Mathematical versus Physical/ Energetics constraints on ocean mixing parameterisations

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Outline

- On the direction of lateral stirring in the ocean and the physical origin of the “neutrality principle”
- Neutral-PV density surfaces and Lorenz reference density
- Is McDougall’s criticism of material surfaces valid?
- Why do we think we need rotated diffusion?

Motion appears to be quasi two-dimensional when written in isentropic coordinates z=ζ(x,y,η,t) if diabatic effects are negligible

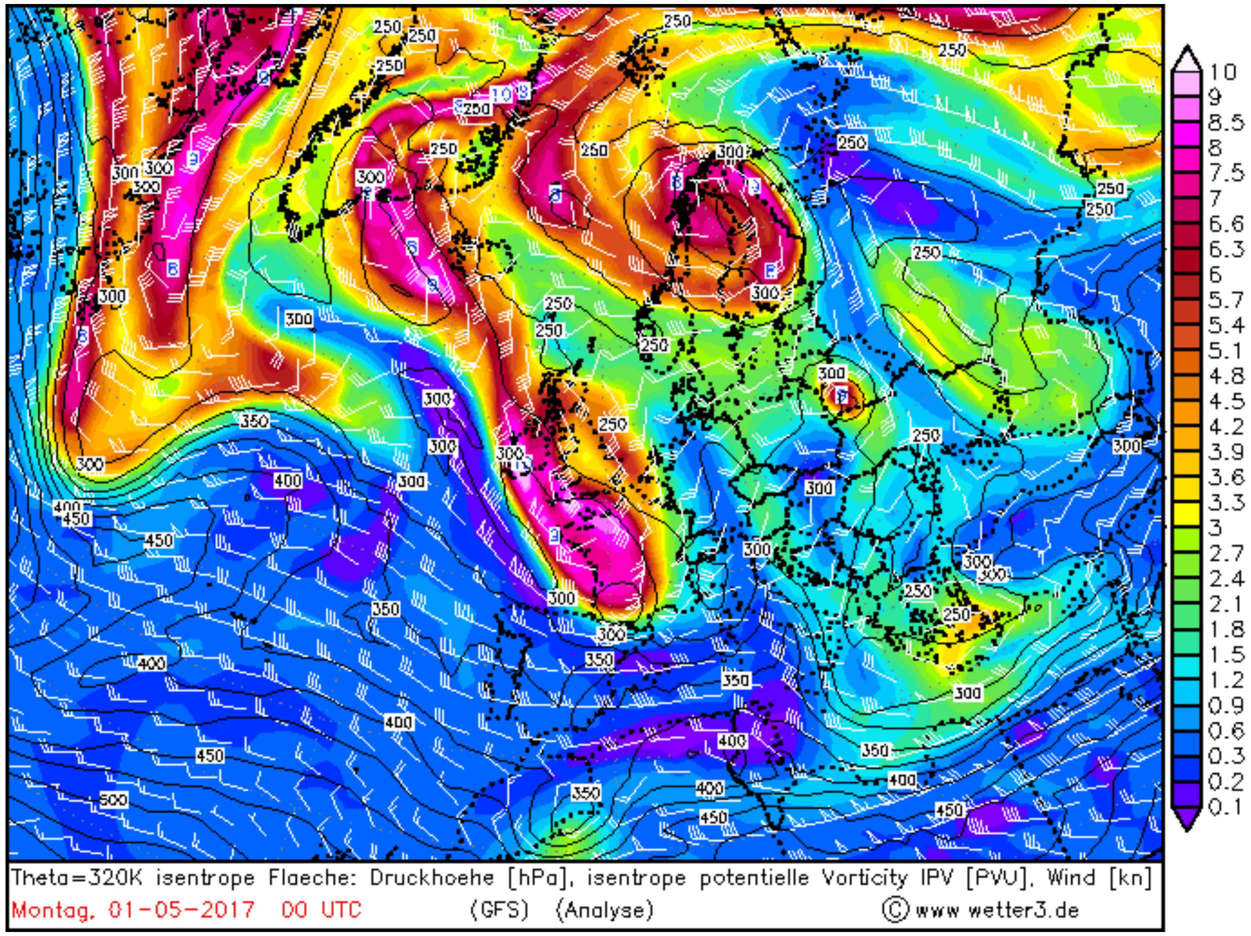
$$\frac{\partial C}{\partial t} + \mathbf{u} \bullet \nabla_{\eta} C + \cancel{\frac{D\eta}{Dt}\frac{\partial C}{\partial \eta}} \approx \kappa \nabla^2 C$$

Ertel Potential Vorticity key dynamical tracer conserved along isentropic surfaces in absence of viscous and diabatic effects

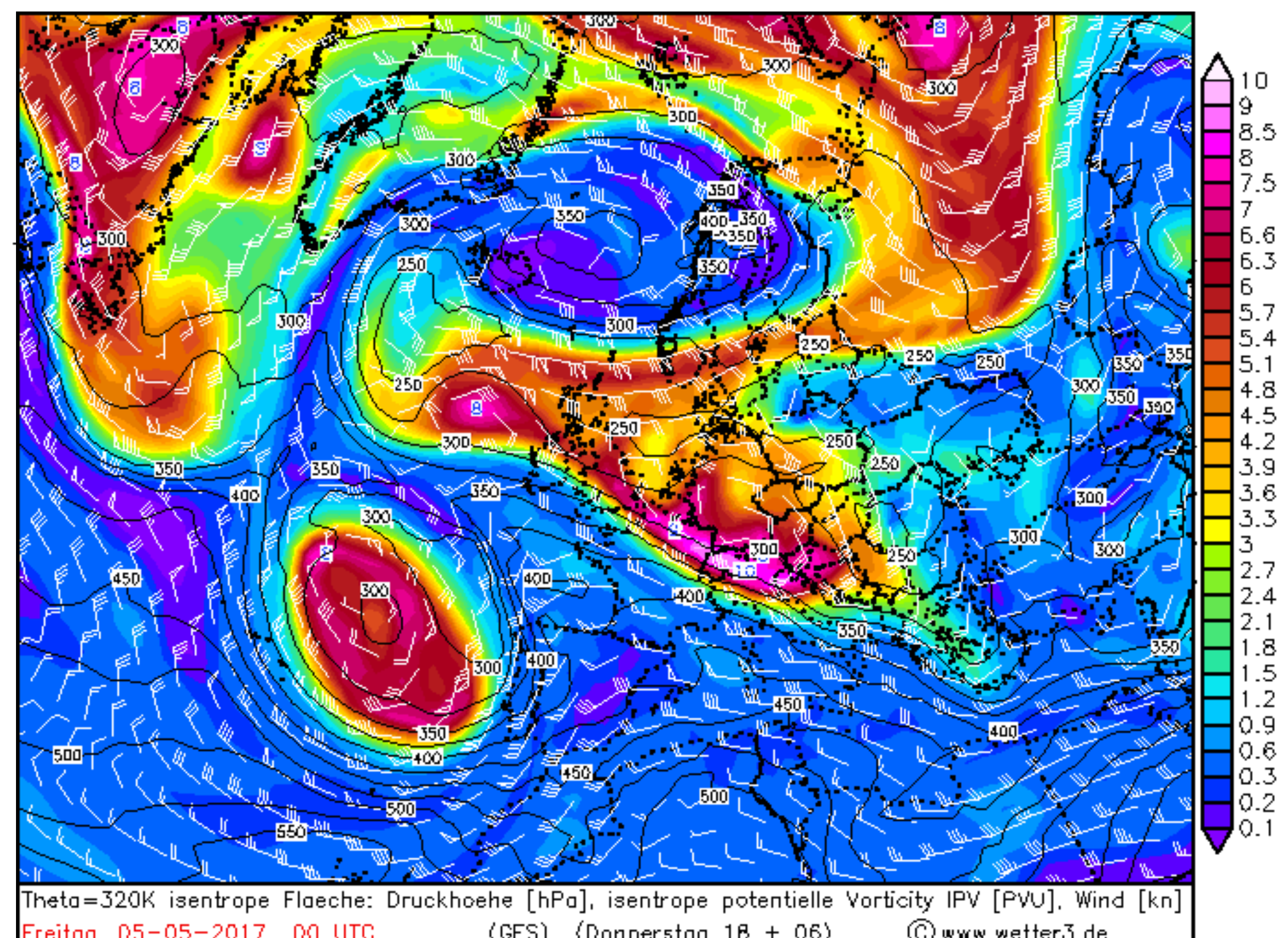
$$\Pi = \frac{\varpi_a \bullet \nabla \eta}{\rho}$$

Isentropic surfaces are generally regarded as associated with observed strong lateral stirring

