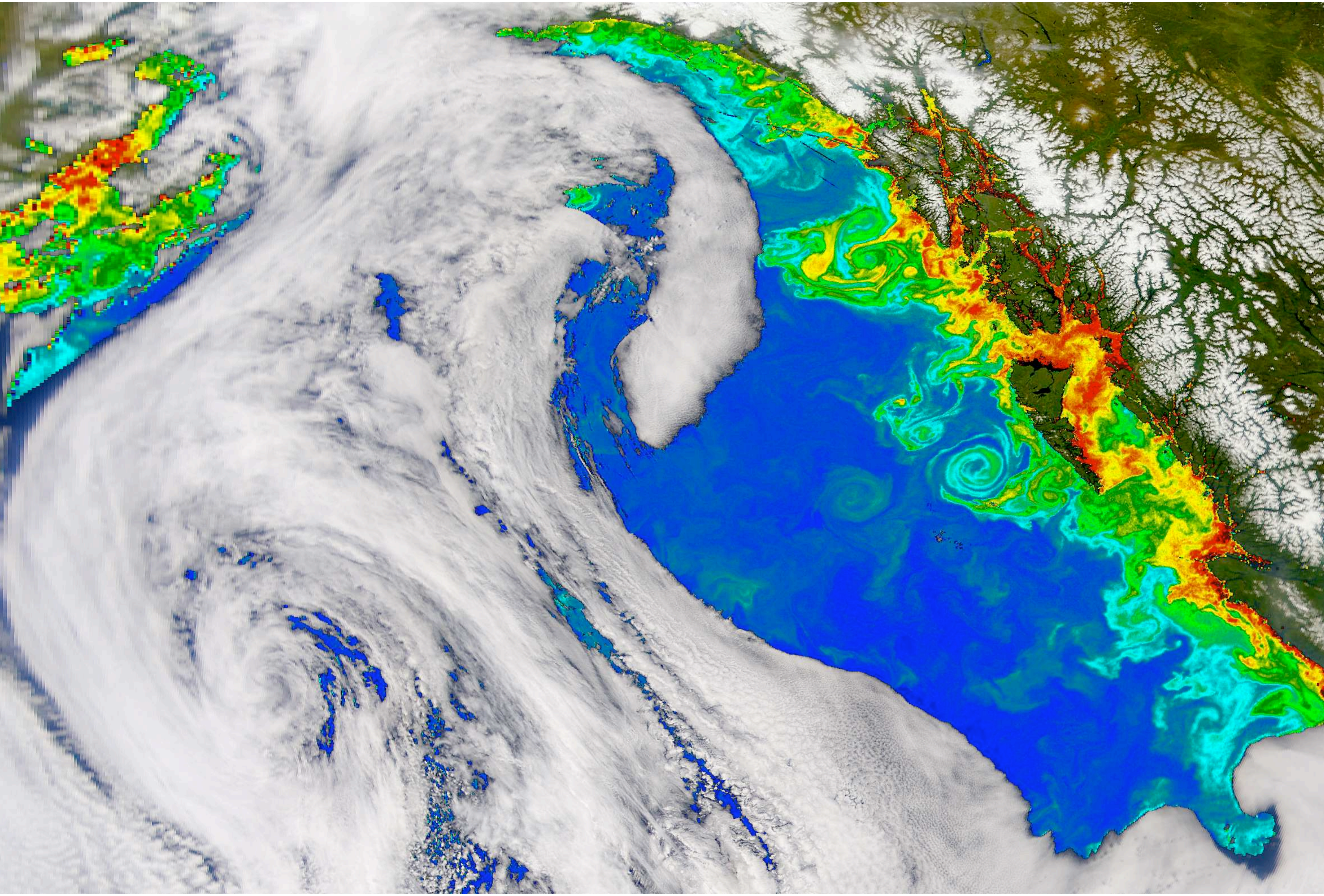
energy growth rate for most unstable mode:

$$0.61 \frac{f}{N} \frac{\partial \overline{u}}{\partial z} \sim \begin{array}{ll} 0.3 \text{ day}^{-1} & \text{- atmosphere} \\ 0.03 \text{ day}^{-1} & \text{- ocean} \end{array}$$

length scale of instability characterised by
Rossby deformation radius:

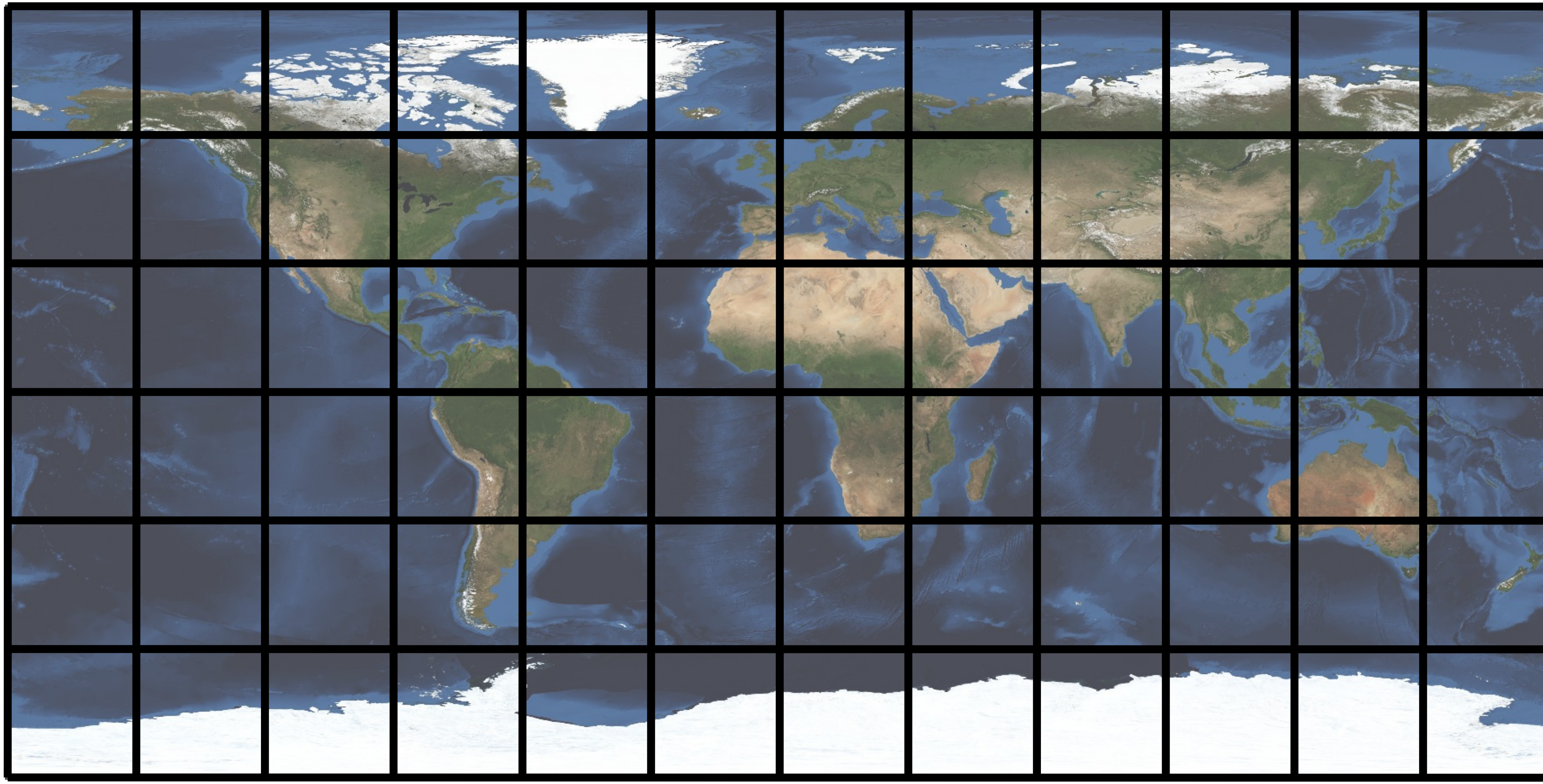
$$L_d = \frac{NH}{f} \sim \begin{array}{ll} 1000 \text{ km} & \text{- atmosphere} \\ 50 \text{ km} & \text{- ocean} \end{array}$$

Composite satellite image showing cloud cover and ocean (pseudo) colour:



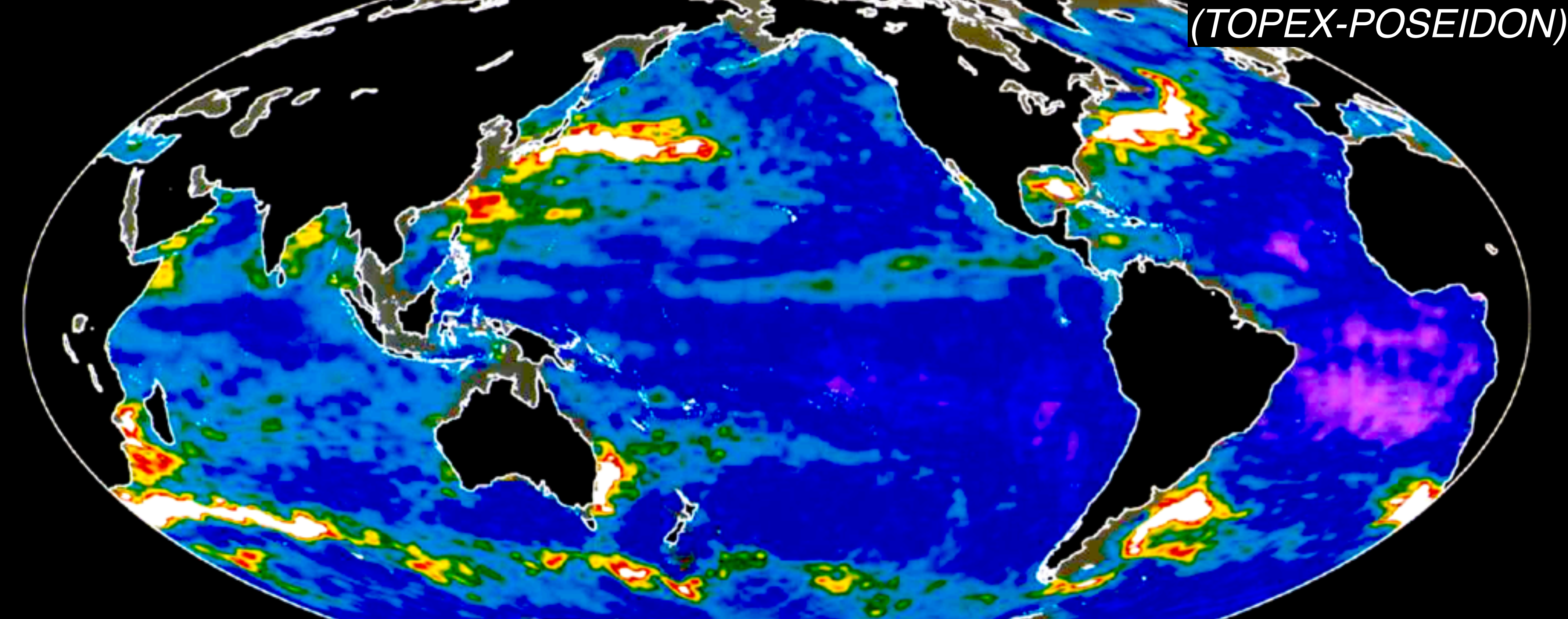
(figure: visibleearth.nasa.gov / SeaWIFS)

In terms of mesoscale eddy resolution, a 1^{o} ocean model ~ 30^{o} atmosphere model:

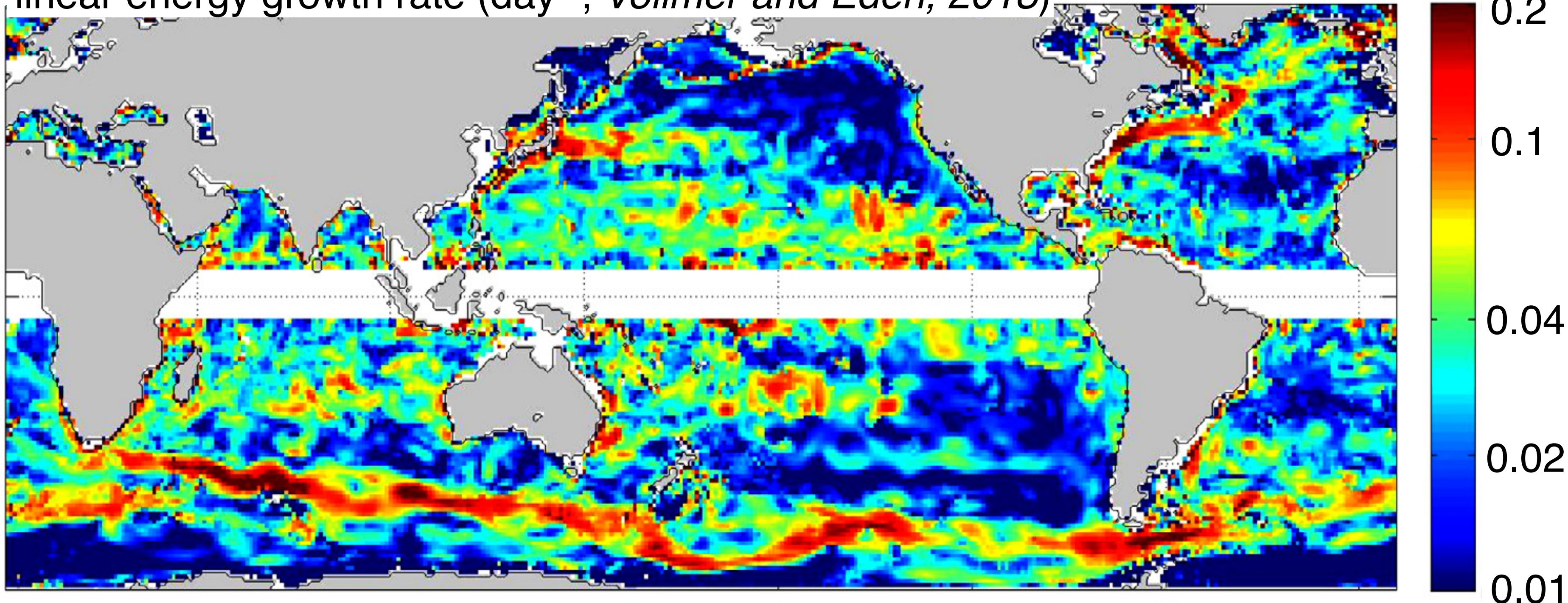


Conversely, a 1^{o} atmosphere model ~ 1/30^{o} ocean model

(after Peter Killworth)



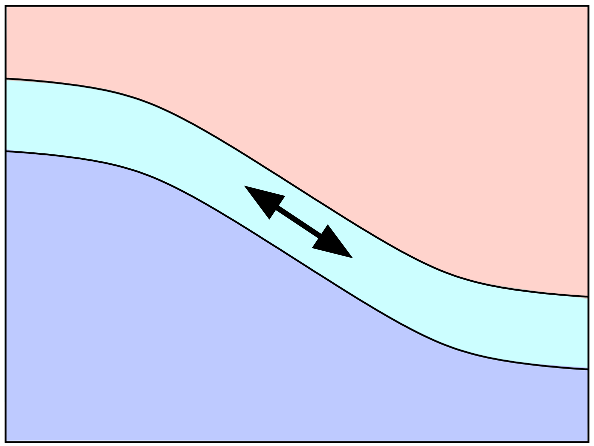
linear energy growth rate (day^{-1}; *Vollmer and Eden, 2013*)



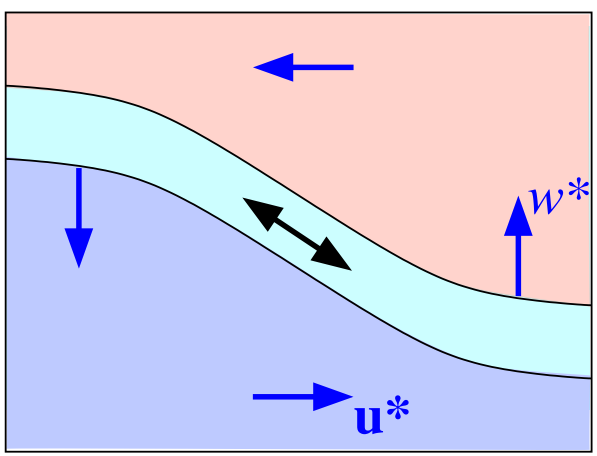
Gent and McWilliams (1990):

adiabatic parameterisation of baroclinic instability

eddies mix along isopycnals (Redi 1982) ...



