Near-inertial damping of a nearly geostrophic flow

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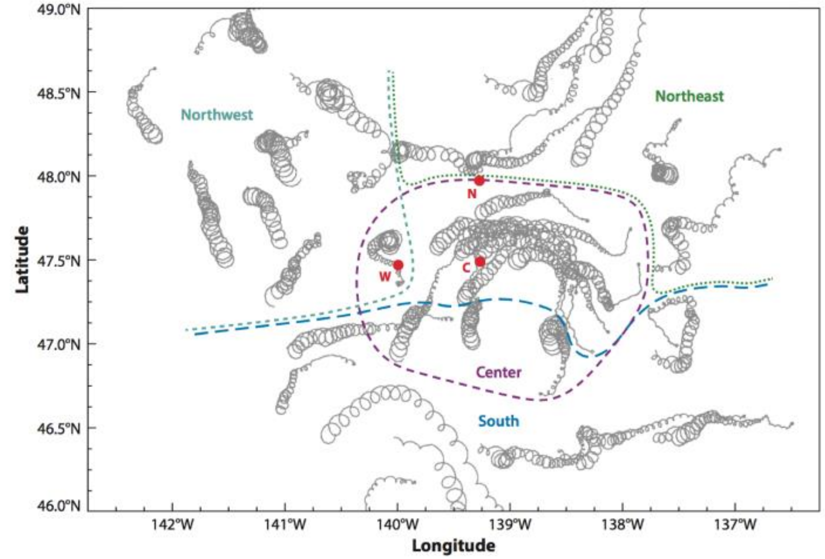
Collaborators: Stephanne Taylor, Aaron Gertz

Geostrophic turbulence is non-dissipitive

- Winds provide an energy source to the geotrophic circulation and eddy field
- So an energy sink is needed:
  - Bottom friction
  - Top friction
  - Loss of balance

Observations show near-inertial motion superposed on geostrophic currents

Want to understand how they might help damp the geostrophic flow



D'Asaro 1995

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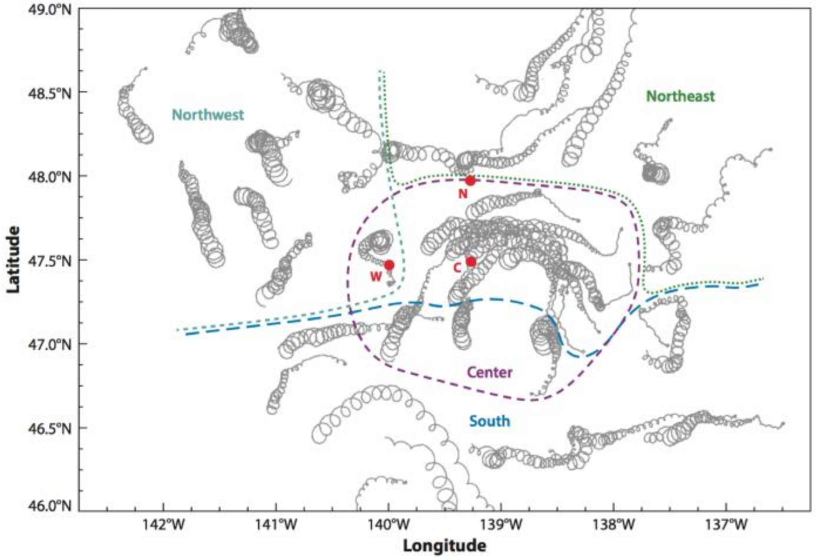
NB: The NIWs are present mainly because of external forcing

Geostrophic turbulence helps to cascade NI wave energy forward

As they cascade forward, NI waves can extract energy from the geostrophic flow



D'Asaro 1995

SLOB
(and other messy jargon)

- Stimulated Loss Of Balance (Xie and Vanneste 2015, Wagner and Young 2016)
- Direct extraction vs Stimulated Imbalance (Barkan et al. 2017)
- Advective Sink (some of my papers)

<u>Comment</u>: unbalanced motion is predominant at small spatial scales. Much literature assumes balance-to-unbalanced transfers are also submesoscale (e.g., Buhler and McIntyre 2003, Guisouard and Thomas 2015). For these studies, the WKB approximation is often useful. My focus is on mesoscale transfers.

Stimulation refers to external forcing of near-inertial motion
(or to NI motion present at significant levels in an IVP

The Xie and Vanneste Model

Separates flow into slow and fast modes
(without assuming WKB)

$$u + iv = M_z e^{-i f_0 t}$$

$$\chi_{zzt} + (\partial(\psi, \chi_z))_z + i\beta y \chi_{zz} + \frac{i}{2}\left(\left(\frac{N^2}{f_0} + \psi_{zz}\right)\nabla^2\chi + \nabla^2\psi\,\chi_{zz} - 2\nabla\psi_z \cdot \nabla\chi_z\right) = 0,$$

$$q_t + \partial(\psi, q) = 0,$$

$$q = \beta y + \Delta\psi + \frac{i f_0}{2}\partial(\chi_z^*, \chi_z) + f_0 G(\chi^*, \chi),$$

$$G(\chi^*, \chi) = \frac{1}{4}\left(2|\nabla\chi_z|^2 - \chi_{zz}\nabla^2\chi^* - \chi_{zz}^*\nabla^2\chi\right),$$