Potential Vorticity on θ=320K Isentropic Surface



Monday 00 UTC



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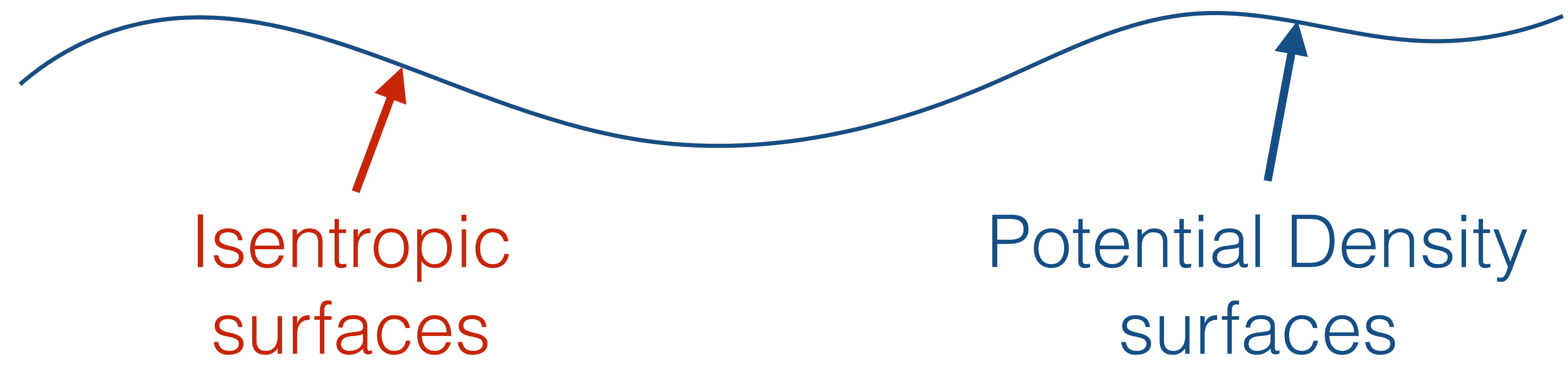
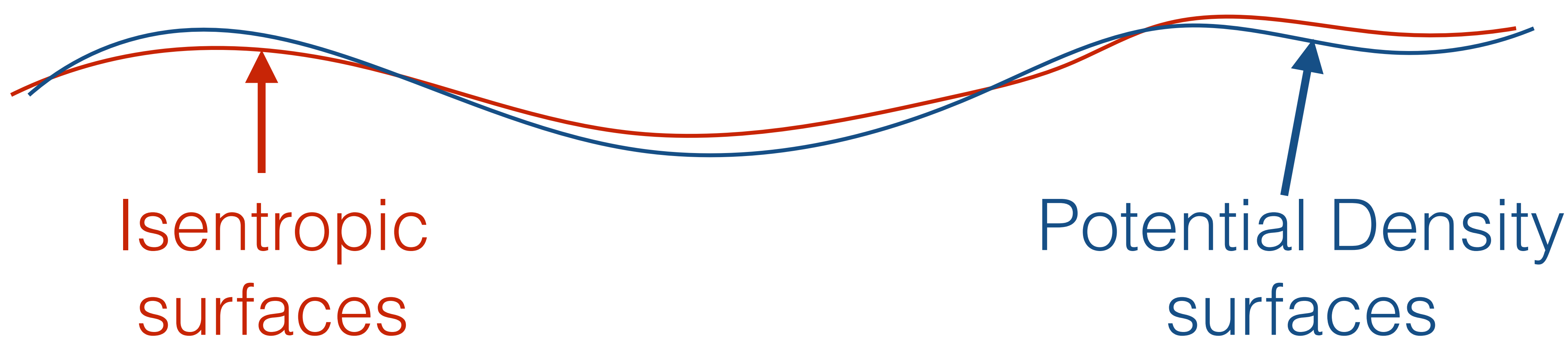
Simple Fluid: S(θ)-ocean or Moist Atmosphere



Complex Fluid: Thermobaric ocean without S(θ) relationship or Moist Atmosphere



From isentropic to Isopycnal Surfaces

potential density

patched potential density
Lynn and Reid (1968)

Material, Continuous
Poor neutrality,
Inversions

Discontinuous,
Non-Material,
improved neutrality

Neutral Surfaces
McDougall (1987)
Neutral Density
Jackett & McDougall (1997)

Orthobaric Density
de Szoeke & Springer (2000)

Material Neutral Surfaces

Eden & Willebrand (1999)

Definition of "Isentropic" Surfaces

$$\gamma(S,\theta) = \text{constant}$$

Definition of Potential Density Surfaces

$$\rho = \rho(S,\theta,p) = \rho^{*}(\gamma,\xi,p)$$ In-Situ Density

$$\rho^{*}(\gamma,\xi,p_r(\gamma)) = \text{constant}$$

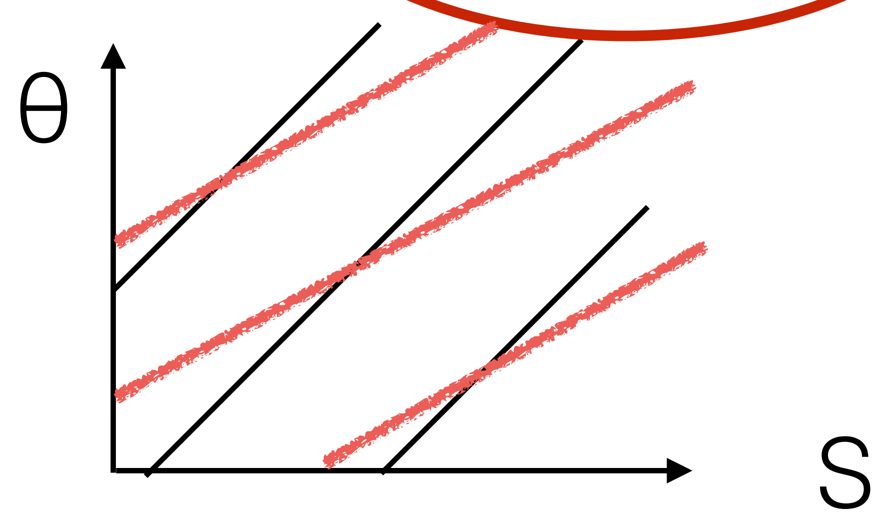
ξ(S,θ)="Spiciness" Variable   ξ(S,θ)=θ for concreteness

Unavoidable dependence on spiciness is what causes the difficulties -> Minimise this dependence

Implication for the Neutral Vector

$$\mathbf{N} = -\frac{g}{\rho}\left(\frac{\partial \rho^*}{\partial \gamma}\nabla\gamma + \frac{\partial \rho^*}{\partial \xi}\nabla\xi\right)$$

$$\frac{\partial \rho^*}{\partial \gamma} = \frac{1}{J}\frac{\partial(\rho,\xi)}{\partial(S,\theta)} \qquad \frac{\partial \rho^*}{\partial \xi} = \frac{1}{J}\frac{\partial(\gamma,\rho)}{\partial(S,\theta)} \qquad J = \frac{\partial(\gamma,\xi)}{\partial(S,\theta)}$$



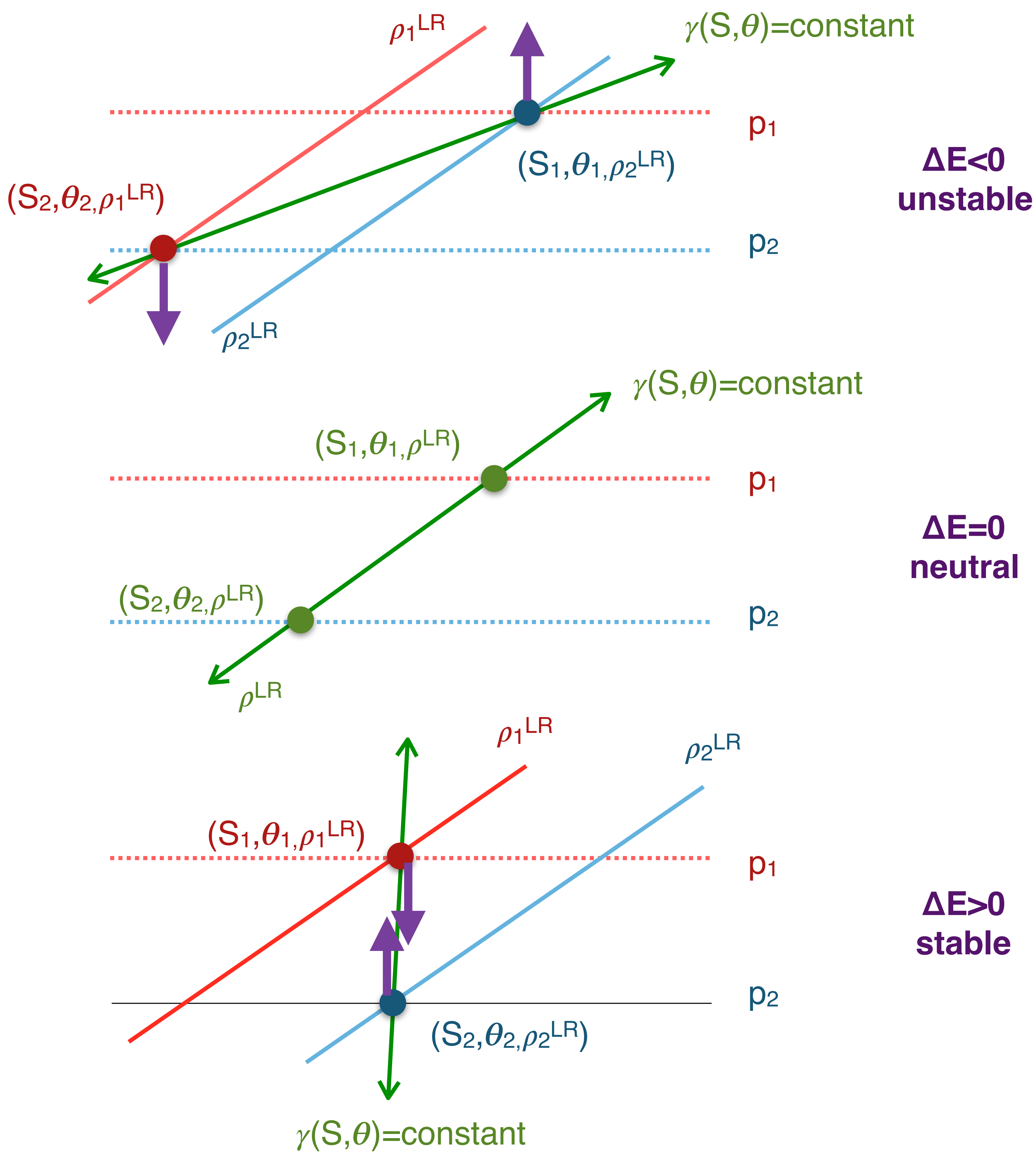
Minimising mismatch between isentropic and potential density surfaces equivalent to maximising neutrality

$$\frac{\partial(\gamma,\rho)}{\partial(S,\theta)} \approx 0$$

Solution is well known locally-referenced potential density

$$\gamma^L(S,\theta) = \rho(S,\theta,p_r(S,\theta)) \qquad \left|p_r(S,\theta) - p\right| \ll 1$$

Note that solution does not need to look like in-situ density at all



Energy cost of adiabatic and isohaline parcel exchanges

Tailleux (2016a)

Energy cost of Adiabatic and Isohaline Parcels exchange (Haine and Marshall, 1998; Olbers et al. 2012, Vallis 2006)

$$\Delta E \approx \frac{1}{\rho^2}\left(\frac{\partial \gamma}{\partial S}\right)^{-1}\frac{\partial(\gamma,\rho)}{\partial(S,\theta)}\Delta\theta\,\Delta p$$