... and advect by an *eddy bolus velocity* - flattens isopycnals (Gent et al. 1995)



$$\mathbf{u}^* = \frac{\partial}{\partial z}\left(\kappa \frac{\nabla_h b}{N^2}\right), \quad w^* = -\nabla_h \cdot \left(\kappa \frac{\nabla_h b}{N^2}\right)$$

removes available potential energy

can relate eddy diffusivity, κ, to mean flow (e.g., Visbeck et al. 1997)
or eddy energy (Eden and Greatbatch 2008)

GEOMETRIC

Goal: Develop a framework for **parameterising** and **interpreting** ocean eddy-mean flow interaction in which the relevant **symmetries** and **conservation laws** are preserved

Consider (quasi-geostrophic) “residual-mean” momentum equation

$$\frac{\partial \overline{\mathbf{u}}_g}{\partial t} = \ldots \boxed{-\nabla \cdot \underline{\underline{\mathbf{E}}}} + \text{forcing - dissipation}$$

eddy force

(Marshall et al, 2012; Maddison and Marshall, 2013)

with an explicit eddy energy equation

$$\frac{\partial E}{\partial t} = \nabla \cdot (\ldots) + \boxed{\overline{\mathbf{u}} \cdot (\nabla \cdot \underline{\underline{\mathbf{E}}})} + \text{forcing - dissipation}$$

- work done by
eddies on mean flow

(cf. Eden and Greatbatch, 2008)

important: generalises to thickness-weighted averaged primitive equations

(Young, 2012; Maddison and Marshall, 2013)

Eddy force is the divergence of the **eddy stress tensor**:

$$\text{eddy force} = -\mathbf{k} \times \overline{q'\mathbf{u}'} = -\nabla \cdot \underline{\underline{\mathbf{E}}}$$

"Taylor identity"

$$\underline{\underline{\mathbf{E}}} = \begin{pmatrix} -M + P & N & 0 \\ N & M + P & 0 \\ -S & R & 0 \end{pmatrix}$$

(Plumb 1986)

where: $M = \dfrac{\overline{v'^2 - u'^2}}{2}$ $\quad N = -\overline{u'v'}$ Reynolds stresses

$$P = \frac{\overline{b'^2}}{2\mathcal{N}_0^2}$$

eddy potential energy

$R = \dfrac{f_0}{\mathcal{N}_0^2}\overline{u'b'}$ $\quad S = \dfrac{f_0}{\mathcal{N}_0^2}\overline{v'b'}$ eddy buoyancy flux / "eddy form stress"

Why?

$$\text{eddy force} = -\mathbf{k} \times \overline{q'\mathbf{u}'} = -\nabla \cdot \underline{\underline{\mathbf{E}}}$$

$$\underline{\underline{\mathbf{E}}} = \begin{pmatrix} -M+P & N & 0 \\ N & M+P & 0 \\ -S & R & 0 \end{pmatrix}$$

1. This is a mathematical identity! (down-gradient flux ≠ divergence of a tensor)

2. Momentum constraints preserved with appropriate boundary conditions:

3. Reduces to Gent and McWilliams (1990) if we only parameterise
the vertical momentum fluxes.

Why?

$$\text{eddy force} = -\mathbf{k} \times \overline{q'\mathbf{u}'} = -\nabla \cdot \underline{\underline{\mathbf{E}}}$$

$$\underline{\underline{\mathbf{E}}} = \begin{pmatrix} -M+P & N & 0 \\ N & M+P & 0 \\ -S & R & 0 \end{pmatrix}$$

4. Suppose we solve an **eddy energy** equation (Eden and Greatbatch, 2008)

$\Rightarrow$ eddy energy is known

This **eddy energy** gives a bound on the **magnitude of the eddy stress tensor**:

$$||\underline{\underline{\mathbf{E}}}|| = \frac{1}{2}\left[(-N)^2 + (M-P)^2 + (M+P)^2 + N^2 + \frac{\mathcal{N}_0^2}{f_0^2}(R^2+S^2)\right] \leq E^2$$

This means there are **no remaining dimensional unknowns**!

Why?

$$\text{eddy force} = -\mathbf{k} \times \overline{q'\mathbf{u}'} = -\nabla \cdot \underline{\underline{\mathbf{E}}}$$

$$\underline{\underline{\mathbf{E}}} = \begin{pmatrix} -M+P & N & 0 \\ N & M+P & 0 \\ -S & R & 0 \end{pmatrix}$$

5. This allows us to rewrite the eddy stress tensor in terms of the **eddy energy**, two non-dimensional **eddy anisotropies**, and three **eddy angles**:

$$M = -\gamma_m E \cos 2\phi_m \cos^2 \lambda \qquad N = \gamma_m E \sin 2\phi_m \cos^2 \lambda \qquad P = E \sin^2 \lambda$$

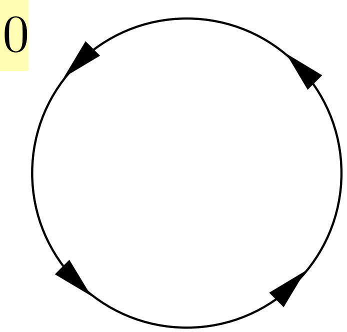
$$R = \gamma_b \frac{f_0}{\mathcal{N}_0} E \cos \phi_b \sin 2\lambda \qquad S = \gamma_b \frac{f_0}{\mathcal{N}_0} E \sin \phi_b \sin 2\lambda$$

horizontal angles

vertical angle

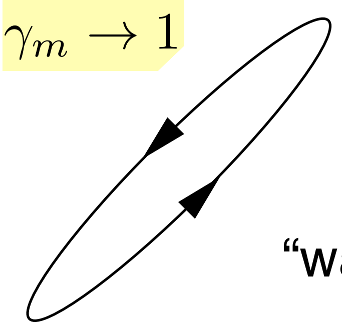
e.g., barotropic eddies:
(plan view)

$\gamma_m = 0$



$\overline{u'v'} = 0$

$\gamma_m \to 1$



"wave-like"

$\overline{u'v'} > 0$

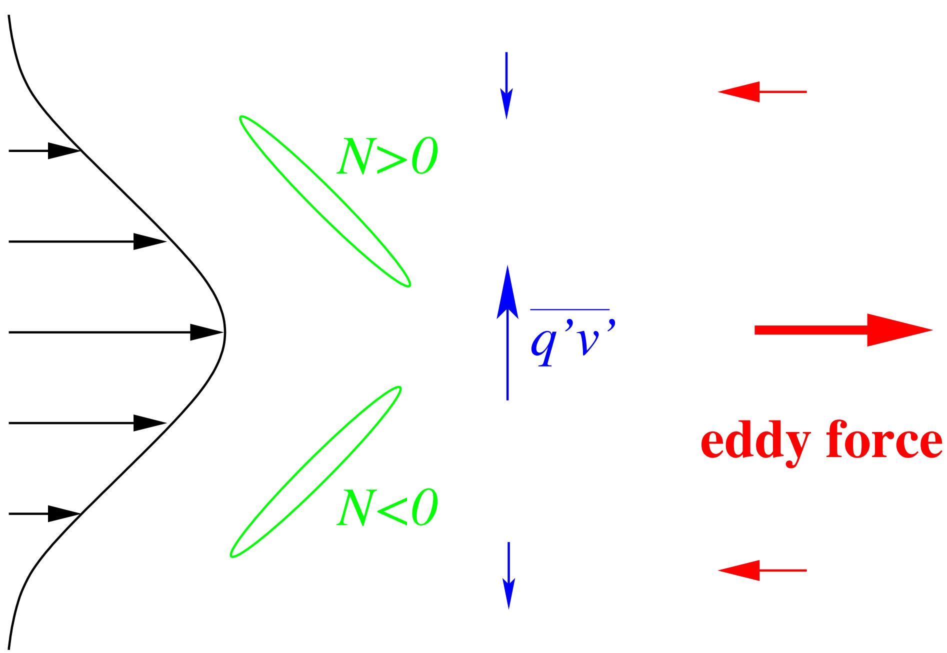
Why?