Conclude that a forward cascade of near-inertial energy implies a sink of geostrophic energy

- Fast KE and total E are both conserved
- A forward transfer of NIWs implies an increase in fast PE
- But since fast KE is conserved, this implies loss of balance

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Wagner and Young (2016) have a similar model, but also include frequencies near 2f

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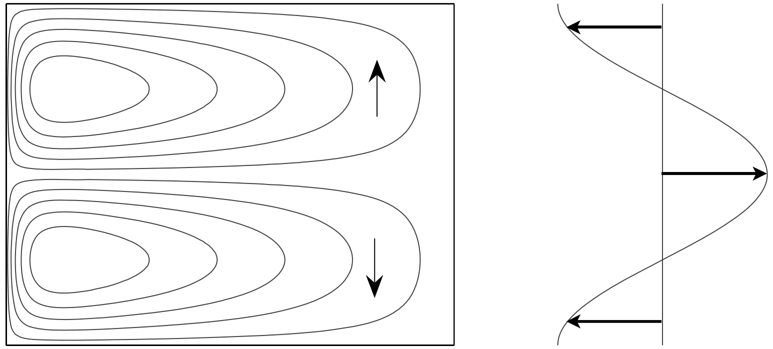
A Ke-only model of SLOB (Gertz and S 2008)

(NB: The Xie and Vanneste mechanism relies on PE)

Unstratified ocean double gyre problem

$$\beta\psi_x = \nabla \times \tau - r\nabla^2\psi$$



$$\frac{\partial \mathbf{u}_h}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{u}_h + \hat{\mathbf{z}}f \times \mathbf{u}_h = -\nabla_h P$$

$$\mathbf{v} \equiv \mathbf{u}_h + \hat{\mathbf{z}}w \quad \nabla\cdot\mathbf{v} = 0 \quad \frac{\partial}{\partial z}P = 0,$$

Hydrostatic dynamics (equivalently, this is an unstratified version of the primitive eqns)

High frequency forcing applied to excite z-dependent near-inertial oscillations

Comments:

- the linear z-dependent modes are inertial oscillations
- their Reynolds stresses feed back onto the depth averaged flow
- This is significant only if the NIWs are externally forced

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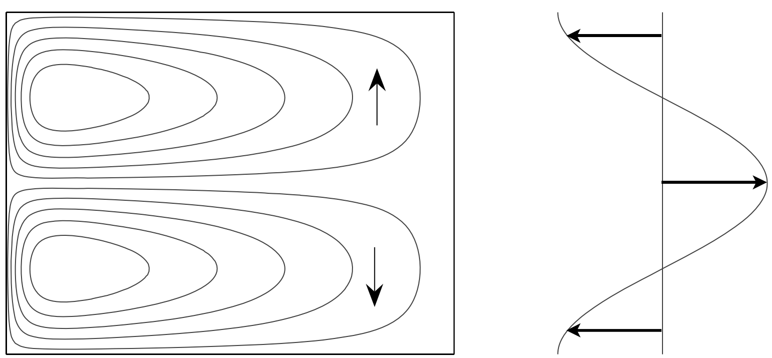

High frequency forcing applied to excite z-dependent near-inertial oscillations

$$\frac{\partial}{\partial t}\boldsymbol{\nabla}_h^2\psi = -J(\psi, \boldsymbol{\nabla}_h^2\psi + \beta y) - \hat{\mathbf{z}}\cdot[\boldsymbol{\nabla} \times \overline{(\mathbf{v}'\cdot\boldsymbol{\nabla})\mathbf{u}'_h}] + \overline{F} - r\boldsymbol{\nabla}_h^2\psi + A\boldsymbol{\nabla}_h^8\psi,$$

$$\frac{\partial}{\partial t}\mathbf{u}'_h + (\overline{\mathbf{u}}_h\cdot\boldsymbol{\nabla}_h)\mathbf{u}'_h + (\mathbf{u}'_h\cdot\boldsymbol{\nabla}_h)\overline{\mathbf{u}}_h + \hat{\mathbf{z}}f \times \mathbf{u}'_h = \mathbf{F}' + A\boldsymbol{\nabla}_h^6\mathbf{u}'_h - (\mathbf{v}'\cdot\boldsymbol{\nabla})\mathbf{u}'_h + \overline{(\mathbf{v}'\cdot\boldsymbol{\nabla})\mathbf{u}'_h},$$

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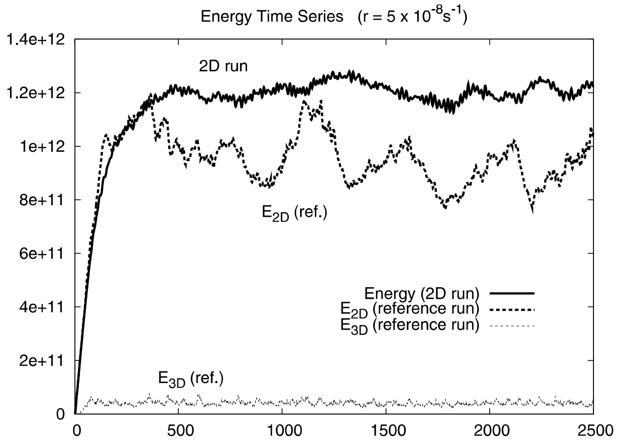


FIG. 1. Energy time series for the reference simulations. Top curve: E_{2D} for the case without 3D forcing. Middle and bottom curves: E_{2D} and E_{3D} energy for the case with our medium value of F_{3D}.