Iso-γ Temperature and Pressure differences

Maximising neutrality = Minimising |ΔE| for finite Δp Δθ

Similar to McDougall (1987) neutrality

Potential Vorticity

$$\Pi = \frac{\varpi_a \bullet \nabla \gamma}{\rho}$$

$$\frac{D\Pi}{Dt} = \frac{1}{\rho^3} \frac{\partial \rho^*}{\partial \xi} \nabla_\gamma p \bullet \left(\nabla \gamma \times \nabla_\gamma \xi \right)$$

Maximising Neutrality minimises thermobaric production of Potential Vorticity, makes it as Lagrangian as possible

Similarities with McDougall (1995)

Neutral PV Surfaces

Construct material density variables that maximises neutrality, minimises the absolute value of the energy cost of adiabatic and isohaline parcel exchanges, and minimises the thermobaric production of Potential Vorticity

Support Eden and Willebrand (1999)'s approach to constructing neutral surfaces. Unlike EW99's construction in physical space, present construction can be done in thermodynamic space

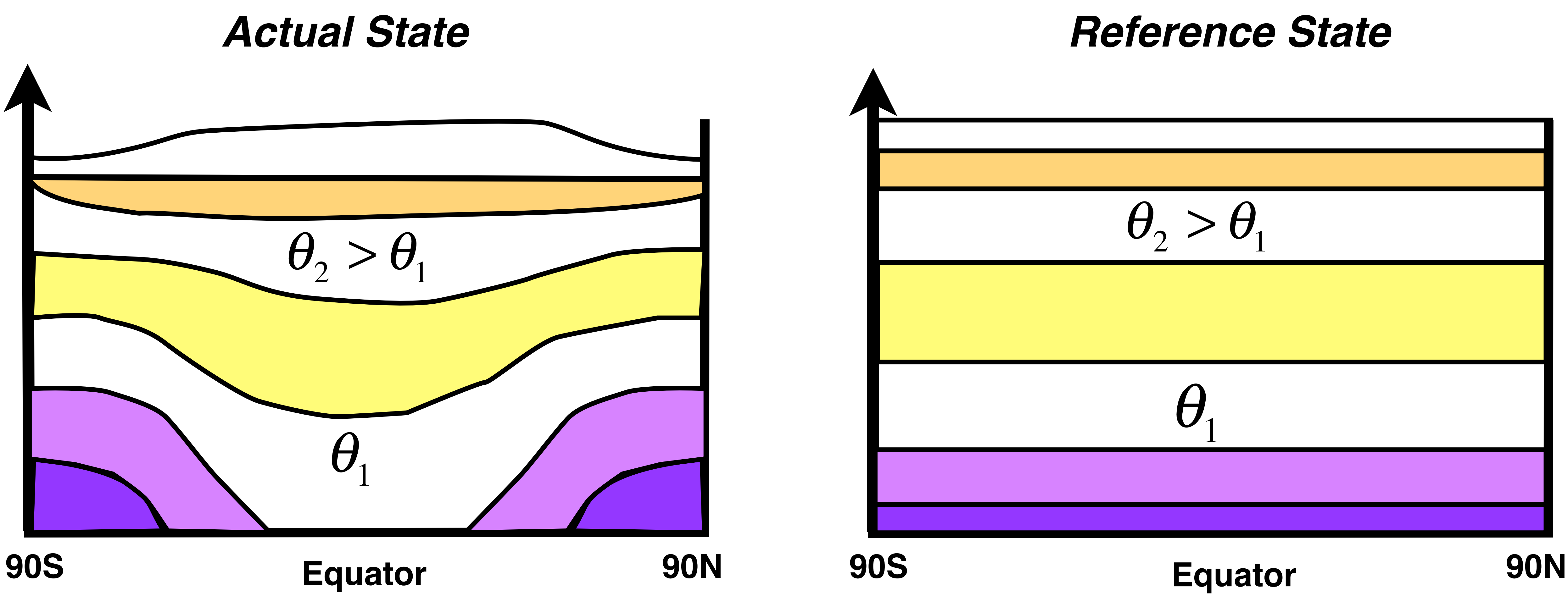
Lorenz's Reference State of Minimum Potential Energy

is obtained from adiabatic rearrangement of mass

= Measure of "heat" of the fluid.

Can only evolve as a the result of diabatic processes

First used by Winters et al. (1995) to diagnose mixing



Probability Distribution Function (P.D.F.) Space

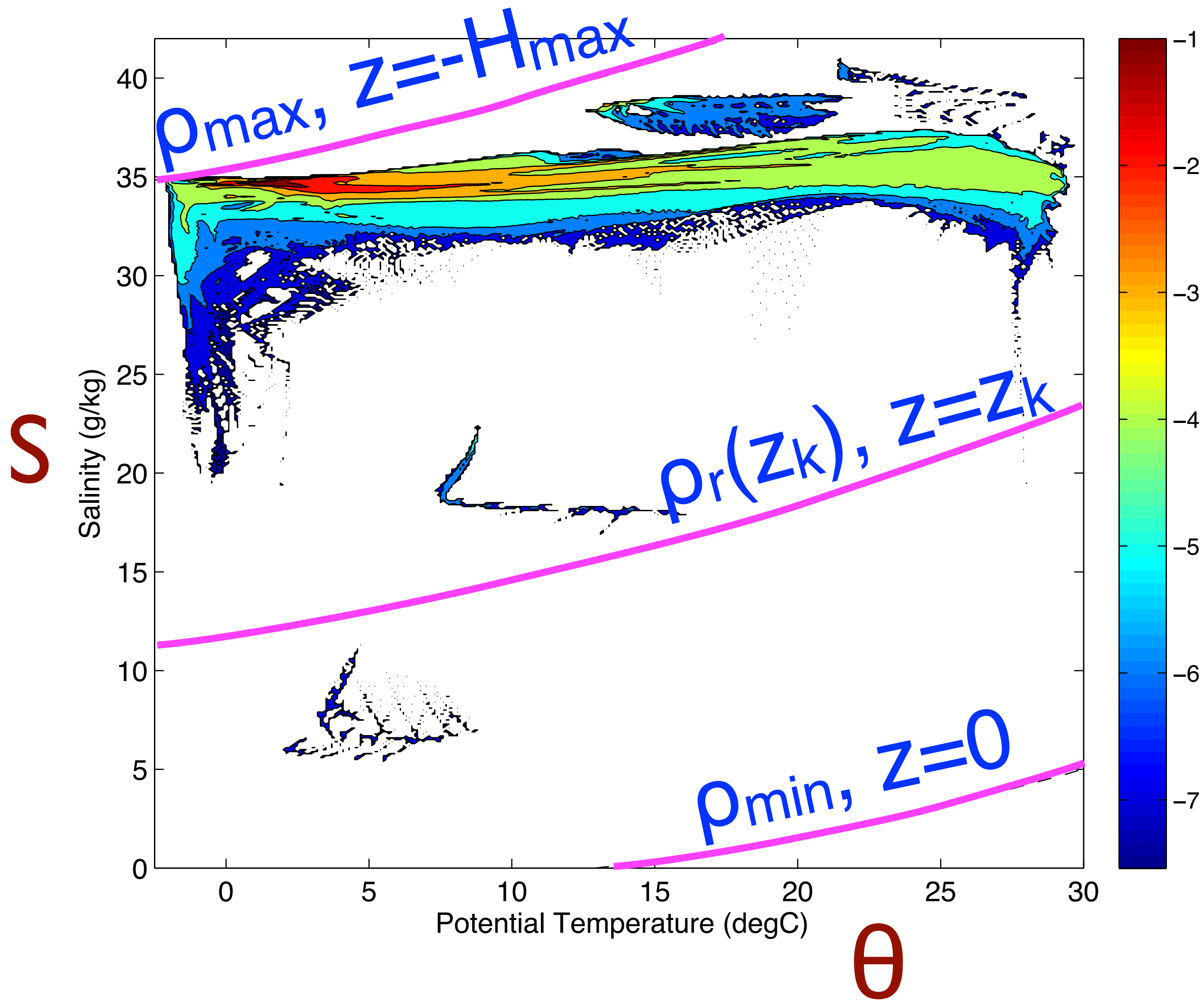
Saenz et al., (JPO, 2015)