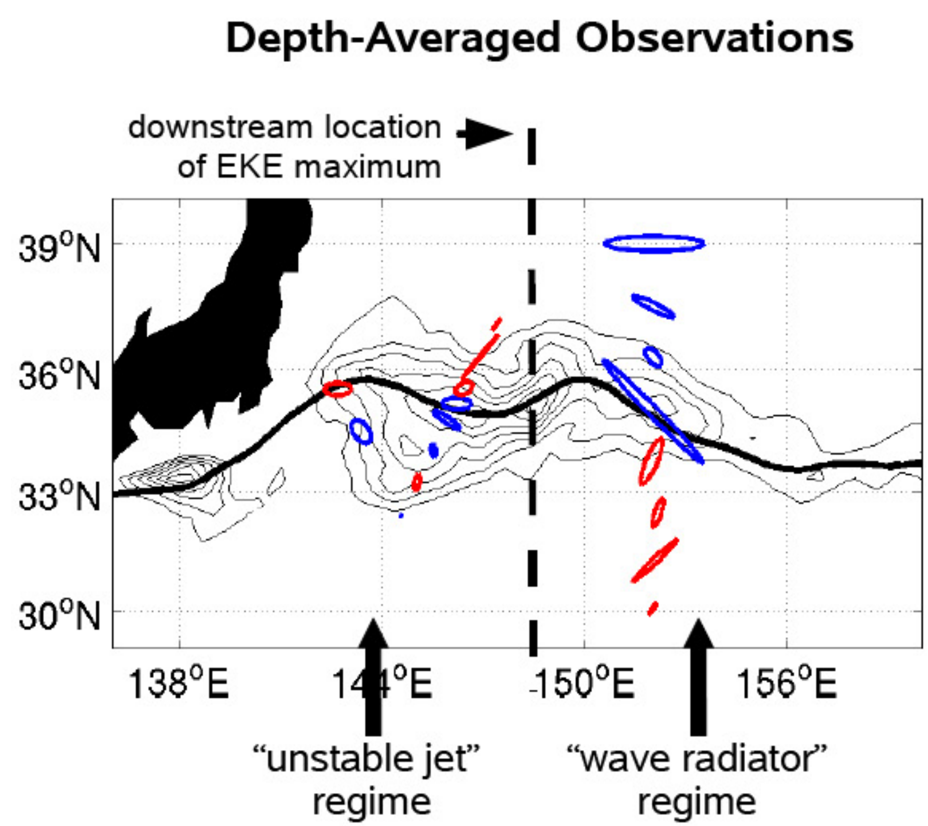
$$\text{eddy force} = -\mathbf{k} \times \overline{q'\mathbf{u}'} = -\nabla \cdot \underline{\underline{\mathbf{E}}}$$

$$\underline{\underline{\mathbf{E}}} = \begin{pmatrix} -M+P & N & 0 \\ N & M+P & 0 \\ -S & R & 0 \end{pmatrix}$$

6. Eddy angles have a strong connection with classical stability theory:

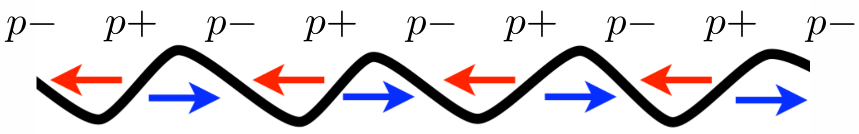
eddies lean “against” mean shear ⇒ extract energy from mean flow - **instability**;
eddies lean “into” mean shear ⇒ return energy to mean flow - **stability**.





(Waterman et al. 2011)

Initial implementation: focus only on vertical “eddy form stress”



eddy form stress is:

$$S = \alpha \frac{|f|}{N} E \qquad (|\alpha| \leq 1)$$

$\Leftrightarrow$

Gent and McWilliams eddy diffusivity is:

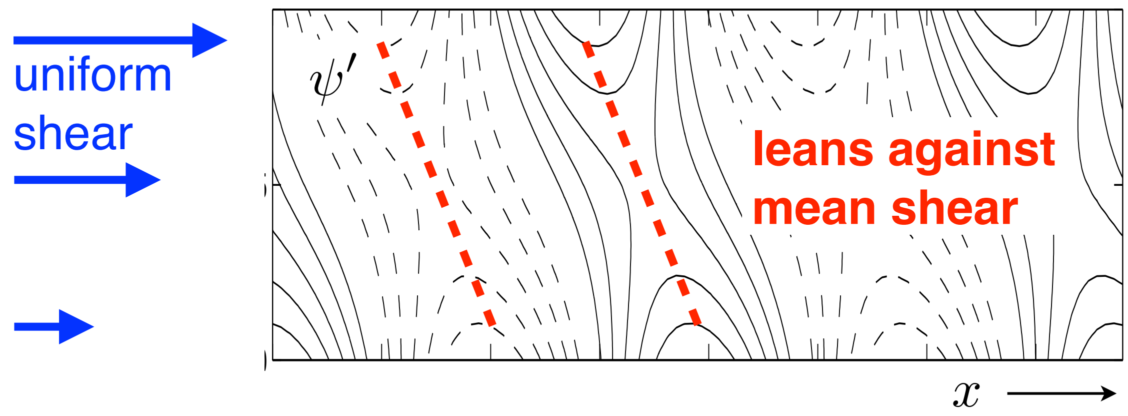
$$\kappa = \alpha \frac{N}{\partial \bar{b}/\partial y} E \qquad (|\alpha| \leq 1)$$

vertical momentum transfer is equivalent to Gent and McWilliams
(Greatbatch and Lamb, 1990; Greatbatch, 1998)

only freedom is to specify the nondimensional parameter, α

Proof of concept 1: Eady problem *(Marshall et al., 2012)*

most **unstable** mode:



Eddy energy budget:

$$\frac{\partial}{\partial t}\iiint E\,dx\,dy\,dz = -\iiint \overline{u}\frac{\partial S}{\partial z}\,dx\,dy\,dz$$ *substitute eddy stress tensor*

$$= \iiint \frac{\partial \overline{u}}{\partial z} S\,dx\,dy\,dz$$ *integrate by parts*

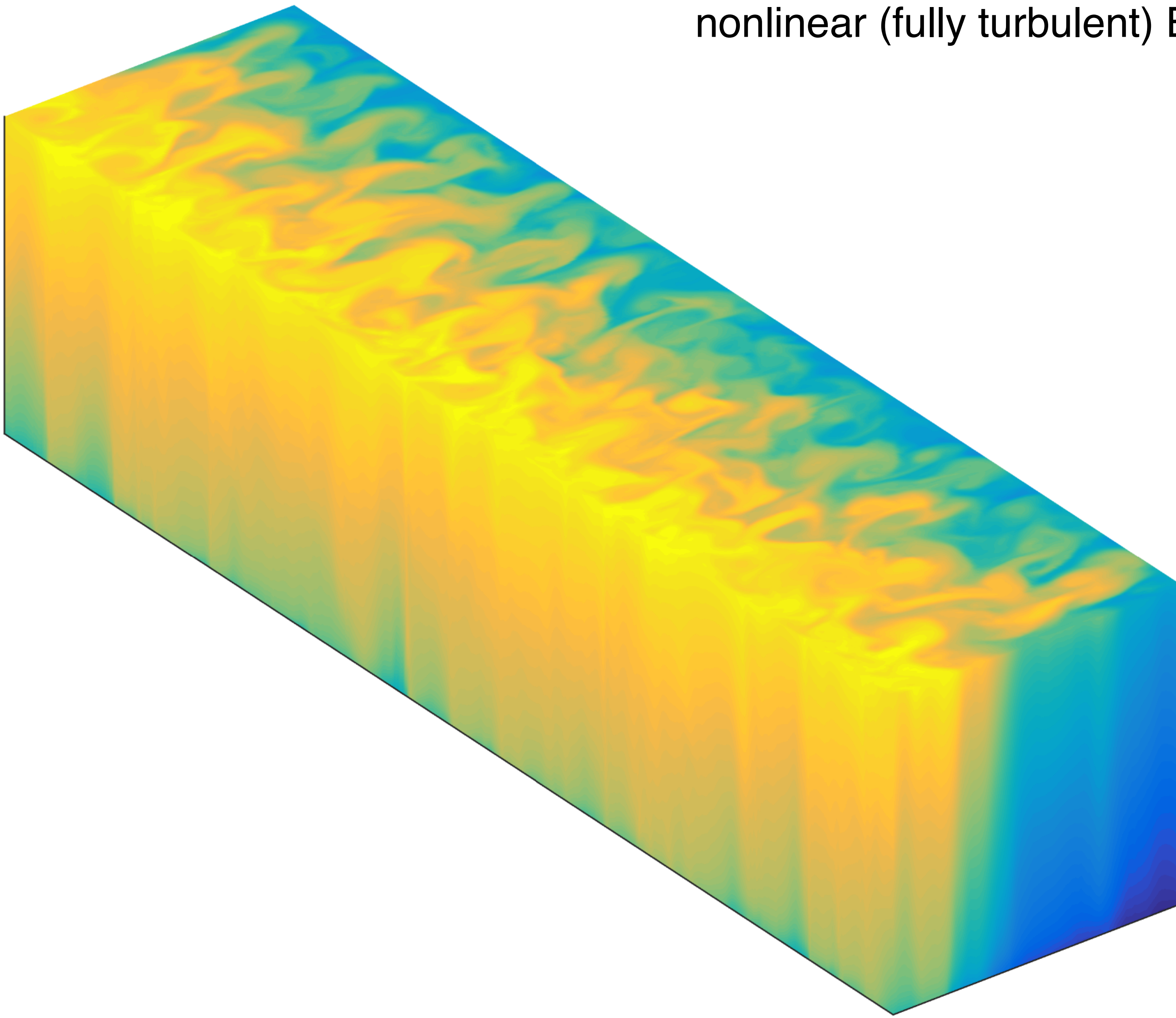
$$= \boxed{\alpha \frac{f}{N}\frac{\partial \overline{u}}{\partial z}} \iiint E\,dx\,dy\,dz$$ *apply energy bound*