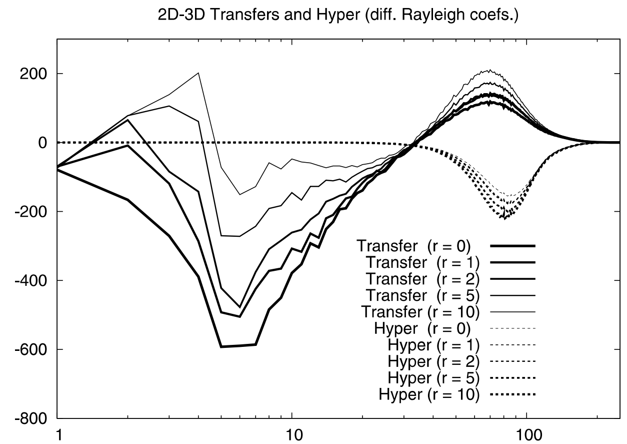


FIG. 6. The terms $\mathcal{A}_{3D}$ and $\mathcal{A}_{hyper}$ for simulations with the medium value of F_{3D} and for different values of the drag coefficient (r = 0, 1, 2, 5, and 10 $\times$ 10^{-8} s^{-1}). Lower r corresponds to more strongly negative $\mathcal{A}_{3D}$ at moderate wavenumbers. At high wavenumbers, both $\mathcal{A}_{3D}$ and $\mathcal{A}_{hyper}$ are reduced for lower r. By contrast, the hyperviscous dissipation of 3D energy is stronger for lower r; not shown. Note that the $r = 5 \times 10^{-8}$ s^{-1} curves also appear in Fig. 3c.

Time series showing geostrophic and near-inertial energy with and without external forcing of the near-inertial modes

Transfer spectra as a function of horizontal wavenumber. Negative values imply geostrophic-to-near-inertial energy transfers

Point: near-inertial modes can feed back on to the balanced flow. *But this feedback is minimal unless the near-inertial modes are externally forced*

More recent work (with S Taylor)

Primitive Eqn eddy-permitting climate model (POP)

Zonally periodic setting with a large-scale meridional ridge

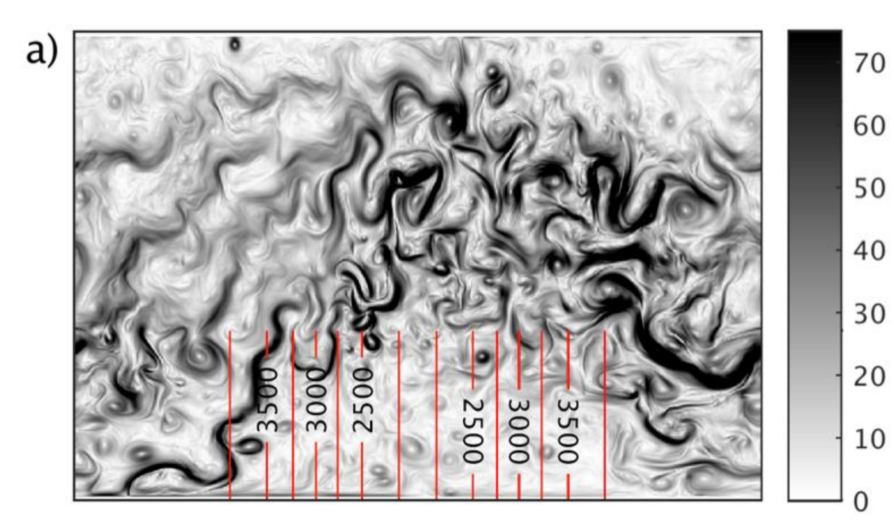
Forcing = sum of

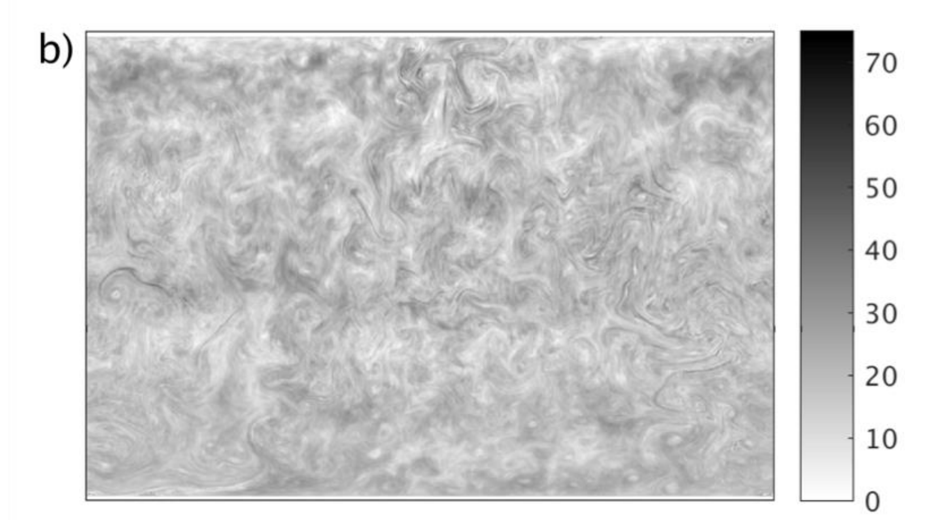
i) a steady zonal wind stress (to force a geostrophic flow)
ii) a large-scale high frequency wind stress (to excite waves)

Dissipation by bottom drag and hyper-viscosity

Base state stratification consistent with observations

About four grid points per Rossby radius and 30 vertical levels
(sub-mesoscale permitting?)





