A proof of concept for scale-adaptive parameterizations: the case of the Lorenz '96 model

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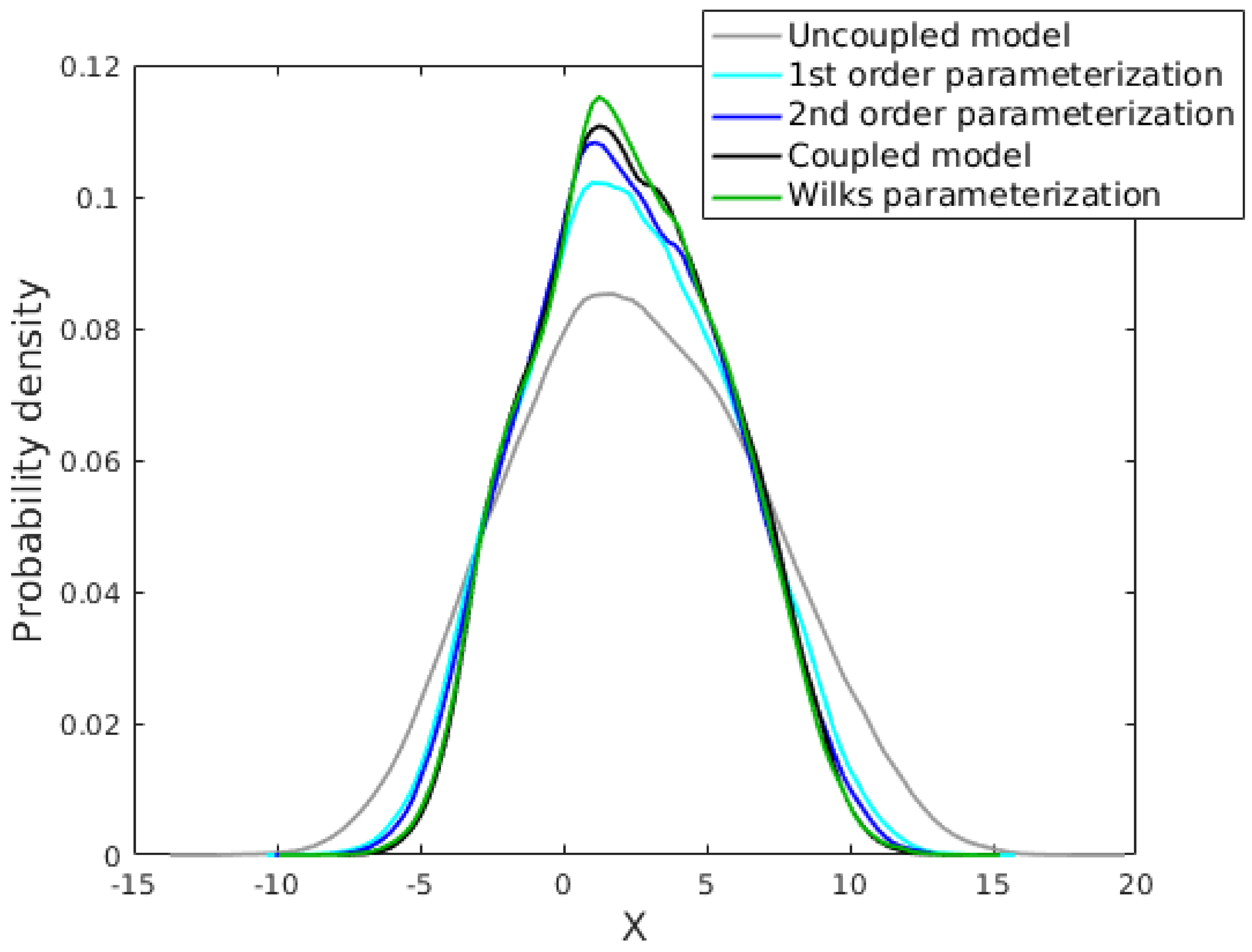
The climate is a forced and dissipative system featuring variability on a vast range of spatial and temporal scales. Convection acts on a lesser time scale with respect to synoptic scale weather phenomena, and they interact with each other through the exchange of energy, which is mathematically represented by the coupling.
In climate models is essential to parameterize the coupling in order to describe the effect of the unresolved variables on the resolved ones.
The approach [1] engaged in this research [2] is based on Ruelle response theory [3] and consists in rethinking the coupling as a perturbation of an otherwise autonomous system and calculating its parameterization up to the second order as a sum of three terms: a deterministic field, a stochastic forcing and a memory term.

$$\frac{dX}{dt} = F_X(X) + \Psi_X(X,Y)$$
$$\frac{dY}{dt} = F_Y(Y) + \Psi_Y(X,Y)$$