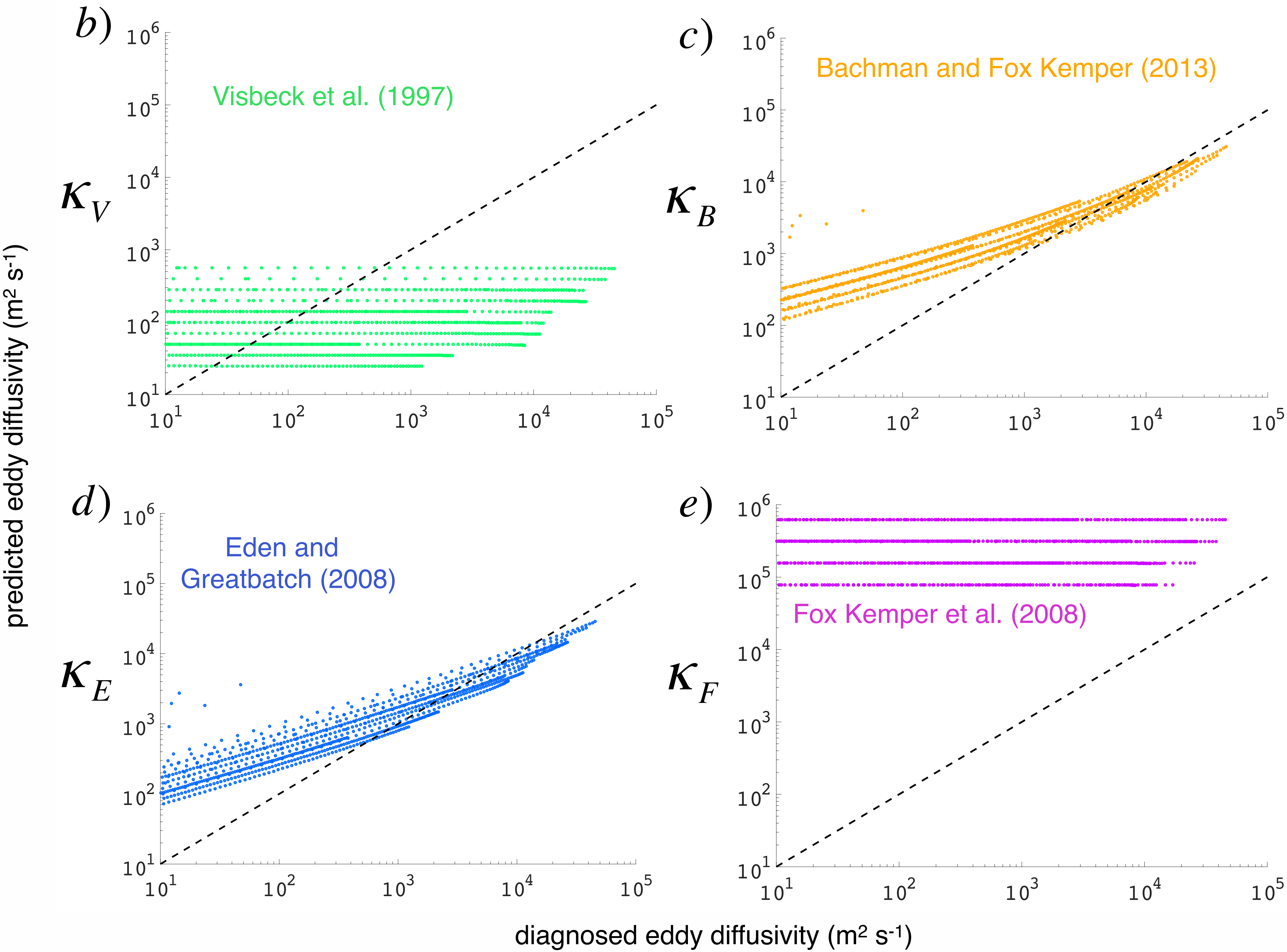
Eady growth rate
if $\alpha = 0.61$

$$(|\alpha| \leq 1)$$

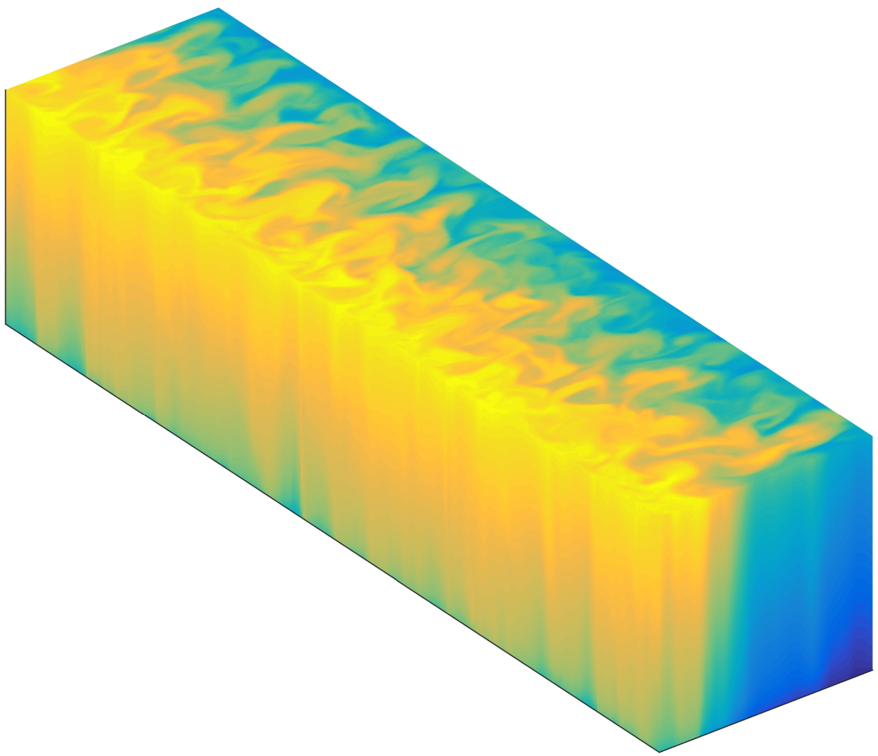
Proof of concept 2: Eddy diffusivity *(Bachman et al., 2017)*