Base state is a nearly geostrophic flow is a S. Hemisphere channel flow in primitive eqn model (POP) forced by steady winds

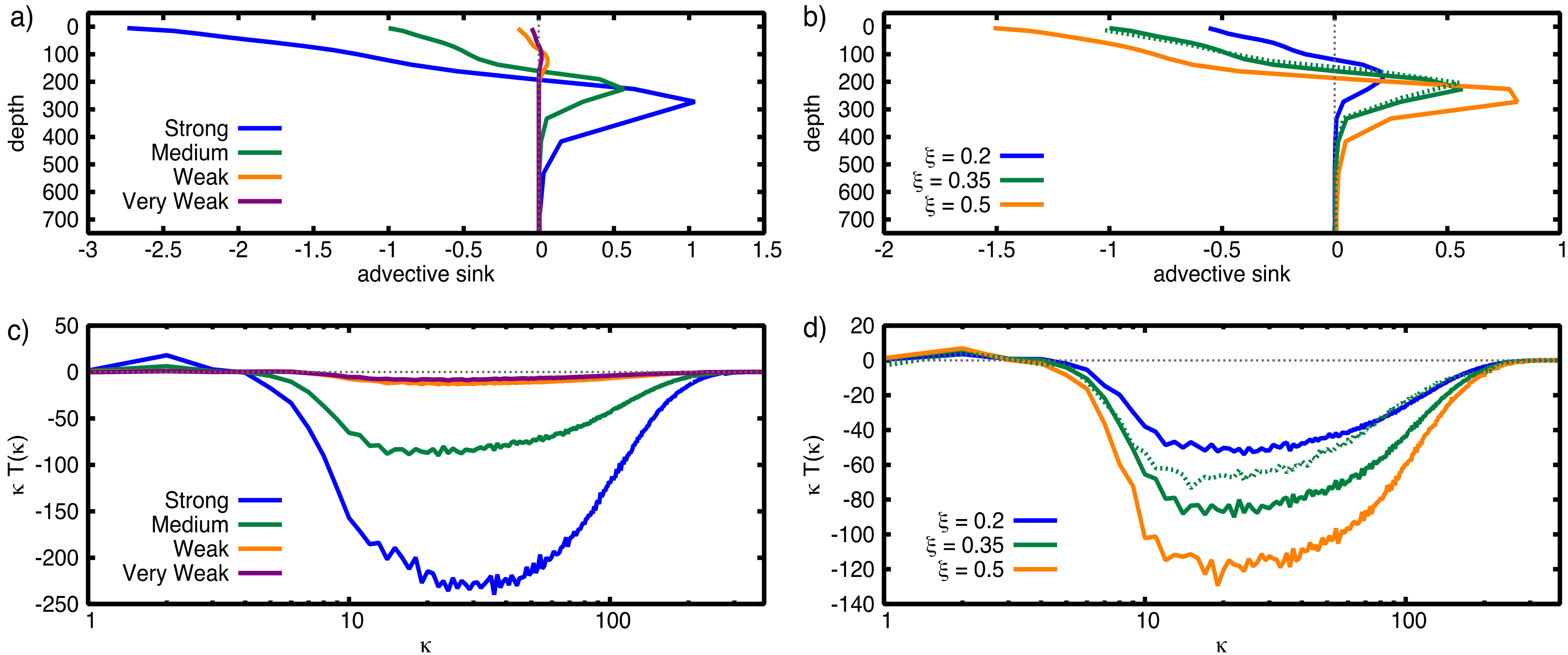
Snapshot of high high-passed flow (shown is surface speed in cm/s)

We considered 4 different base states (varying the amplitude of th steady forcing)

And several levels of high frequency forcing for each base state

$$\tau_{\text{steady}} \equiv \tau_0\Big(1+\cos(2\pi(\theta-\theta_0)/\Delta\theta)\Big)$$

$$\tau^x_{\text{NI}}(t) \equiv \xi\,\tau_0 \sum_{n=1}^{N} N^{-0.5} \exp\!\left(-\frac{(\omega_n - f_0)^2}{2\sigma^2}\right) \sin(\omega_n t + \phi_n)$$