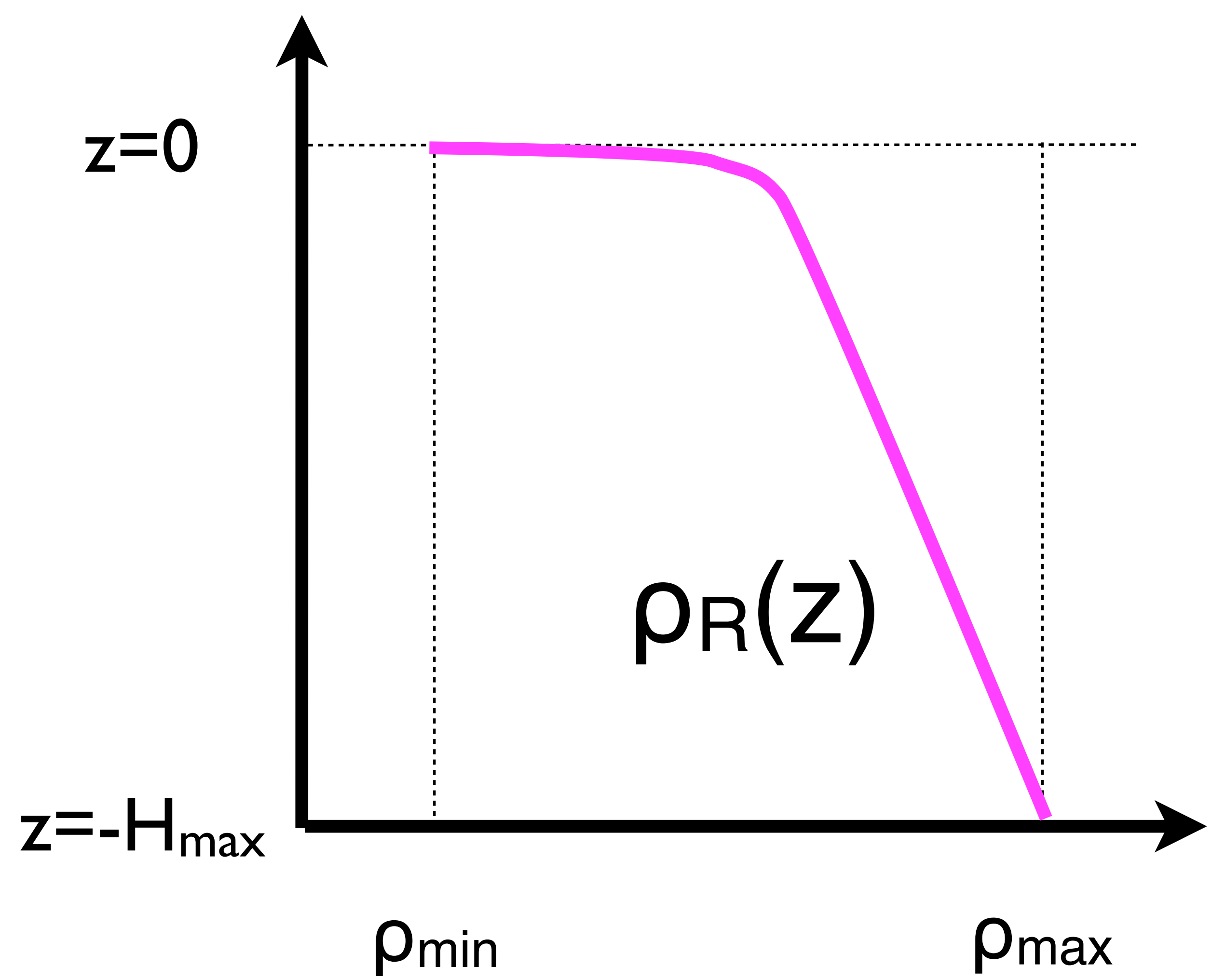
Maps onto

Physical Space

State of minimum potential energy

Link to Lorenz APE theory

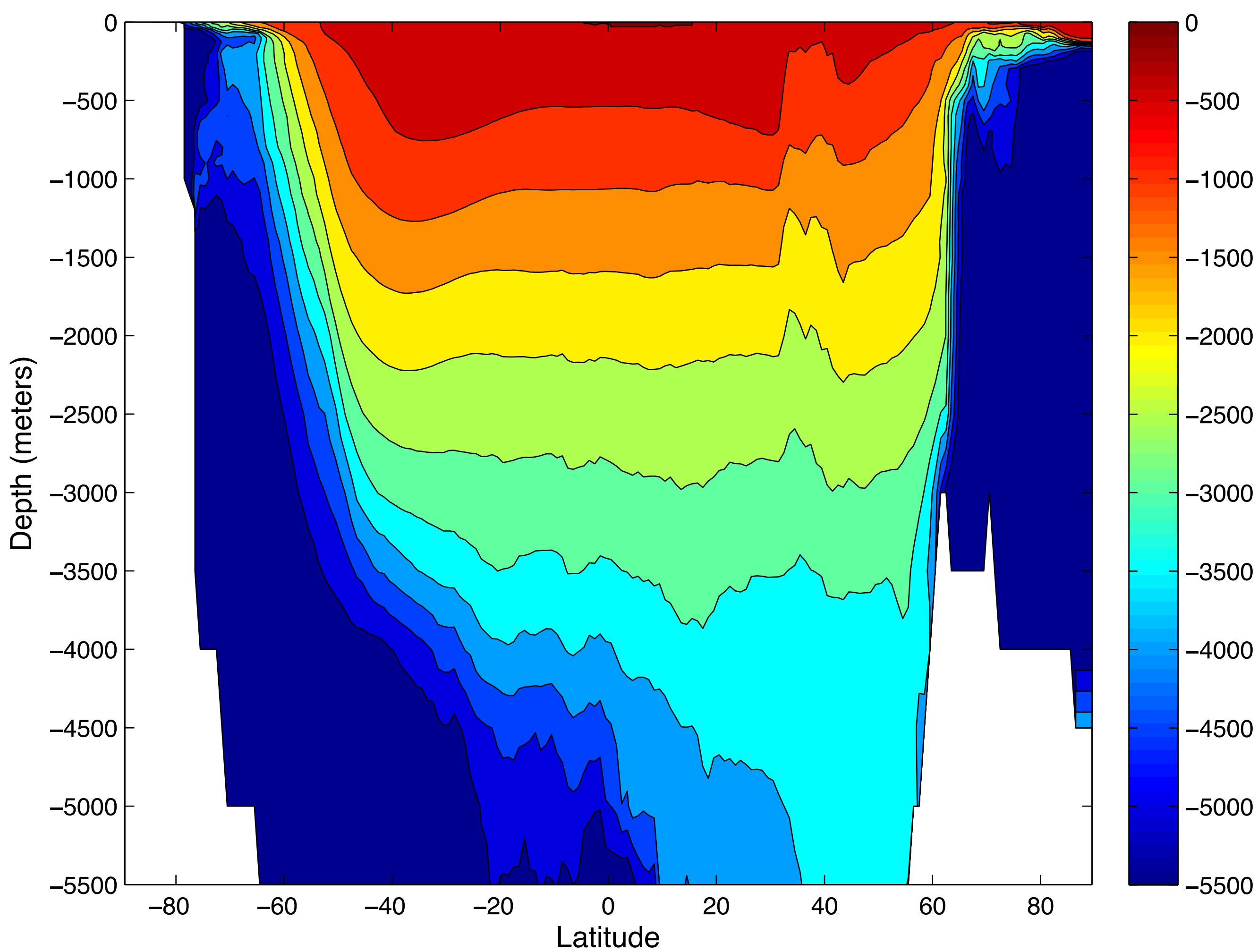




$$\int_{S_{\min}}^{S_{\max}} \int_{\theta_{\min}}^{\theta_{\max}} \pi(S,\theta)\,\mathrm{d}S\mathrm{d}\theta = 1$$

Level of Neutral Buoyancy (LNB) = Reference Level

Tailleux (2013, JFM)