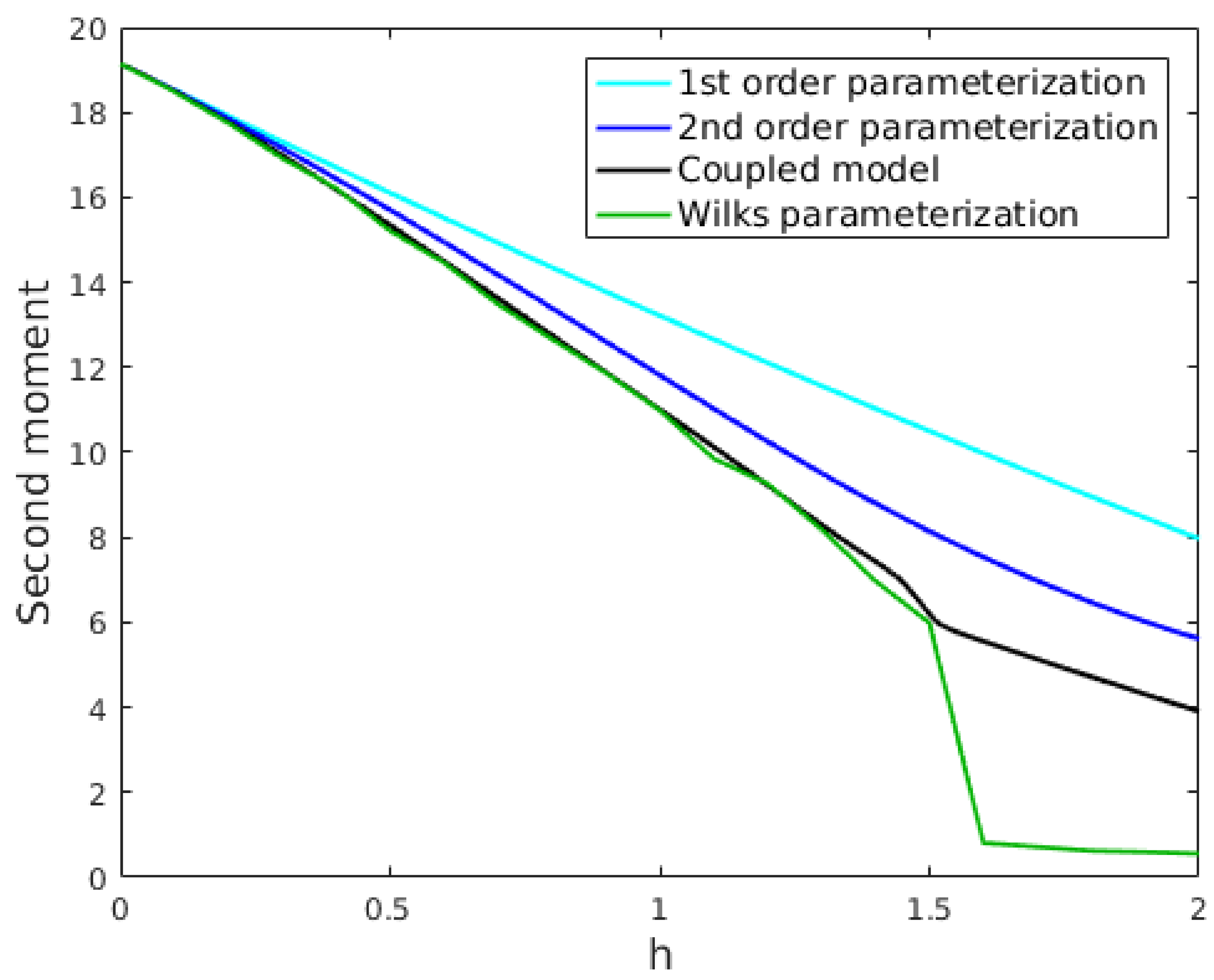
$$\frac{dX}{dt} = F_X(X(t)) + D(X) + S(X) + M(X)$$

Application on modified Lorenz 96 model



$$\frac{dX_k}{dt} = -X_{k-1}(X_{k-2} - X_{k+1}) - X_k + F_1 - \frac{hc}{b}\sum_{j=1}^{J} Y_{j,k}$$
$$\frac{dY_{j,k}}{dt} = -cbY_{j+1,k}\left(Y_{j+2,k} - Y_{j-1,k}\right) - cY_{j,k} + \frac{hc}{b}X_k + \frac{c}{b}F_2$$

Lorenz 96 [4] is a dynamical system that shows chaotic behaviour on two different time scales, a feature which makes it suitable for studies about nonlinearity in geosciences; it was built as a simplified model of the atmosphere and it could work as a starting benchmark for weather and climate investigations.
The X variables represent slow dynamics which change on synoptic scales, while Y variables portray fast, convection related dynamics.
Here we compare the results obtained applying Wouters-Lucarini's and Wilks's [5] parameterizations to the uncoupled version of Lorenz 96 model, used as a reference to assess the improvement obtained.