Influence of high-passed Reynolds stresses on low-passed KE

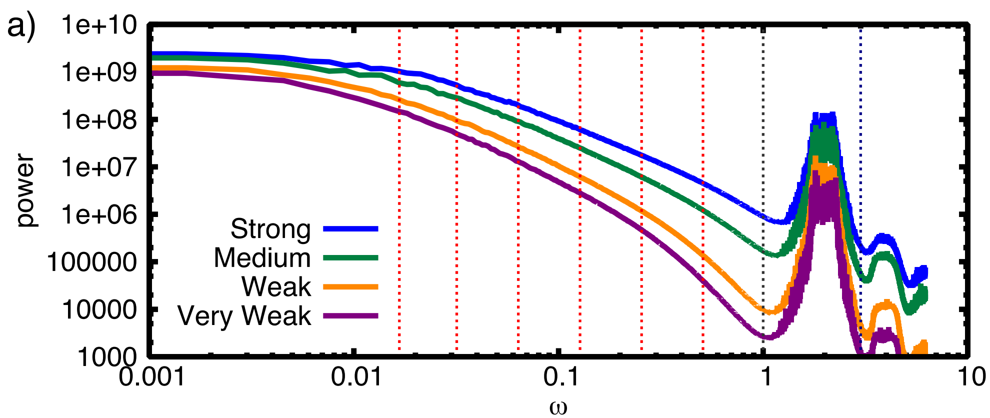
$$\chi = -\mathbf{u}^{<} \cdot \left[\nabla \cdot \left(\mathbf{v}^{>}\mathbf{u}^{>}\right)\right]^{<} = -u^{<}\left[\nabla \cdot \left(\mathbf{v}^{>}u^{>}\right)\right]^{<} - v^{<}\left[\nabla \cdot \left(\mathbf{v}^{>}v^{>}\right)\right]^{<}$$

Serves mainly to redistribute KE vertically

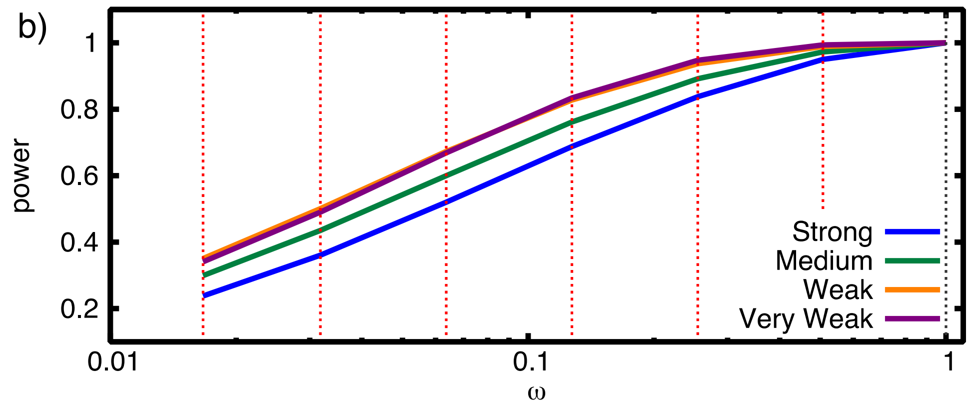
But also comprises a net sink
(centered in the ocean mesoscale)

Stimulated Imbalance vs Direct Extraction

(jargon from Barkan et al.)



KE frequency spectra for different base state forcing levels



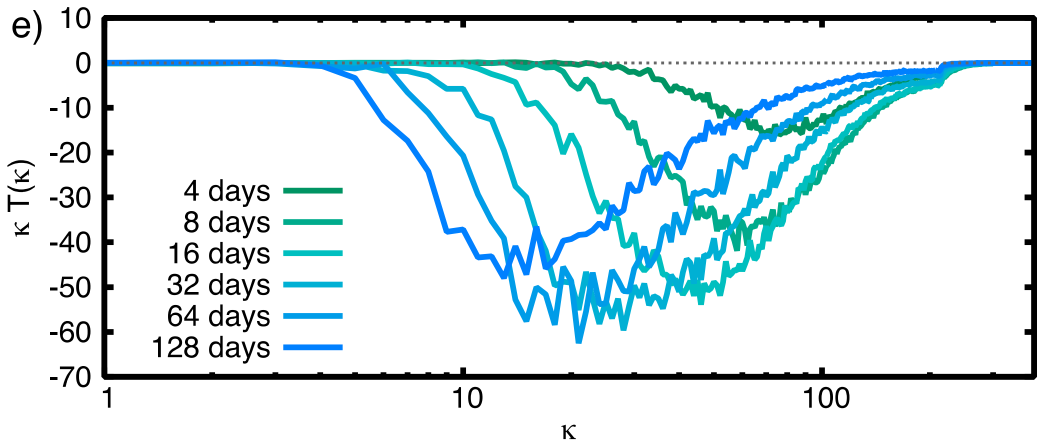
High freq defined as periods greater than 2 days

Used different cutoffs to define the low-pass fields:

2,4,8,16,32,64,128 days

Frequencies from which ‘balanced’ KE is extracted

Stimulated Imbalance vs Direct Extraction
(Barkan et al.)