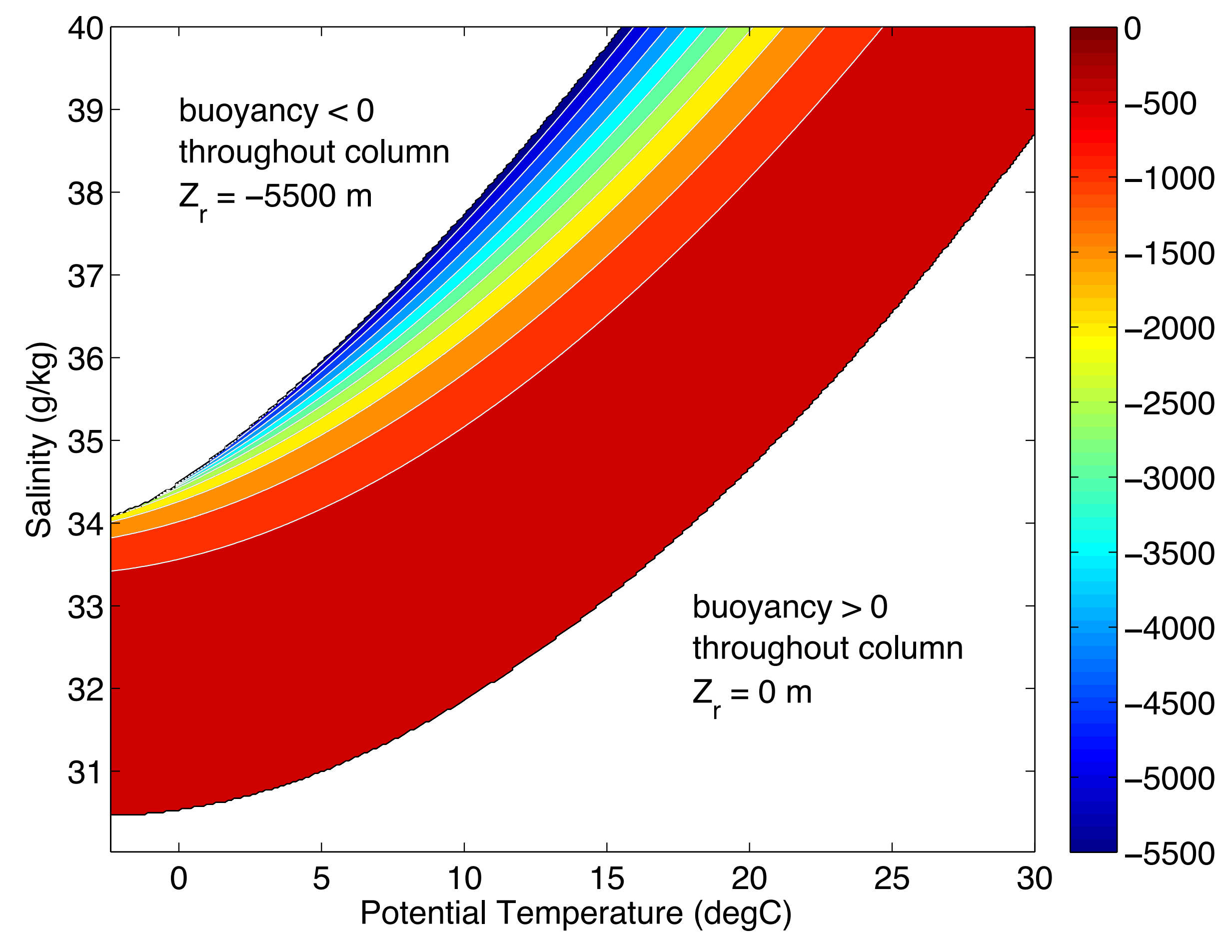


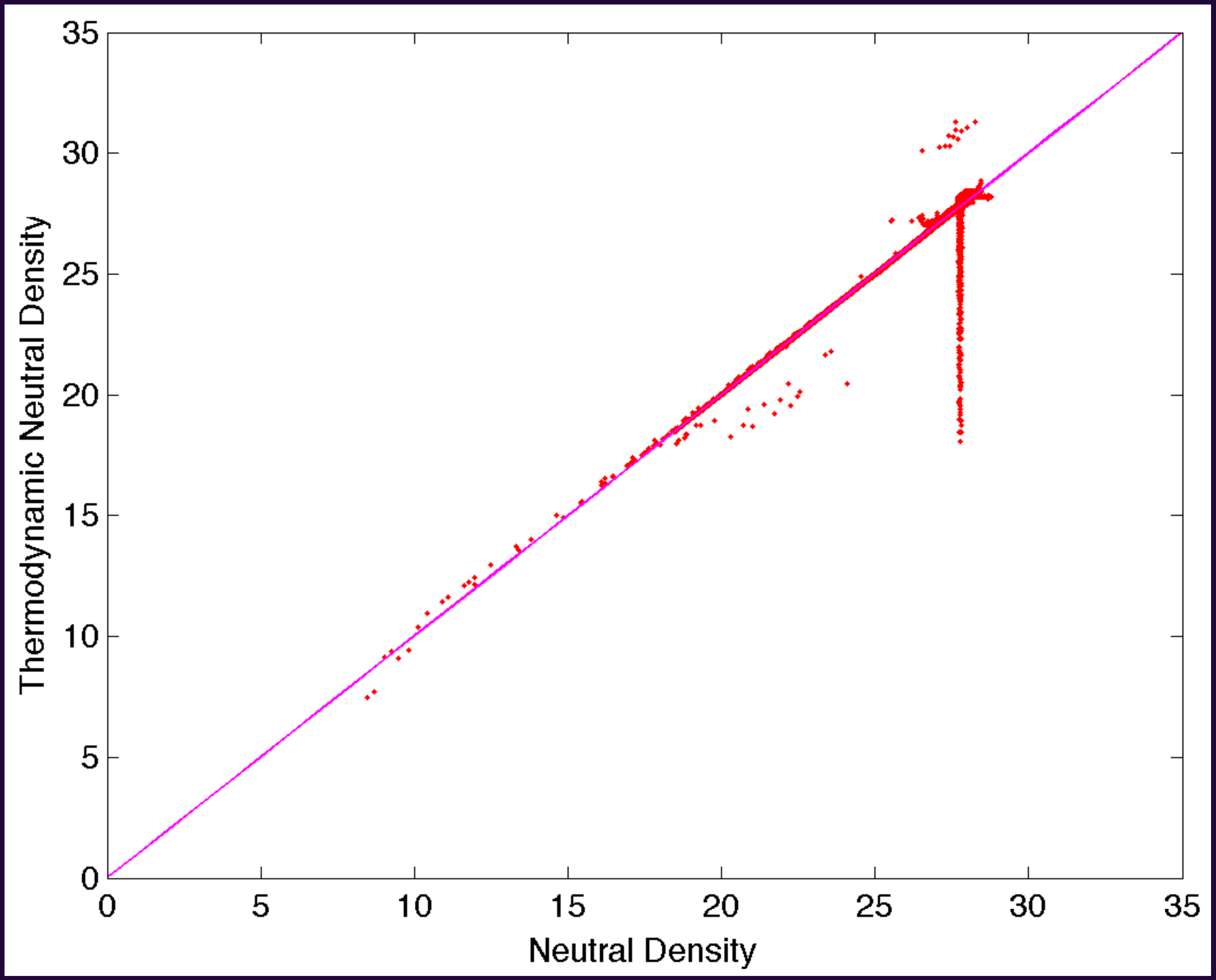
Implicit Solution of LNB equation

$$\rho\big(\theta,S,Z_R(\theta,S)\big)=\rho_R\big(Z_R(\theta,S)\big)$$

World Ocean Atlas 2009

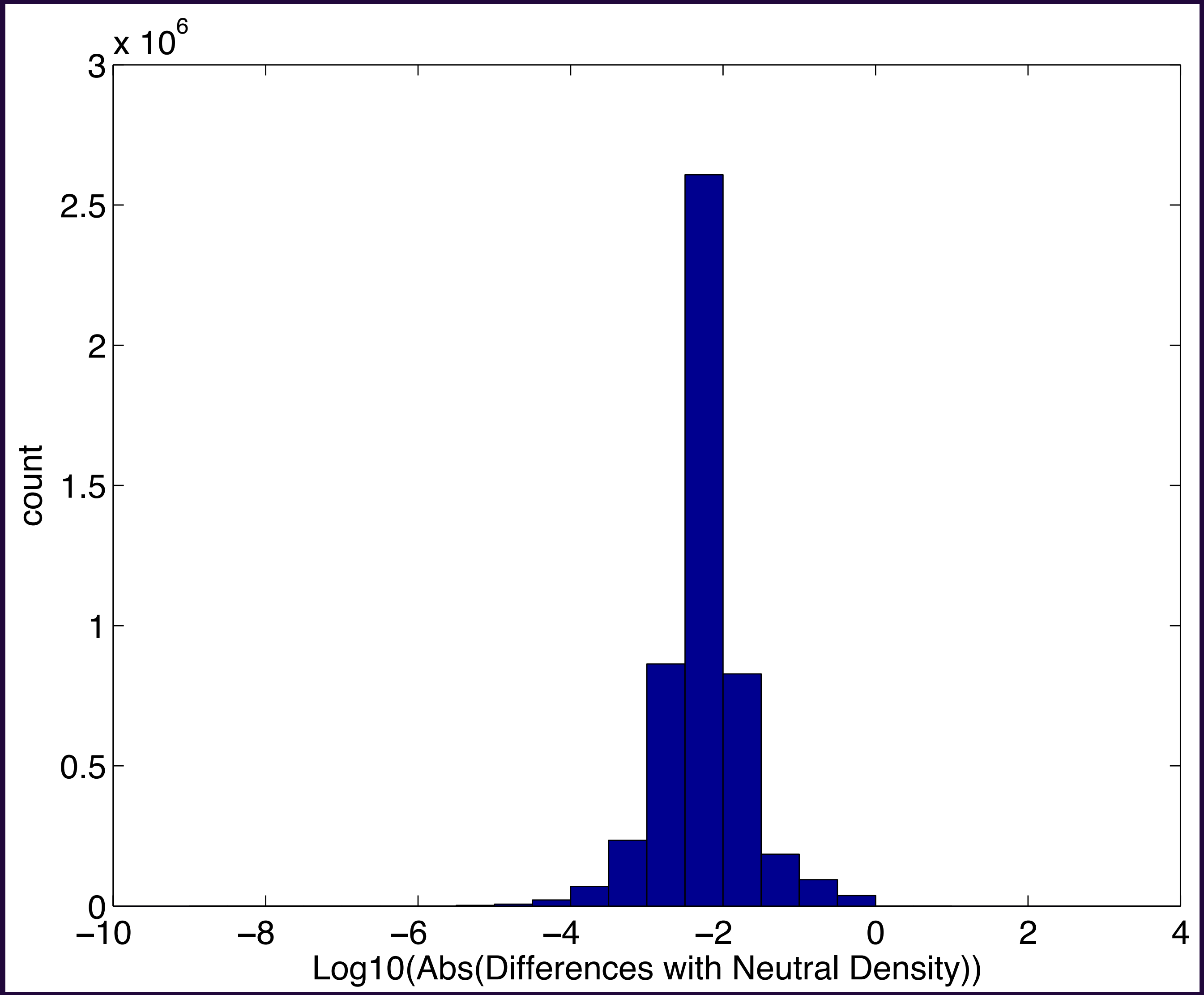
As a function of and S and Θ



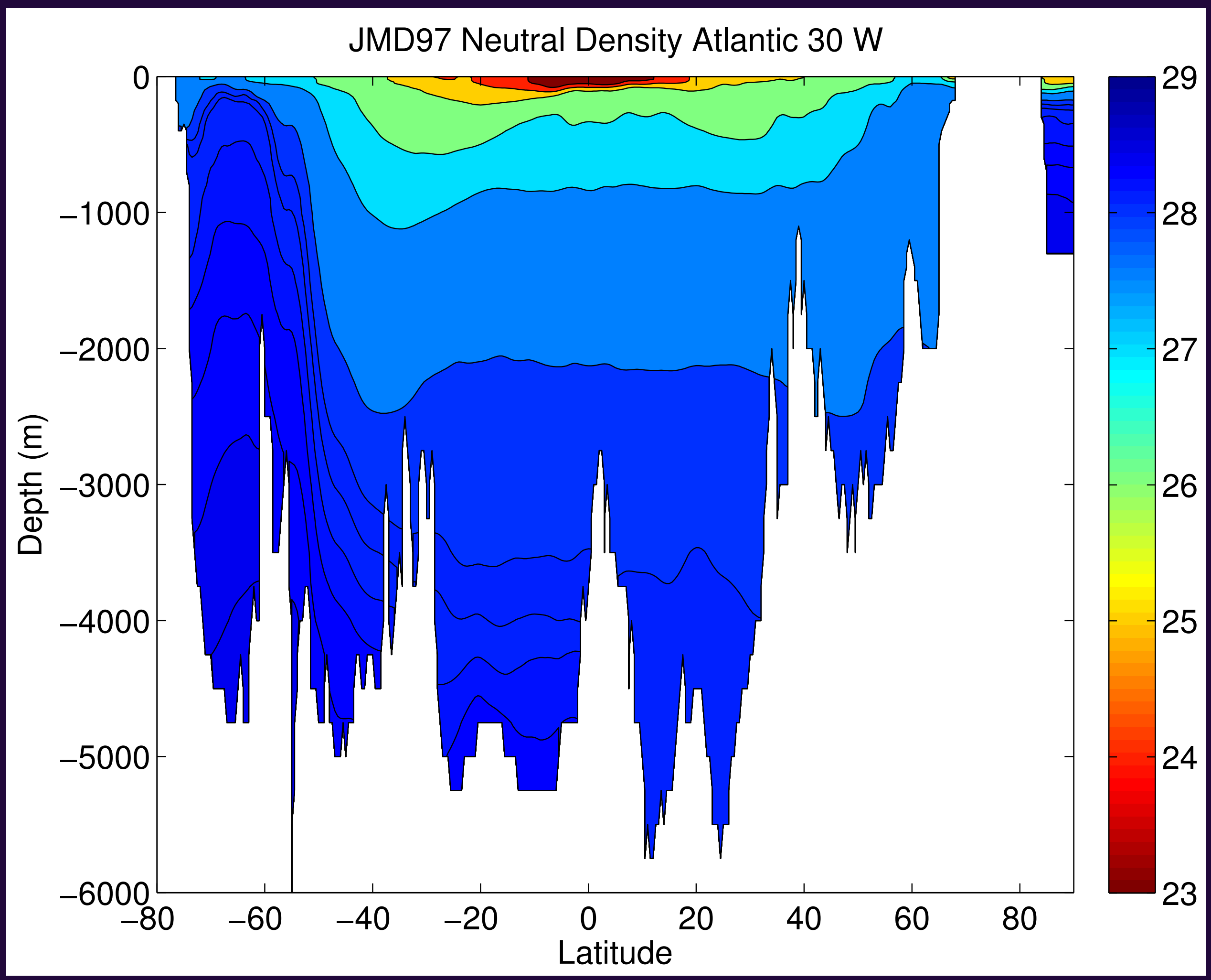


Neutral density strongly correlated to thermodynamic neutral density almost everywhere