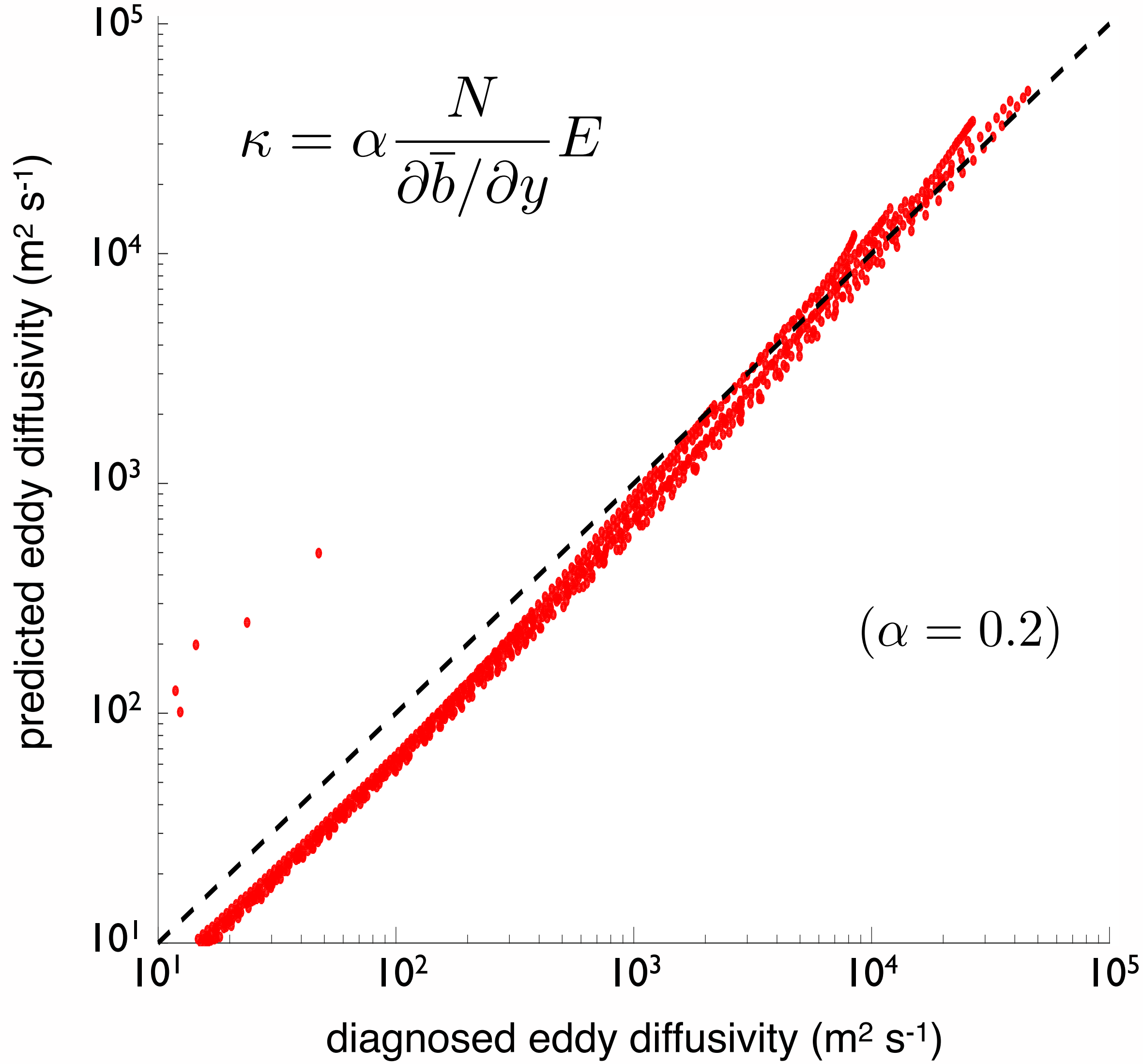
nonlinear (fully turbulent) Eady spin-down





b)
Visbeck et al. (1997)
κ_V
c)
Bachman and Fox Kemper (2013)
κ_B
d)
Eden and Greatbatch (2008)
κ_E
e)
Fox Kemper et al. (2008)
κ_F
predicted eddy diffusivity (m^2 s^{-1})
diagnosed eddy diffusivity (m^2 s^{-1})

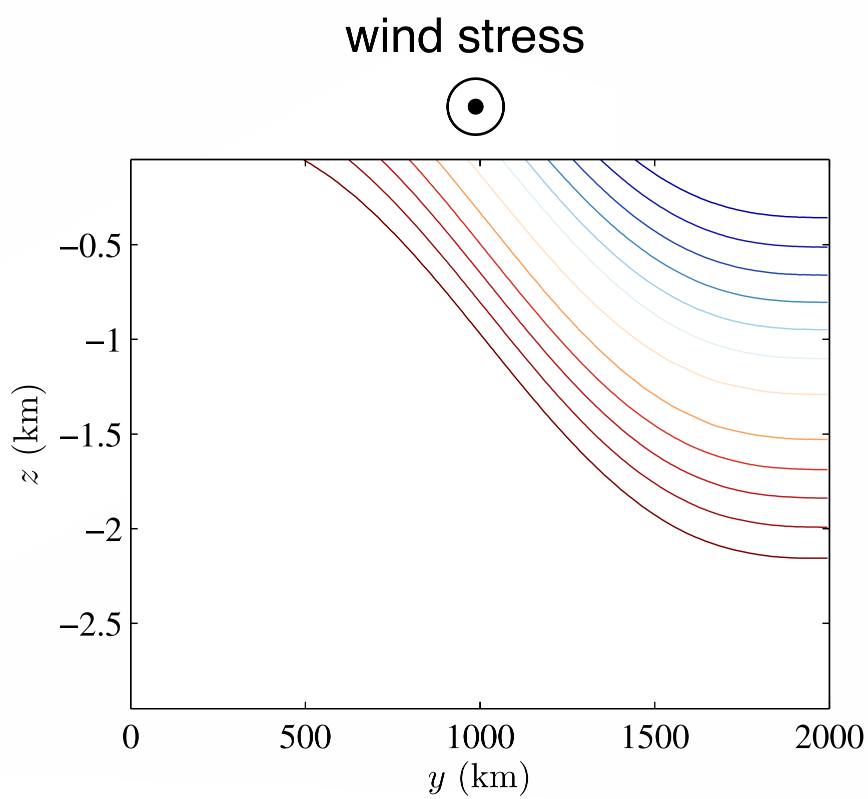


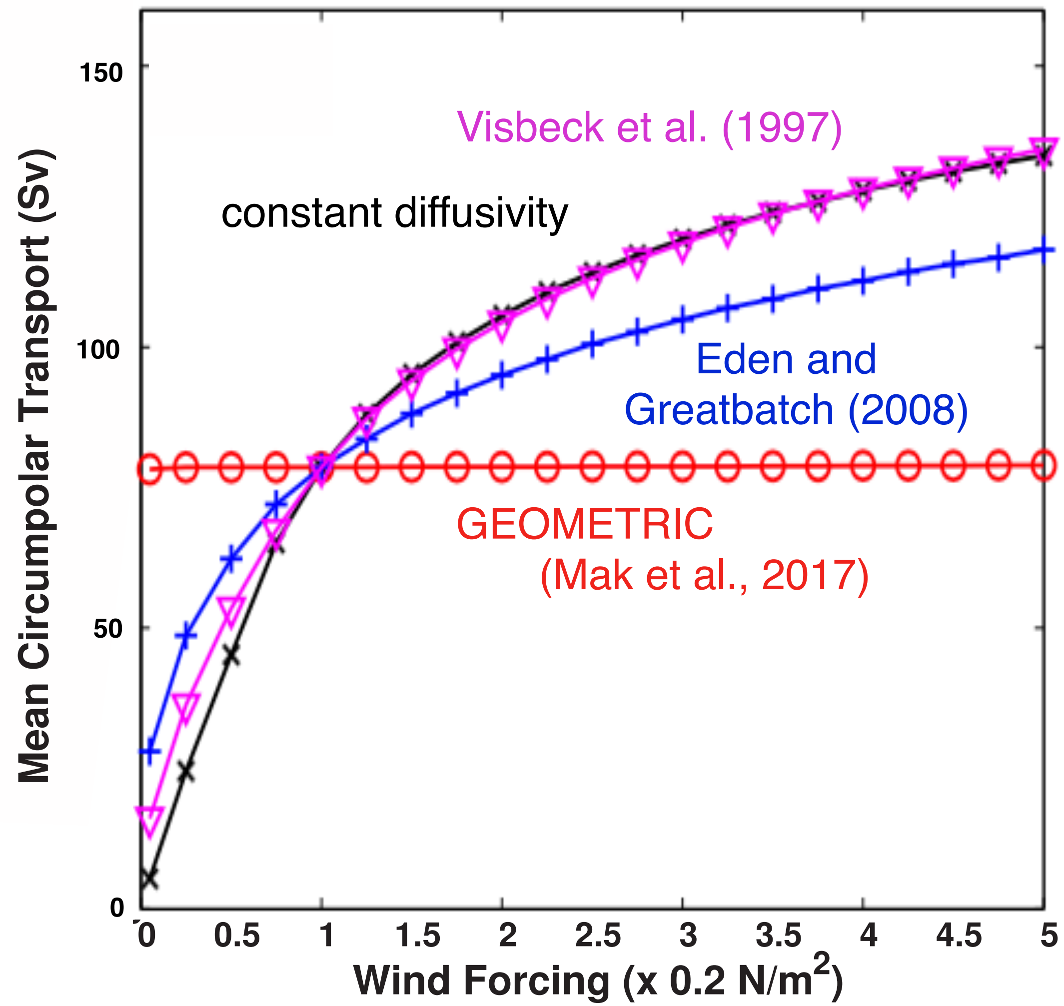


(Bachman et al., 2017)