Surface transfer spectra. Different curves show energy extracted from low-passed freqs corresponding to periods of 2-4, 4-8, 8-16, 16,32,32-64, 64-128 days

Shown are results for the medium base state

High freq defined as periods greater than 2 days

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Potential energy and V-WW transfers

For low Ro, Fr (Bartello 95)

V-WW is small

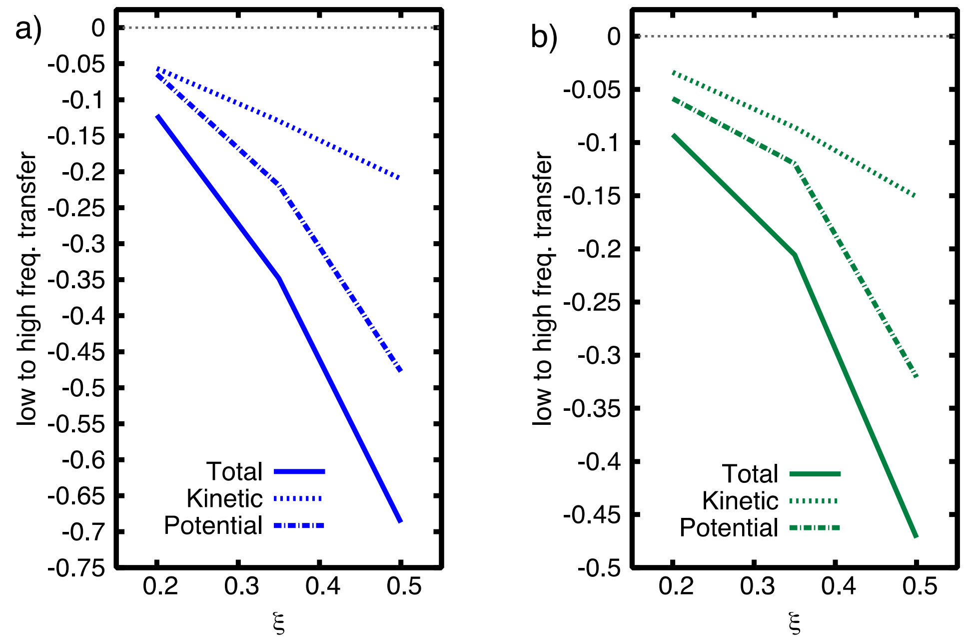
W-VW → forward cascade of wave E

The Reynolds stress transfers we consider are low-to-high freq KE transfers.

If they can be thought of as (mainly) part of an overall V-WW transfer then…

we should expect low-to-high freq PE transfers

V ie, because the ratio of PE to KE in the modes is given by L/L_d



Shown are total, PE and KE sinks of low-passed energy for the strong and medium base states (as a function of the high freq forcing amplitude)

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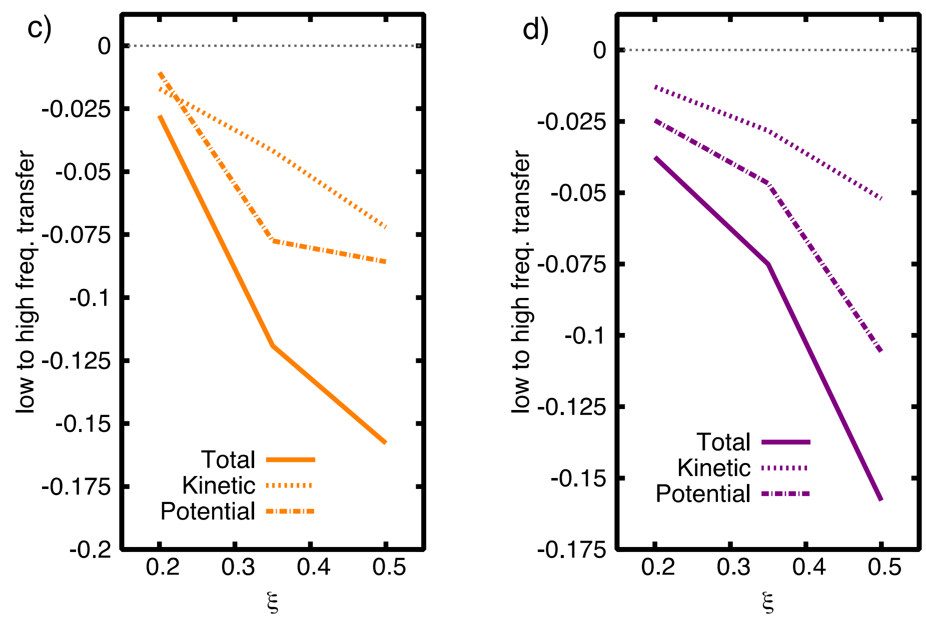