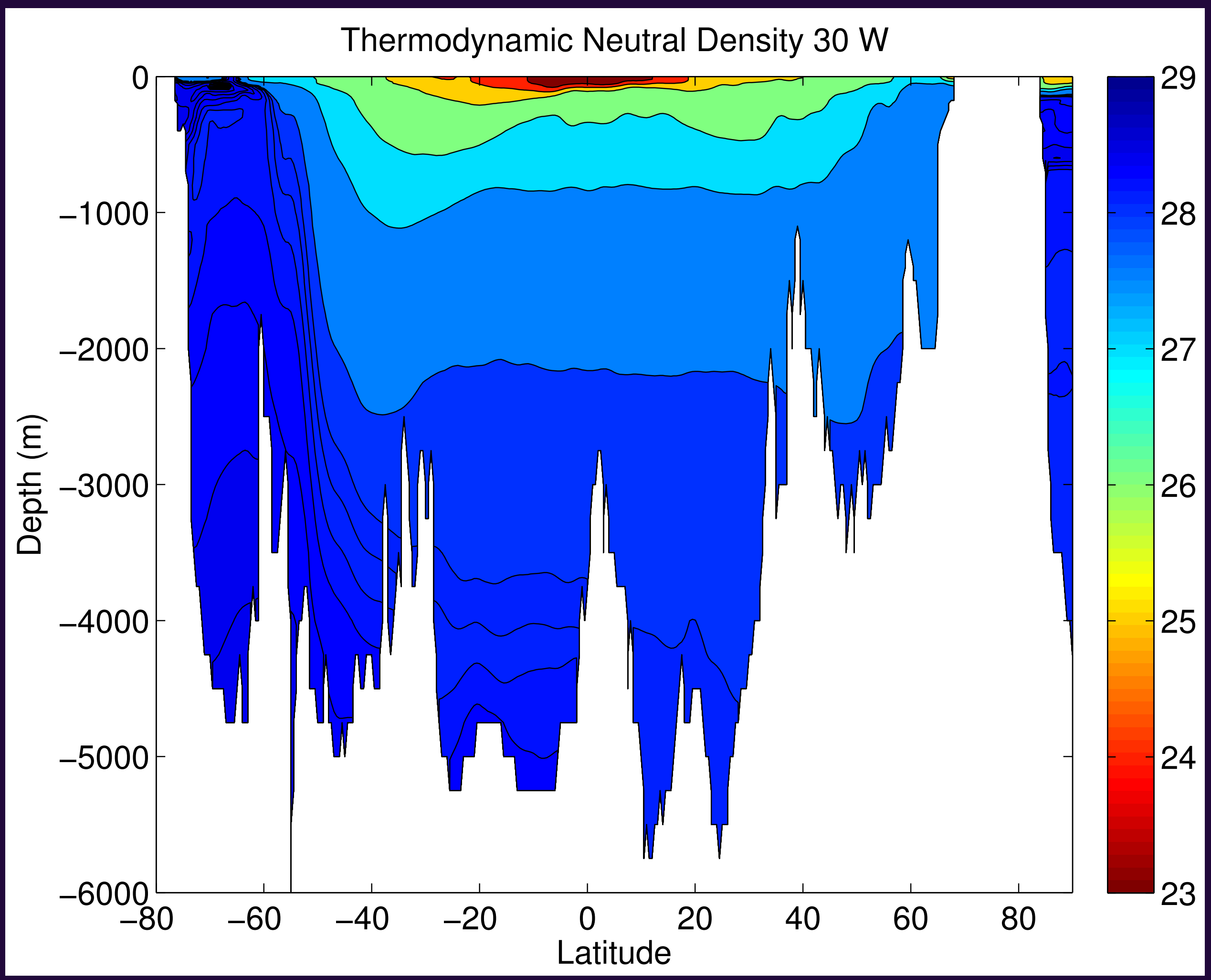
Differences between two variables less than 0.01 kg/m^3 almost everywhere



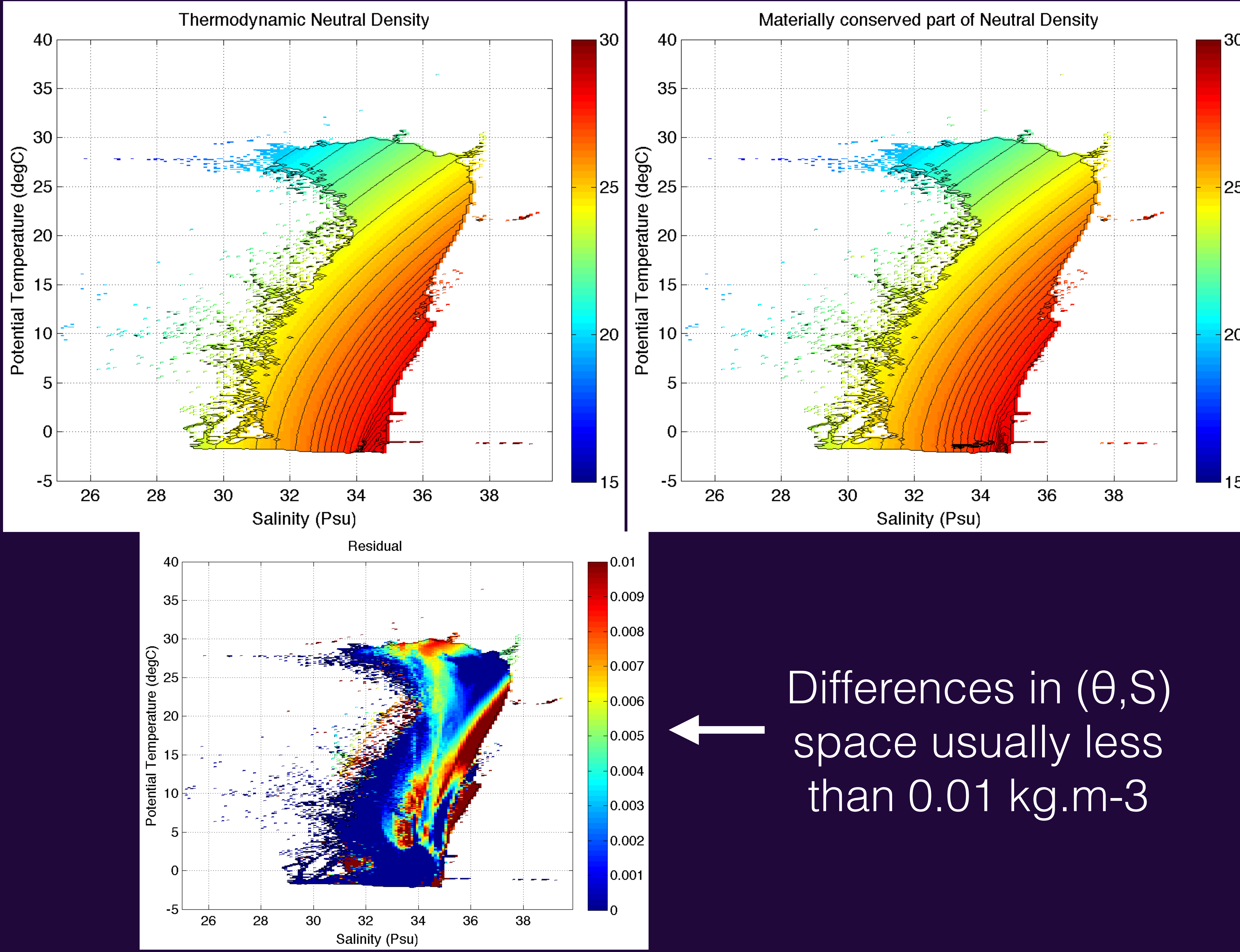
Jackett & McDougall (1997)
Neutral Density
Atlantic WOCE 30W