Proof of concept 3: Eddy saturation *(Mak et al., 2017)*

2-d model; solve prognostic equation for domain-averaged eddy energy





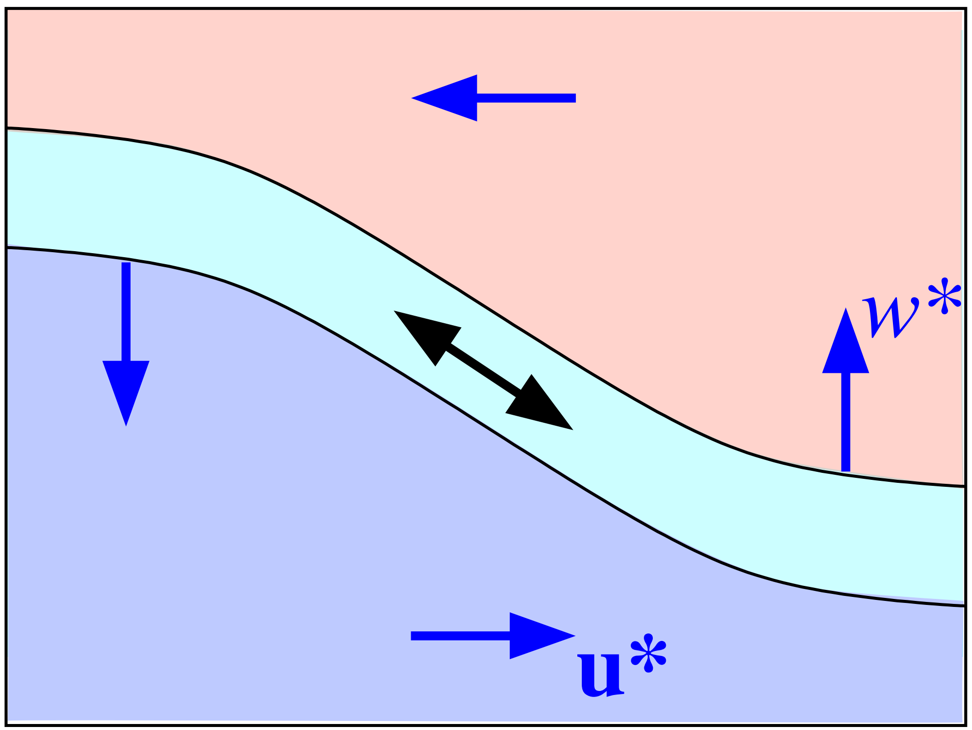
eddy saturation is an emergent property (more in James' talk)

Recipe for implementation in an ocean climate model

1. Employ existing Gent and McWilliams code with prescribed eddy diffusivity profile

2. Solve a prognostic equation for the depth-integrated eddy energy

3. Rescale eddy diffusivity profile at each latitude/longitude to match energetic constraint.



$$\boldsymbol{\psi}_0 = \kappa_0(z)\frac{\nabla b}{N^2} \qquad \lambda = \frac{\int E\, dz}{\int N|\boldsymbol{\psi}_0|\, dz}$$

$$\boldsymbol{\psi} = \lambda\, \kappa_0(z)\frac{\nabla_h b}{N^2}$$

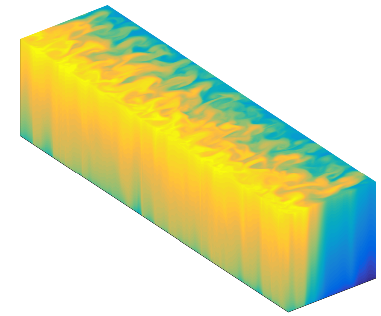
$$\mathbf{u}* = \frac{\partial \boldsymbol{\psi}}{\partial z}, \qquad w* = -\nabla_h \cdot \boldsymbol{\psi}$$

- proposing to put into NEMO ocean model (partner with Carsten/TRR 181)

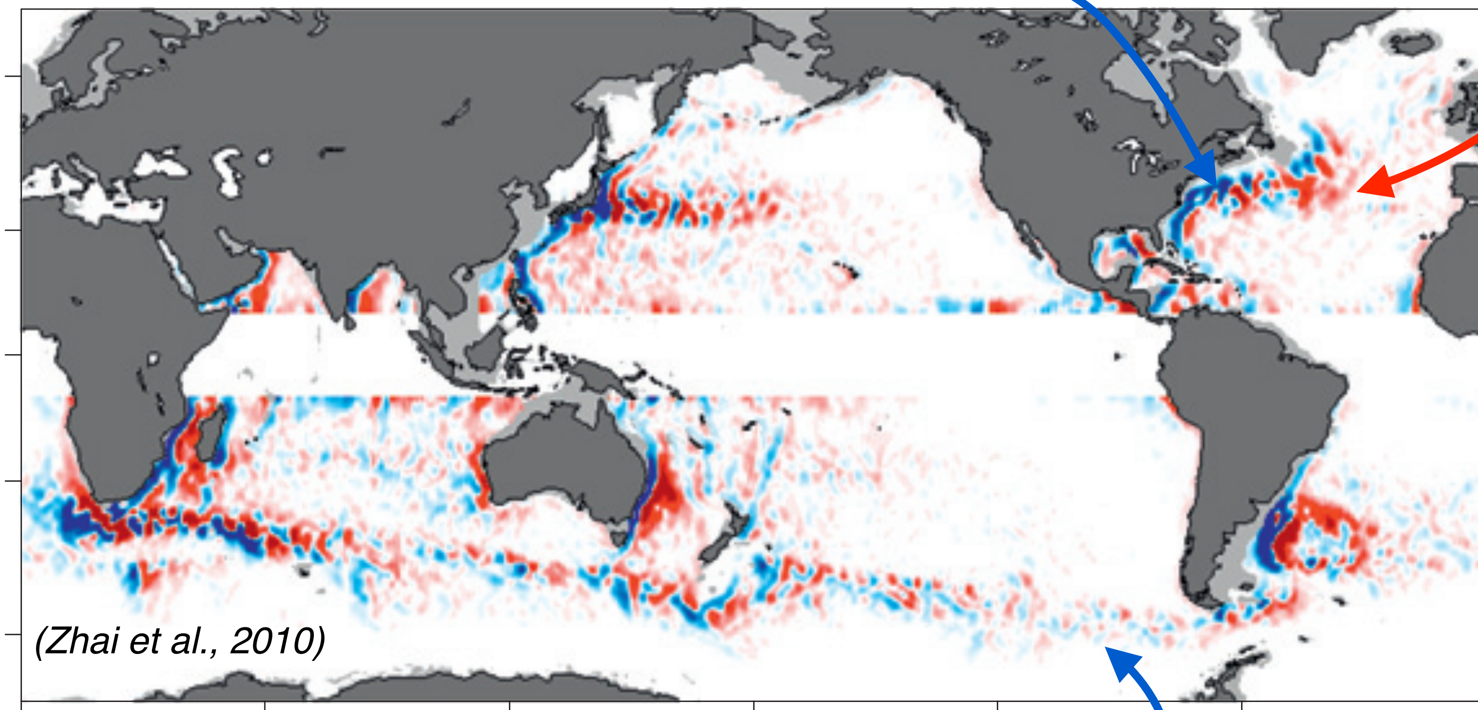
Prognostic depth-integrated eddy energy equation:

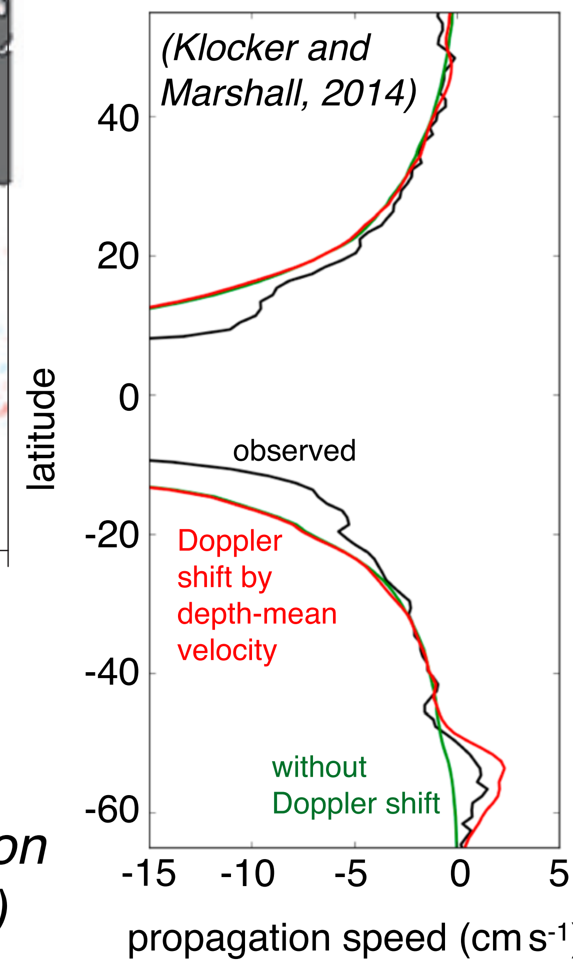
- already attempted by Eden and Greatbatch (2008) in 3-d with some success
- simpler here as only need 2-d depth-integrated eddy energy

eddy energy source: baroclinic instability



eddy energy sink: western boundaries (Zhai et al., 2010)