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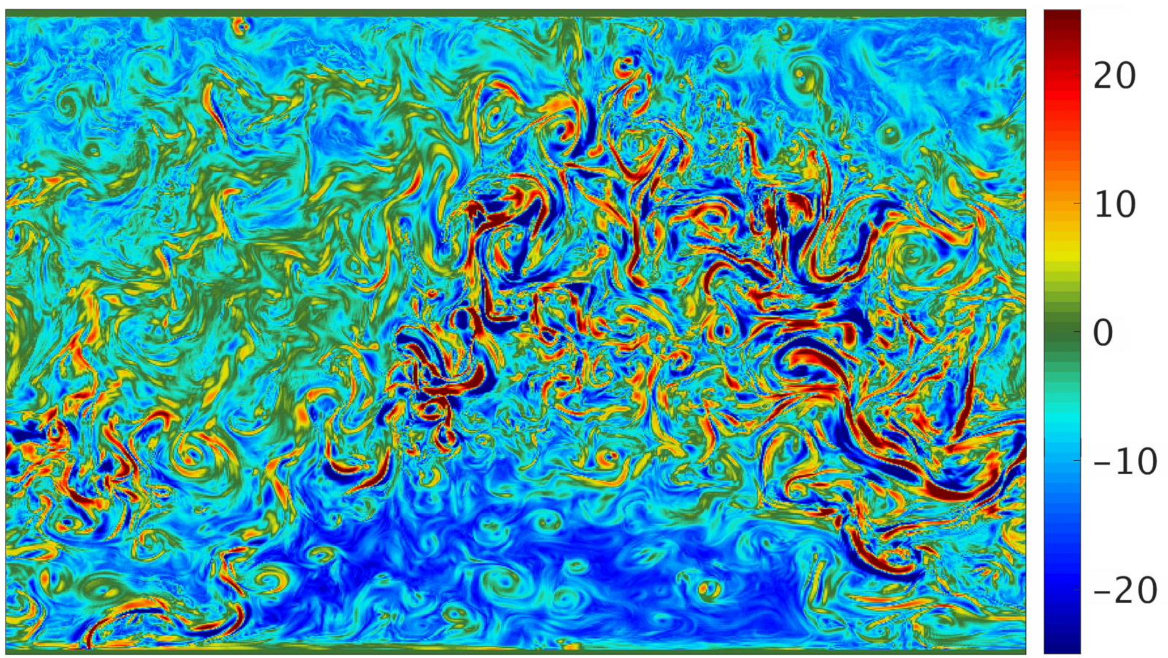
My view of SLOB

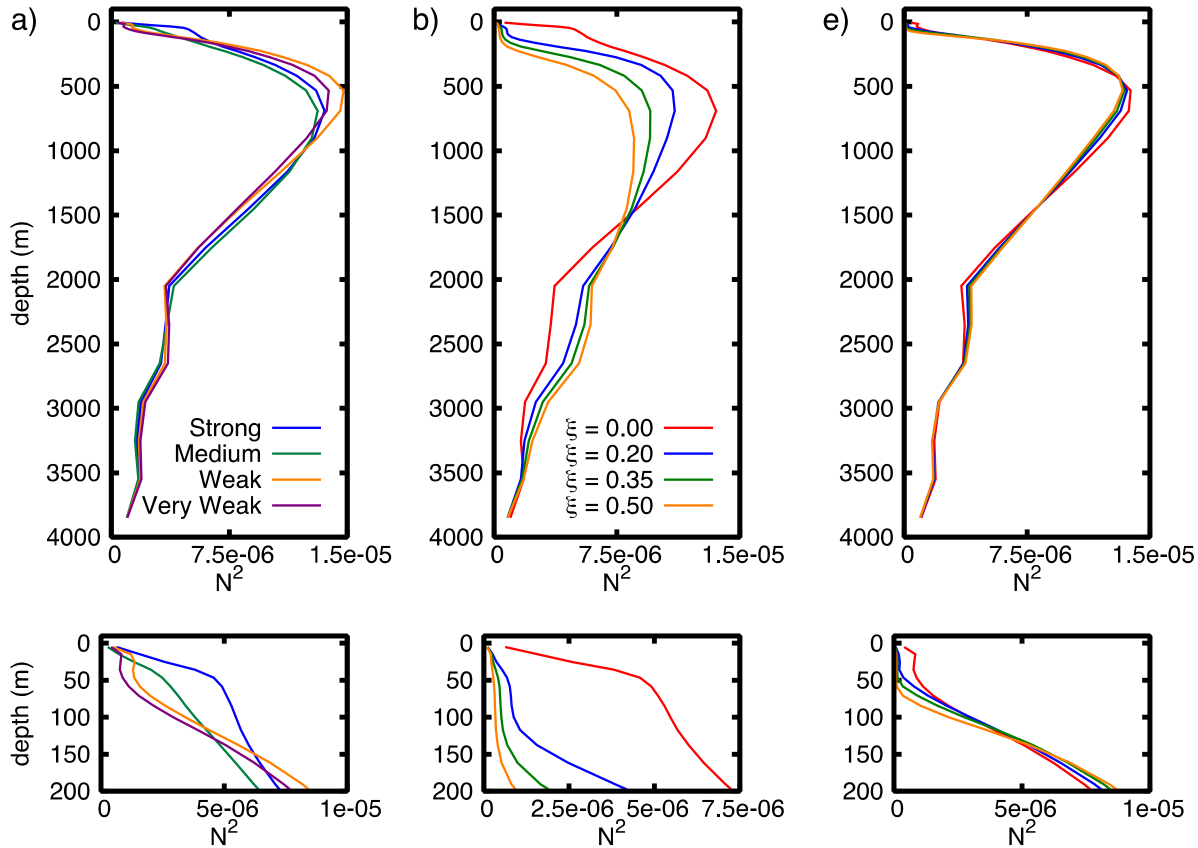
- Balanced flows are weakly unstable to unbalanced motion
  - How weak depends (loosely) on the base state Rossby number
- At equilibrium, balanced-to-unbalanced transfer depends on
  - The base state Rossby number *and* the saturation value of the unbalanced modes
  - For a given base state, more NI forcing implies higher saturation levels (and hence more transfer)

Conclusions (and short animation of surface current speed)