Thermodynamic
Neutral Density
Atlantic WOCE 30W

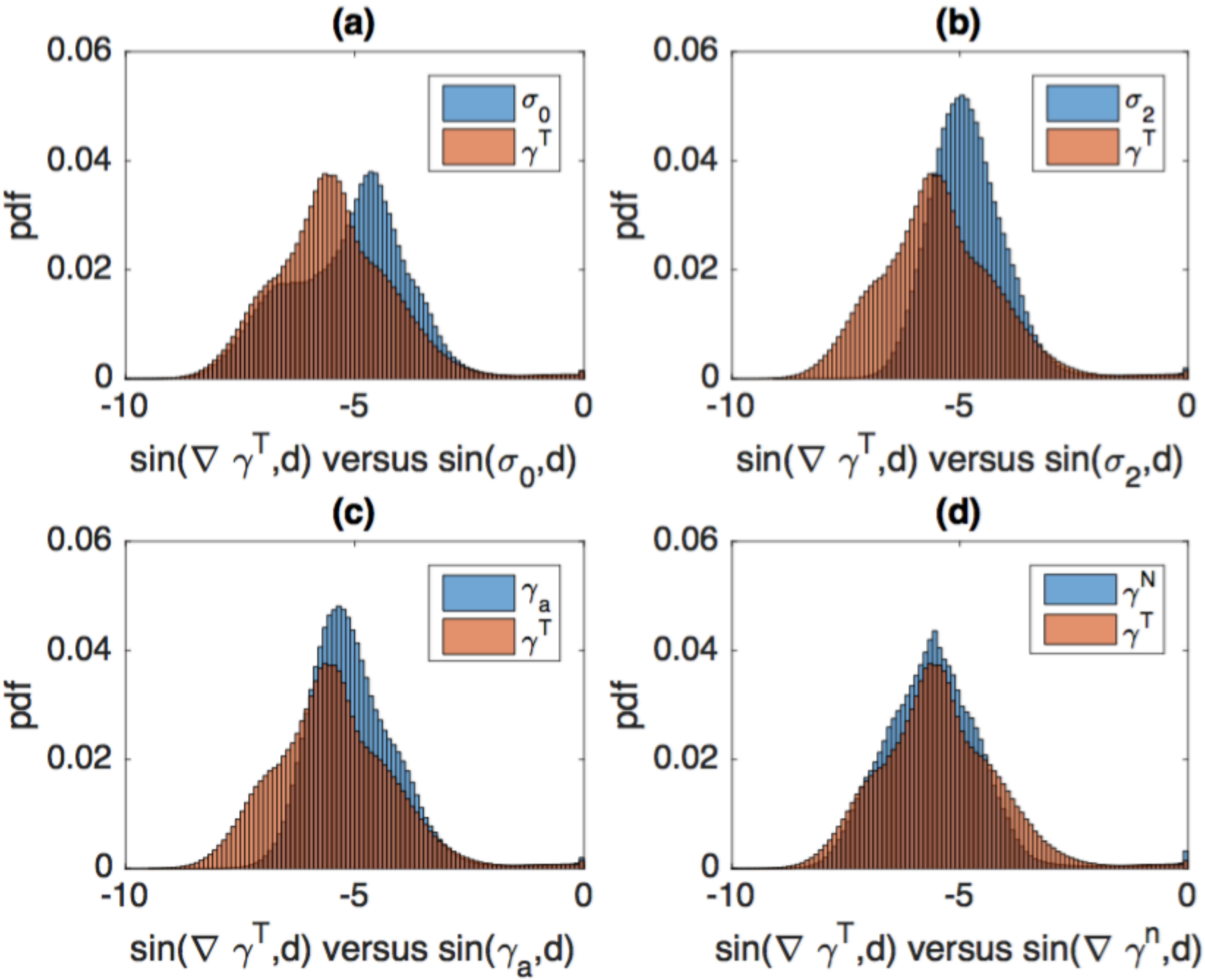


$f(p_r(S,\theta))$ constructed to minimise difference with Jackett and McDougall (1997) neutral density on WOCE dataset



Differences in (θ,S) space usually less than 0.01 kg.m-3

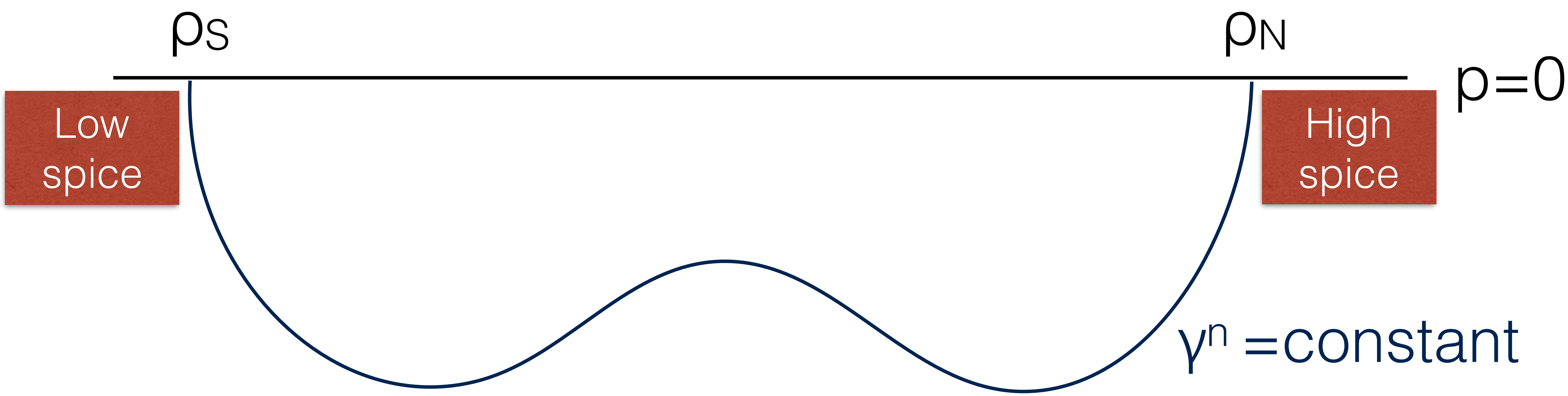
Neutrality performances



McDougall's criticism of purely material density variables

- North-South Density Differences of outcropping isopycnal surfaces
- Representation of thermobaric effects
- Fictitious diapycnal mixing

North-South In-Situ Density differences

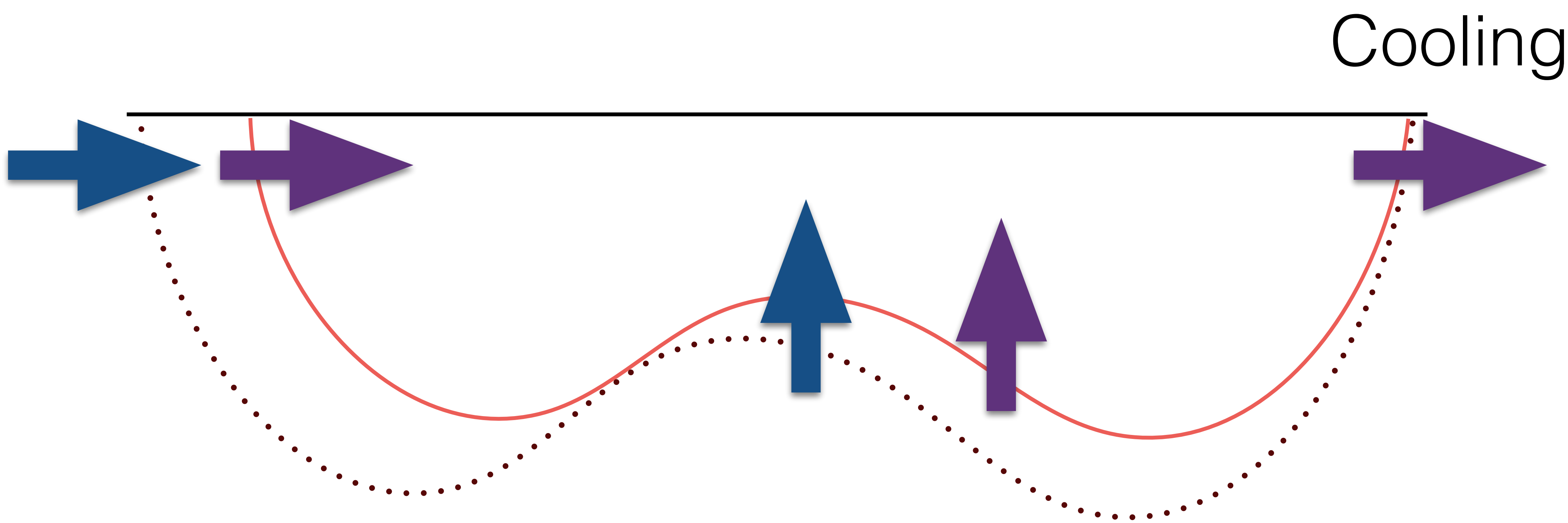


McDougall & Jackett (2005) assert that material density variables cannot exhibit North-South density differences, unlike γ^n or Pached Potential Density

$$\rho_N - \rho_S = \rho^*(\gamma, \xi_N, 0) - \rho^*(\gamma, \xi_S, 0) \neq 0$$

FALSE: Non-zero density difference controlled by North-South contrast in spiciness

North-South In-Situ Density differences (ct'd)



Important to test "adiabatic" theories of the Atlantic Meridional Overturning Circulation (e.g., Wolfe and Cessi, 2011)

Different constructions will a priori give more importance to Southern Ocean compared to interior diapycnal mixing to balance the northern light-to-dense water mass conversion. Maximising neutrality = Minimising interior mixing?

Representation of Thermobaric Effects

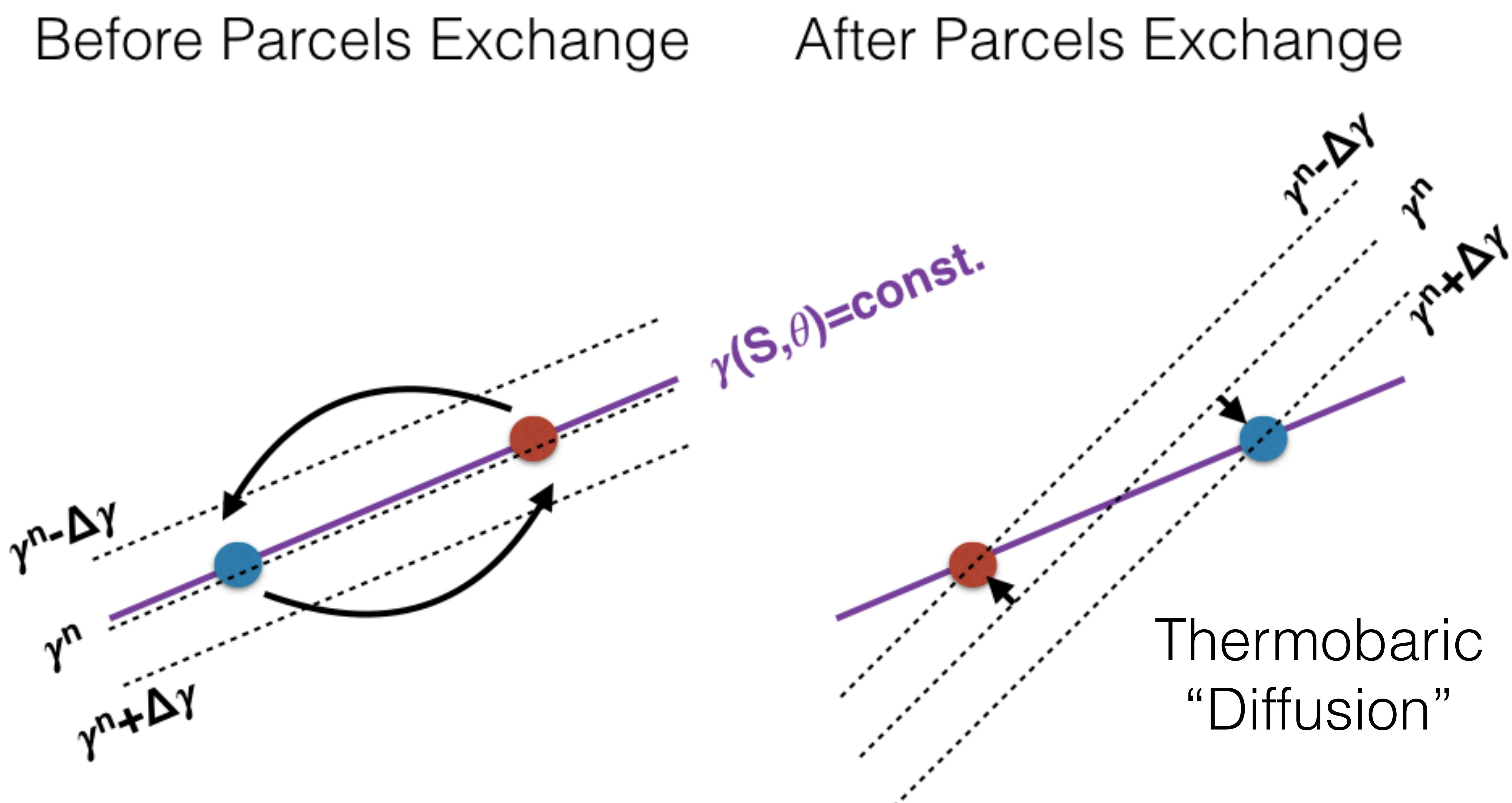
McDougall et al. (2017) assert that material functions cannot represent thermobaric effects, unlike γ^n