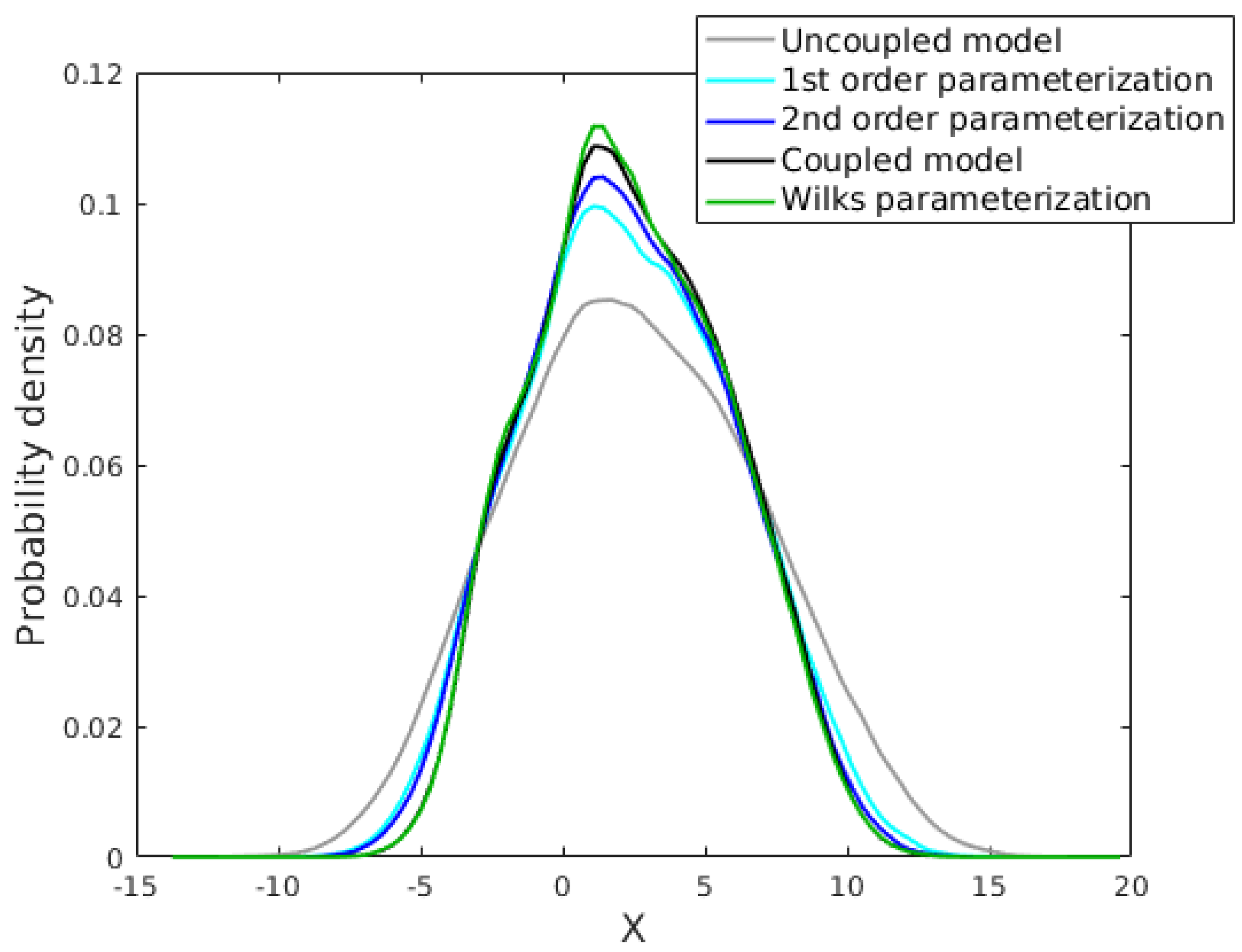
As it can be seen from the first plot, Wouters-Lucarini's parameterization reproduces the distribution of the X as efficiently as the Wilks's approach, while in the case of the second centered moment, plotted with respect to changes in coupling strength, the fair behaviour of our method is outmatched by the latter.
This is due to the empirical nature of Wilks's parameterization, which provides an accurate approximation of a dynamical system given a particular set of parameters like time scale separation, relative amplitude of the fluctuation of the variables and coupling strength.

Nevertheless, this represents the most restrictive limitation of this kind of parameterization: every single slight change in these parameters requires a new application of the method, with new simulations and computations.

Scale adaptivity

The WL parameterization guarantees a complete scale adaptivity: the calculation of the three terms must be performed only once, since it is possible to get new values for different cases through simple transformations.

In the last figure we show the distribution of the slow variables obtained with completely different time scale separation, relative magnitude of the variables and coupling strength.
While for the empirical approach a whole new computation was necessary, Wouters-Lucarini's method was applied straightly from the standard case above showed, obtaining the same result of the direct application and therefore demonstrating its flexibility and reliability.



References:
[1] Wouters, J., and V. Lucarini (2012), Disentangling multi-level systems: averaging, correlations and memory, Journal of Statistical Mechanics: Theory and Experiment, 2012 (03), P03,003.
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