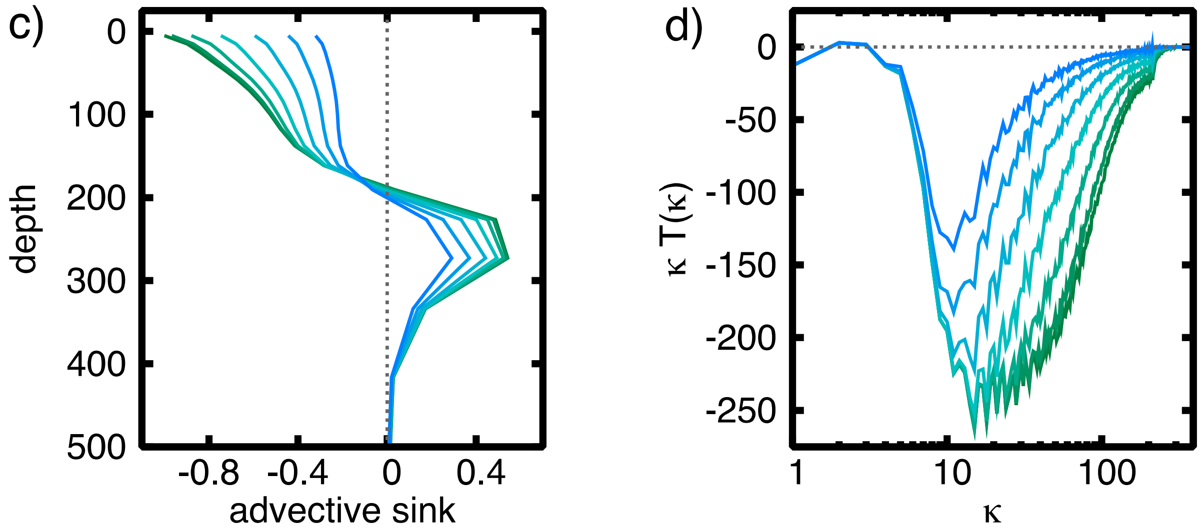
- Dominant sink of balanced flow probably bottom drag
- But some form of SLOB merits an honorable mention
- More work needed to determine which SLOB is the biggest
- NOTE: Forced NI motion also has other more mundane, but perhaps more important consequences



Changes in stratification due to application of NI forcing in the different cases



Stimulated Imbalance vs Direct Extraction
(Barkan et al.)



Left: vertical structure of the `advective sink' as a function of the cutoff defining the low passed flow

Right: Surface transfer spectra

Shown are results for the strong base state

High freq defined as periods greater than 2 days

Used different cutoffs to define the low-pass fields: 2,4,8,16,32,64,128 days

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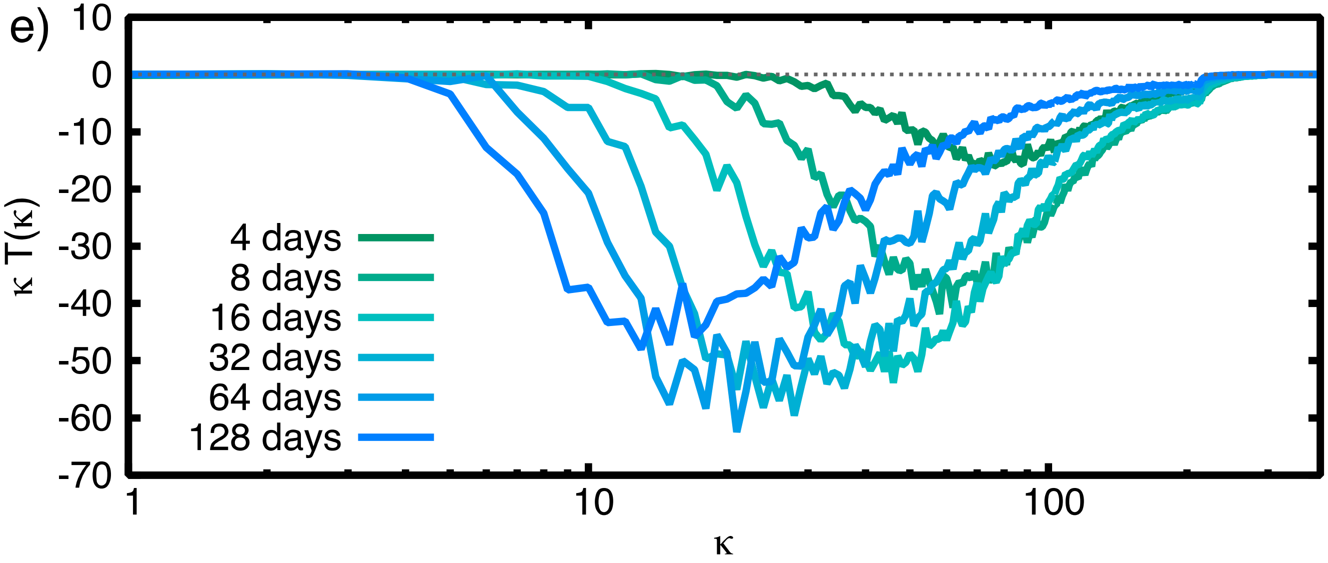

Portion of surface transfer spectrum coming from 2-4, 4-8, 8-16, 16-32, 32-64, and 64-128 day periods