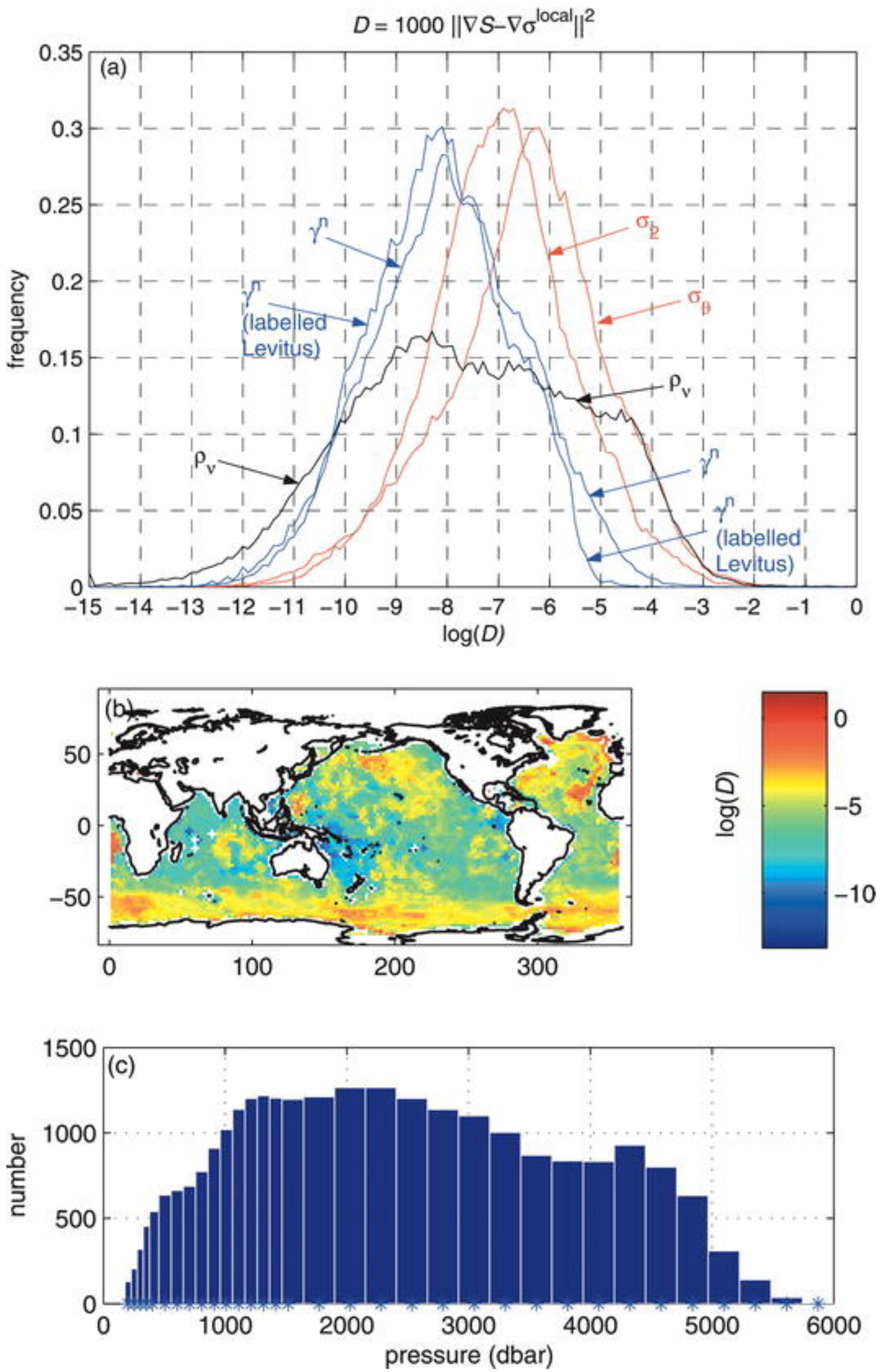
True for standard potential density variables

$$\frac{D\sigma_2}{Dt} = -\rho_0 \alpha_2 \frac{D\theta}{Dt} + \rho_0 \beta_2 \frac{DS}{Dt}$$

Not true for Lorenz Reference Density $\rho_{LZ}(S,\theta) = \rho(S,\theta,p_r(S,\theta))$

$$\frac{D\rho_{LZ}}{Dt} = -\rho_0 \alpha_R \frac{D\theta}{Dt} + \rho_0 \beta_R \frac{DS}{Dt} + \frac{1}{c_R^2} \frac{Dp_R}{Dt}$$


Before Parcels Exchange
After Parcels Exchange
γ(S,θ)=const.
γn-Δγ
γn
γn+Δγ
γn-Δγ
γn
γn+Δγ
Thermobaric
"Diffusion"



McDougall and Jackett (2005) argue that departure from neutrality induces fictitious mixing

$$K_{fictitious} = K_i \sin^2(\nabla\gamma, \mathbf{N})$$

Related to so-called Veronis effect, responsible for spurious upwelling in western boundary currents of early OGCMs with no GM, mixing horizontally/vertically

The Inversion Problem

$$-F_\theta = \psi_{eddy} \times \nabla\theta + \left[K_i \left(\mathbf{I} - \mathbf{dd}^T \right) + K_{T\gamma} \mathbf{dd}^T \right] \nabla\theta + \mathrm{Gauge}_\theta$$

$$-F_S = \psi_{eddy} \times \nabla S + \left[K_i \left(\mathbf{I} - \mathbf{dd}^T \right) + K_{S\gamma} \mathbf{dd}^T \right] \nabla S + \mathrm{Gauge}_S$$

Mathematically, the mesoscale eddy potential, and all three diffusion coefficients can be determined uniquely from knowledge of the fluxes and of the Gauge terms

Gauge terms can be obtained from solving two global elliptic problems, e.g., Roberts and Marshall (2000), or following Eden et al. (2007)

The Inversion Problem (ct'd)

$$-F_\theta = \psi_{eddy} \times \nabla\theta + \left[K_i \left(\mathbf{I} - \mathbf{dd}^T \right) + K_{T\gamma}\mathbf{dd}^T \right] \nabla\theta + \mathrm{Gauge}_\theta$$

$$-F_S = \psi_{eddy} \times \nabla S + \left[K_i \left(\mathbf{I} - \mathbf{dd}^T \right) + K_{S\gamma}\mathbf{dd}^T \right] \nabla S + \mathrm{Gauge}_S$$

A priori, inversion can be done for any arbitrary direction, but in general requires different diffusivities for heat and salt. Imposing equal T/S diffusivities makes problem ill-posed.

Inverting the problem yields mesoscale eddy velocity potential with more terms than in classical GM, e.g., Eden et al. (2007)

The Inversion Problem for diffusivities

Production of Temperature Variance

$$-F_\theta^* \bullet \nabla\theta = K_i \left|\nabla_i \theta\right|^2 + K_{T\gamma} \,|\, \nabla_d \theta \,|^2$$

Production of Salinity Variance

$$-F_S^* \bullet \nabla S = K_i \left|\nabla_i S\right|^2 + K_{S\gamma} \,|\, \nabla_d S \,|^2$$

Production of T/S Covariance

$$-F_S^* \bullet \nabla\theta - F_\theta^* \bullet \nabla S = 2K_i \nabla_i S \bullet \nabla_i \theta + (K_{T\gamma} + K_{S\gamma}) \nabla_d S \nabla_d \theta$$

Rotated Diffusion Tensors

- Rotated diffusion tensors do not require the use of neutral directions. However, diffusivities used depend on the choice of isopycnal/diapycnal direction, and a priori must be different for T & S.
- A priori, there is no harm in mixing horizontally/vertically if done right. Rotated diffusion is therefore not a solution to the Veronis effect. Supported by Boning et al. (1995), Lazar et al. (1999), Huck et al. (1999) suggesting that it is GM that cures Veronis effect.

Conclusions

- Neutral-PV surfaces defined to be material surfaces that minimises mismatch between isentropic and potential density surfaces. Clarifies physics of the “neutrality principle” and of “locally-referenced potential density”

- Lorenz reference density naturally appears to be the best neutral-PV surface constructed so far

- McDougall’s criticism against material density variables does not appear to be valid

- A priori no requirement to use neutral directions in rotated diffusion tensors. Mixing horizontally/vertically is fine if done properly. Rotated diffusion cannot cure Veronis effect.