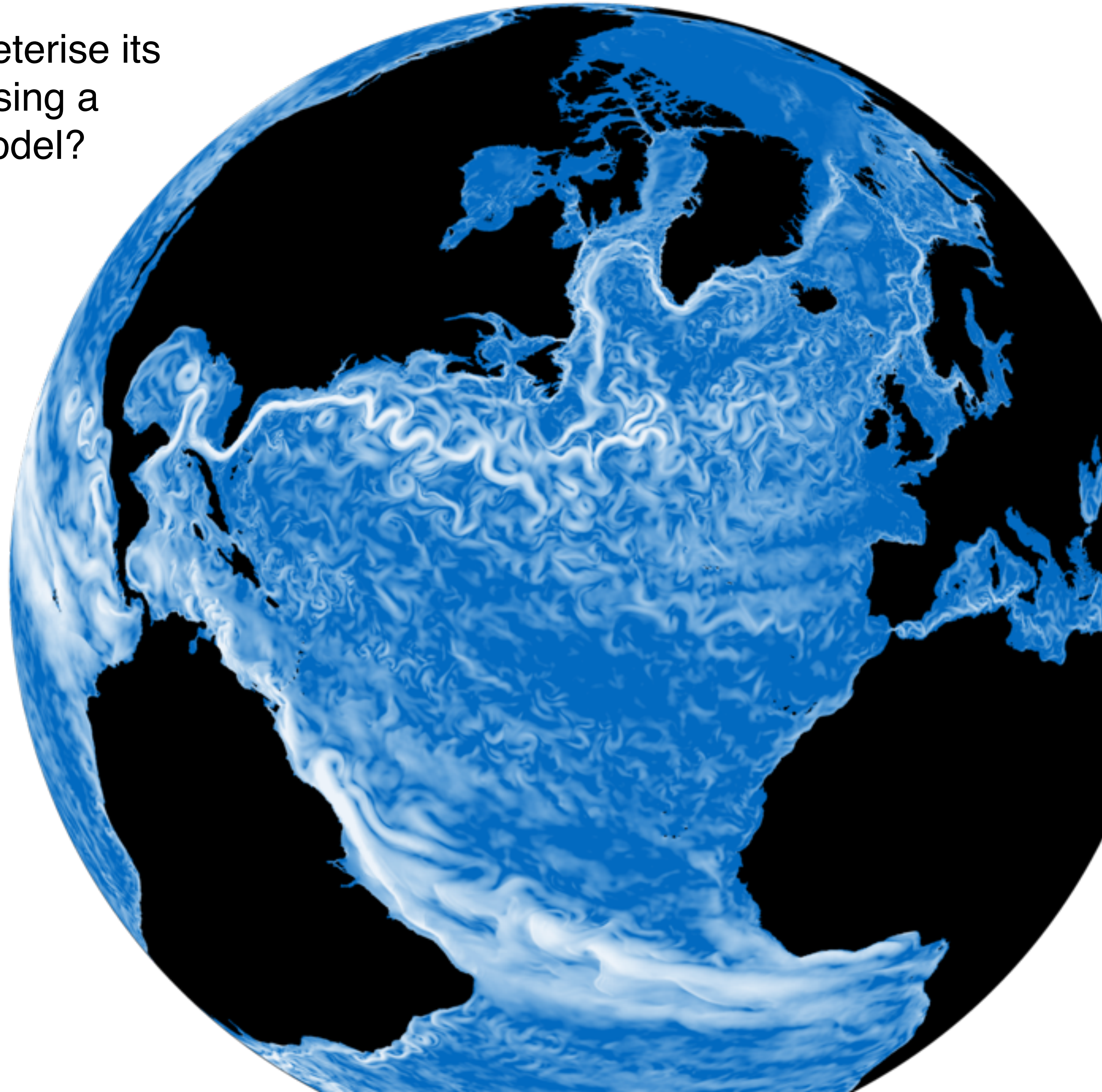
eddy energy sink: bottom drag (Sen et al., 2008); lee wave radiation (Naveira Garabato et al., 2004, ...)

diffusion of eddy energy (Eden and Greatbatch, 2008)

westward propagation (Chelton et al. 2007)

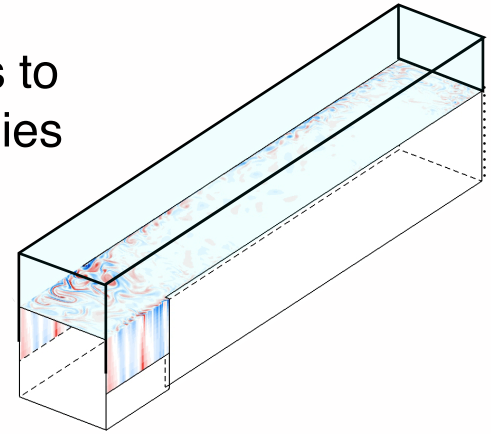
Depth-integrated eddy energy budget in NEMO 1/12° model - “model truth”

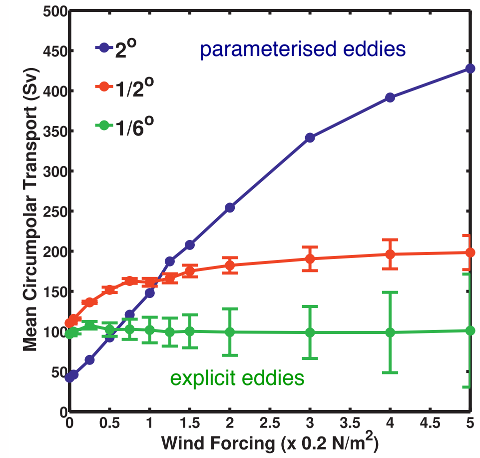
- how well can we parameterise its eddy energy budget using a non eddy-resolving model?



Concluding remarks

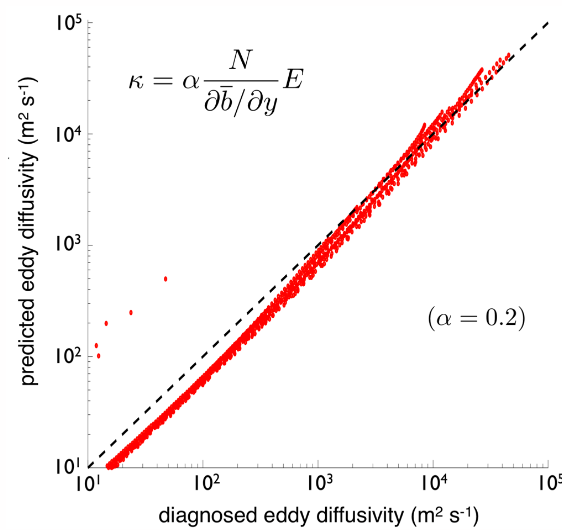
- Ocean models give very different sensitivities to forcing with parameterised and explicit eddies

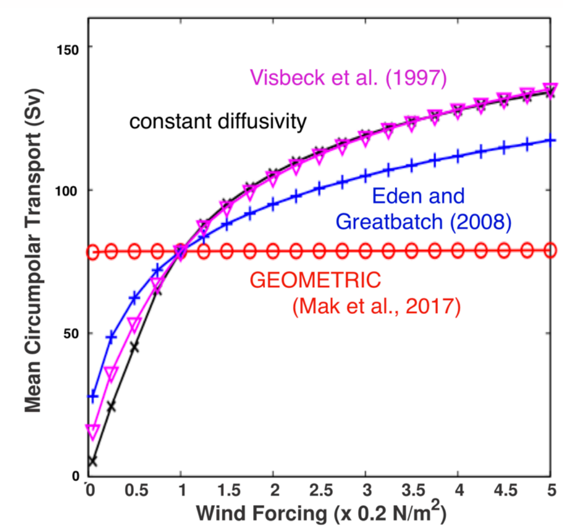




- GEOMETRIC
  - conserves momentum and energy by construction
  - solve prognostic equation for eddy energy
  - remaining unconstrained parameters are dimensionless
  - physical interpretation via stability properties of flow

- Proofs of concept
  - Eady growth rate
  - eddy diffusivity
  - eddy saturation





- Implementation
  - eddy energy budget
  - otherwise simple modification to Gent and McWilliams

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